1st Interdisciplinary Symposium on NonLinear Problems
21-22 January 2000, NTU, Athens, Greece

# Instability and Nonlinear Dynamics in Ship Motions

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## ABSTRACT

Some recent developments in the field of nonlinear ship dynamics are presented, beginning with an overview of the current topics of research worldwide. Then, the basic aspects of ship instability and capsize are presented for various angles of wave encounter. The main purpose the paper is to demonstrate the importance of nonlinearity for the onset of ship instability.

*Keywords:* ship, nonlinear, dynamics, roll, capsize, yaw, broaching, chaos, bifurcation, homoclinic.

## INTRODUCTION

In the field of nonlinear ship dynamics we explore the onset and evolution of the unfamiliar dynamic responses beyond the linearity regime which are not amenable to the conventional techniques of seakeeping theory. Nonlinear phenomena have been found to underlie in several instances the ship dynamics, much as they do for the dynamics of several other engineering systems (Thompson & Bishop, 1994; Moon 1999). Scientific curiosity apart, their particular engineering significance stems from their frequent connection with safety-critical behaviour. Nonlinearities can be purely geometric or they may be a feature of the flow around the ship. A well known example for the first type is the restoring moment in roll (referring to a rotation around the longitudinal axis of the ship, see Fig. 1) which is a strongly nonlinear function of the rotation angle. This nonlinearity can give rise to complicated *heteroclinic* phenomena (discussed in more detail in later Sections), leading to reduced stability when a ship is exposed to near resonant waves with steepness higher than some critical level. Extra complications may arise in longitudinal waves where large fluctuations of restoring are sometimes possible, leading to dangerous parametric-type responses.

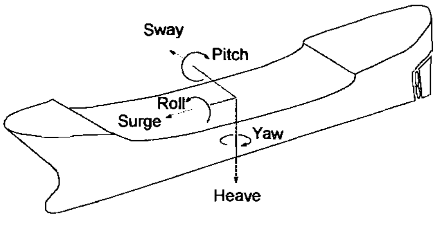

**Fig. 1**: Ship motion terminology

In recent years there has been a very notable interest for the study of ship behaviour under extreme and safety critical environmental conditions where often the nonlinearity is quite strong. A well-known success story of the nonlinear methodology in ship dynamics concerns the rolling behaviour of ships in waves: Significant new insights have been achieved about the mechanism of ship *capsize* which is perhaps the most fearful of all types of ship instability. Nonlinearities have also been shown to play an important role in the horizontal as well as in the vertical-plane dynamics. A number of phenomena of motion instability, such as *broaching*, the sudden loss of control that triggers a forced turning motion often ending with capsize; *surf-riding*, the capture of a ship by a single wave; and *porpoising*, the self-sustained pitching (rotation around the transverse axis) and heaving (up and down motion) at high speed, have been explained as constituting manifestations of some local or global *bifurcation*. Furthermore, recently we have seen experimental evidence of *chaotic* ship responses. Due to their robustness, these are relatively easily reproduced experimentally on a scaled model excited by "beam" (i.e. vertical to it) waves when a quantity of water is contained in a high compartment of the model.

Some of the popular topics of nonlinear ship dynamics are listed below:

Ship capsize in regular "beam" seas,
Development of transient capsize criteria based on Melnikov's method,
Criteria of ship capsize in random waves,
Large amplitude rolling and capsize in longitudinal waves,
Single-degree and coupled surge dynamics; especially the occurrence of asymmetric surging and surf-riding,
Broaching and its connection with capsize,
Coupled rolling with liquid motion in an internal compartment,
The behaviour of high-speed planing craft; especially the heave-pitch oscillations known as the porpoising condition,
Oscillations of moored, anchored or towed ships under the effect of currents or waves,
Path control of marine vehicles and directional instability in strong wind.

The growing interest for the subject is evidenced by the publication of a Theme Issue by the Royal Society of London devoted to it (Spyrou & Thompson 2000). This follows a similar publication last year of a Theme for the neighbouring field of flight dynamics. In the current paper we shall attempt to present an overview of some current developments in the field of ship stability, focusing especially on those cases where the effect of nonlinearity is catalytic for the onset of instability. The paper is written in a descriptive style, addressing a wider audience than experts of the field. For this reason, all mathematical expressions have been omitted and an effort is made to explain all the technical terms used in the paper.

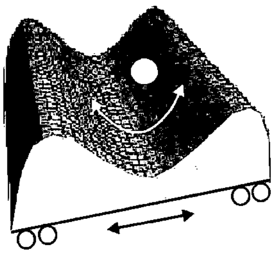

**Fig. 2**: A simple analogue of ship roll in beam waves.

## THE BASICS OF SHIP ROLLING AND CAPSIZE IN WAVES

At a very fundamental level, the uncoupled rolling of a ship in waves corresponds to the motion of a lightly damped rotational oscillator, tied to the wave slope by a rotational spring. The restoring and damping functions are nonlinear although restoring's nonlinearity is the stronger one, playing a critical role for the dynamics. In beam waves the external excitation is proportional to the wave slope. A useful analogy of beam-sea rolling is the movement of a ball at the interior of a potential well shaken back and forth, as shown in Fig. 2. The escape of the ball from the well corresponds to a capsize. If the direction of the waves coincides with that of the ship (longitudinal waves) there will be a different situation because in that case there is no direct external forcing from the waves. However, an "internal" excitation is likely to arise if the waves are steep and long, due to a modulating roll restoring moment (Fig. 3). This moment (which is a function of the ship's underwater geometry) becomes in this case dependent on the instantaneous positioning of the ship relatively to the wave, e.g. whether the ship's center of gravity lies near to a crest or near to a trough. Combinations of internal and external forcing are the norm for intermediate angles of encounter.

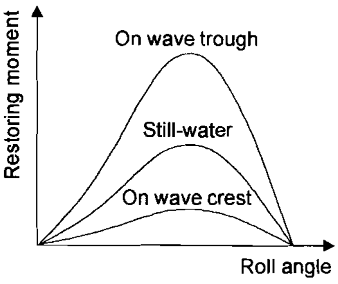

**Fig. 3**: Variation of restoring

As for any type of dynamic response, ship rolling can be studied in a steady-state or in a transient sense. Studies of the first type, often analytical, were most popular in earlier days. They focus on the "skewed" characteristic (to the left for a softening nonlinearity) of the frequency-response curve, the possibility of a jump from smaller to larger amplitude roll motions due to folding of the frequency response curve and the onset of sub-harmonic response on the upper limb of the same curve (period-doubling). Such qualitative changes in the character of behaviour are manifestations of *bifurcations* which are very common phenomena in nonlinear systems and they can be realised according to a small number of classified generic patterns.

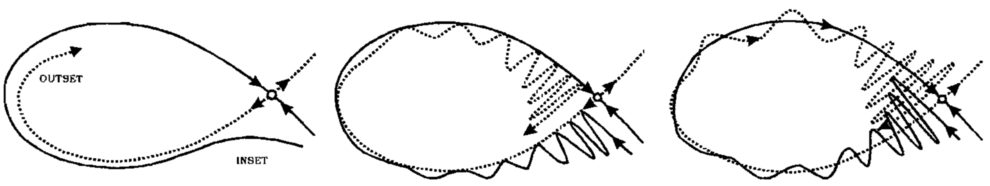

**Fig. 4**: Development of a homoclinic tangling as the forcing is increased for a biased ship. The horizontal axis can be assumed to be the roll angle and the vertical axis the roll velocity.

Ship capsize is by its very nature a transient phenomenon and as such it cannot be adequately described through a steady-state analysis. A key concept required for the study of capsize is the *basin* (*domain*) *of attraction*, representing the set of initial conditions which lead to a certain stable steady-state (*attractor*) for fixed control parameters' settings. Conversely, the basin of attraction may also be defined in the control space for fixed initial conditions. For a simple 2-d system the boundary of the basin of attraction is determined by those special orbits which approach (in infinite time) nearby saddle-type states (i.e those states which attract in one direction and they repel in another, Fig. 4). Together with their "twin" orbits that leave these saddles they are usually called respectively, the *inset* (if entering a saddle) and the *outset* (if leaving a saddle). These special orbits can become sometimes entangled in a very intricate way, producing boundaries which are not sharp. The critical event leading to such a development is the tangency of an inset with an outset (followed by successive intersections) originating from the same (this is what we call a "homoclinic" event) of from different ("heteroclinic") saddles. This is a *global bifurcation* phenomenon that incurs a major change in the system's phase-space. In Fig. 4 is shown the process of development of a homoclinic tangling for a simple oscillator allowing escape only from the one side (see again Fig. 2 but assume that, say, the left hill-top is raised well above the right one). For a ship, this could correspond to a situation where the rolling is asymmetric e.g. due to cargo-shift or a lateral strong wind.

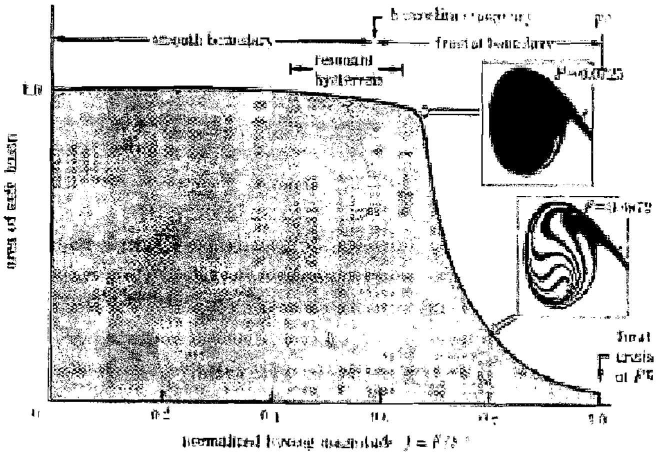

**Fig. 5**: A sharp reduction in system safety (measured by the area of the basin) follows the occurrence of a homoclinic event (diagram originally produced by Thompson & Ueda 1989).

The important point here is that the heteroclinic event is followed by rapid loss of the area of the safe basin which determines the degree of safety ("integrity") of a system. In other words, there is a threshold value of wave slope (which depends naturally on the restoring and damping functions) beyond which (i.e. with any further increase of the wave slope amplitude) the stability of the ship vanishes very rapidly (Fig. 5).

## INSTABILITIES DUE TO WAVES APPROACHING A SHIP FROM THE STERN

When a group of steep waves meets a ship from behind (this situation is known as a "following/quartering sea") a capsize is possible to occur according to the following three fundamental instability mechanisms:

(a) *Pure-loss of stability*

This is a sudden, non-oscillatory type capsize taking place around a wave crest due to slow passage of the ship from a region of the wave where roll restoring has become negative. The distance of ship ship from to the wave celerity and the degree of loss of restoring capability at the crest determine whether, in a particular circumstance, a capsize becomes likely.

(b) *Parametric instability*

This is the gradual build-up of excessively large rolling created by a mechanism of internal forcing, the

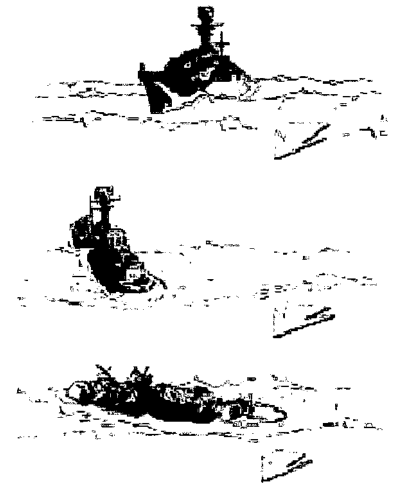

**Fig. 6**: Broaching of a frigate (from Connolly 1972).

result of a fluctuating restoring that depends on where the ship lies in relation to the wave. It is usually treated with a nonlinear Mathieu-type equation of motion, sometimes with coupling terms included.

(c) *Broaching*

This instability begins on the horizontal plane with a sudden loss of controllability. However, sometimes it can lead to the development of a large heel on the down-slope of the wave, as the ship performs a tight turn soon after the control is lost (Fig. 6). Depending on the intensity of the phenomenon and the ship's roll restoring capability, ship capsize is possible to happen.

## RECENT NEW INSIGHTS ABOUT BROACHING

Recent studies of broaching have revealed a rich content in terms of nonlinear dynamics (Spyrou (1996a, b & c, 1997). In waves a ship normally performs an oscillatory motion which engages, to a greater or lesser extent, all their degrees of freedom. However, in steep following/quartering seas the ship motion pattern may depart from the ordinary periodic response. This raises a number of issues for ship safety which are analysed next:

### The following sea

In Fig. 7 we have considered the changes in the geometry of surge (this is the motion in the direction of the longitudinal axis of the ship) under the gradual increase of the nominal Froude number, $Fn$. This number is a nondimensional version of the steady speed obtained in still water, it is therefore a control variable. Let us assume a group of steep sinusoidal waves approaching our ship from behind. For a low $Fn$ there is only a simple harmonic periodic response shown qualitatively in the phase-plane (a) of Fig. 7. However, as the speed approaches the wave celerity the response becomes more and more asymmetric [planes (b) and (c)]. An important characteristic of the response is that the ship stays longer in the crest region of the wave and it passes quickly from the trough.

In parallel an alternative, stationary behaviour starts to coexist, owed to the fact that the resistance force which opposes the forward motion of the ship in the water, can be balanced by the sum of the thrust produced by the propeller of the ship and the wave force along the ship's longitudinal axis. The condition realised is known as *surf-riding*. Its main feature is that the ship is forced to advance with a speed equal to the wave celerity. On a plane having as axes the position and velocity of the ship is the surge direction, the surf-riding states appear in pairs: one is nearer to the wave crest which is always unstable and another nearer to

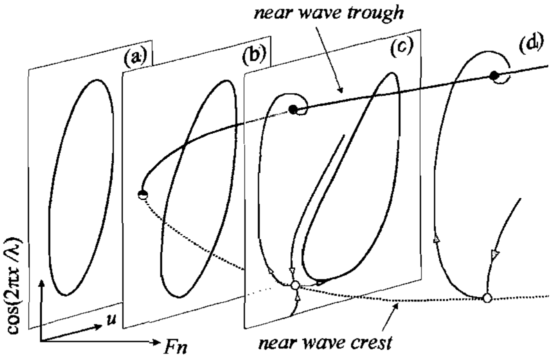

**Fig. 7**: Transformations of a ship's forward motion. On the planes, the horizontal axis is the instantaneous speed and the vertical is the ship's relative position on the wave. Effect of a gradual increase of nominal Froude number.

the trough. Stable surf-riding can be realised only in the vicinity of the trough.

Surf-riding is characterised by two speed thresholds: the first is where the balance of forces becomes possible [plane (b)]. The second threshold flags the complete disappearance of the periodic motion. This is a critical for safety event and in fact it represents manifestation of a *homoclinic saddle connection*, a global bifurcation phenomenon. This occurs when a periodic orbit collides in state-space with an unstable equilibrium (the stationary state near to the wave crest).

It has been shown that the underlying dynamics of surge resemble those of a pendulum under an external constant torque. The periodic motions of a ship overtaken by waves corresponds to full rotations of the pendulum whereas surf-riding corresponds to the equilibrium states. The key nonlinearity is the sinusoidal nature of the stiffness term.

**The quartering sea: continuation studies**

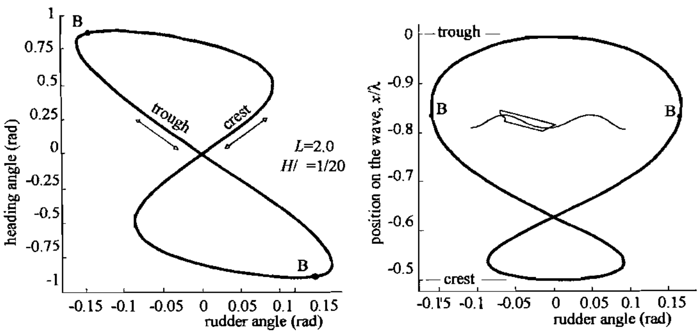

Fig. 8: Surf-riding in quartering seas: a steady-state continuation study

For continuity reasons one should expect surf-riding to be realisable also for a range of non-zero angles of heading. This range however should not extend too far; because as a beam sea situation is approached, the wave force in the surge direction will diminish and equilibrium should become impossible to reach. We have found that when the heading of the ship does not coincide with the wave direction, the points of surf-riding equilibrium belong to a closed curve. 2-D projections of this curve are shown in Fig. 8. The following symbols are used: $\lambda$ is the wave length; $x$ is the longitudinal position of the ship measured from a wave trough; and $H$ is the wave height. The curves were obtained on the basis of a coupled mathematical model that allows motions in surge, sway (sideways), yaw (rotation on the horizontal plane) and roll. Equations for the rudder and its control law were also incorporated. The curve of steady-states is identified by connecting this mathematical model with a continuation algorithm which is a rather standard bifurcation analysis tool. The stability of the identified states is concurrently examined. We investigated stability with the rudder: (a) fixed at certain angles, and, (b) controlled by an autopilot. For the latter feedbacks based on the yaw angle (the heading error from the desired course) and the yaw rate were used.

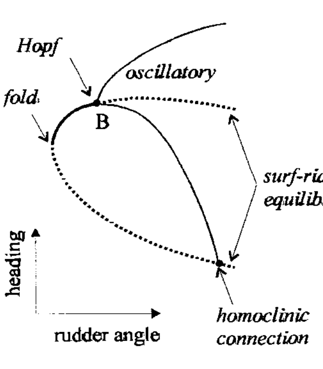

Fig. 9: Domain of oscillations.

Very interesting is the occurrence of "Hopf" bifurcations at points B, B' of Fig. 8 creating stable oscillatory surf-riding whose domain is shown in Fig. 9. It is notable that such oscillatory motion on a single wave has been experimentally observed in Japan (Kan 1990). These self-sustained oscillations can become chaotic, as explained next:

## Chaos and homoclinic events

We examined the evolution of the steady-state oscillations, created at a point near to the middle of the wave's down-slope, as the angle of the rudder was varied. These oscillations went through a period doubling cascade after a slight increase of the rudder angle $\delta$. A projection of the chaotic attractor is shown in Fig. 10 together with the corresponding power spectrum showing the frequency content of the response. The existence of chaos was confirmed also through calculation of Lyapounov exponents. These are basically time averages of the logarithms of the moduli of eigenvalues. This information is useful because it indicates whether initially close-by orbits tend to diverge as time progresses with an exponential rate.

Further increase of the angle $\delta$ brought the response back to periodicity; but due to the increasing $\delta$ the oscillatory response came nearer to the (unstable) points of surf-riding (see Fig. 9). This had as a result a new homoclinic connection (unrelated with the one discussed earlier). This event ended the oscillations performed by the ship while it was carried forward by the wave. The ship was then forced to return to a more usual motion pattern where it is overtaken by the waves.

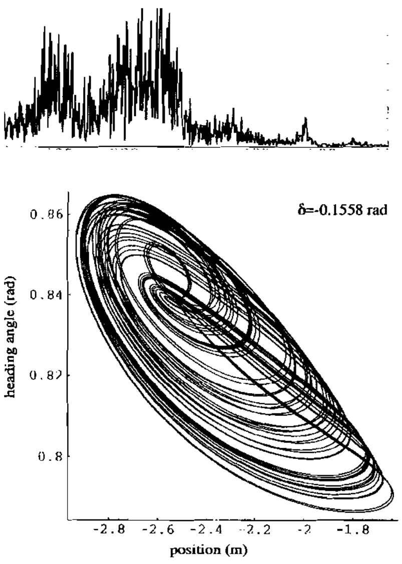

Fig. 10: Chaotic attractor and power spectrum

## The connection between surf-riding and broaching

The key for explaining why surf-riding is conducive to broaching lies in the role of the nonlinear surge motion dynamics. As outlined in earlier Sections, in large and relatively long waves a ship does not approach smoothly the condition of zero frequency of encounter but it jumps to it as soon as a certain speed threshold is exceeded. If the ship's longitudinal axis lies at some small angle relatively to the direction of the waves, the dynamic effect due to the transition that takes place in surge is 'imported' into yaw. At the ensuing stage, the ship passes from a trough, where it should experience a tendency to turn because around the trough the wave yaw moment produces a destabilising effect. The homoclinic connection shown in Fig. 3 is therefore the

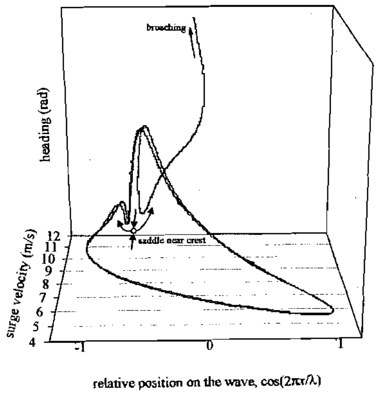

Fig. 11: Simulation of broaching

key event behind broaching. Once the periodic response is vanished, the likely types of motions are, either surf-riding or broaching. Which of the two should prevail depends on whether the steering system is capable to check the turn before the yaw angle becomes too large.

A broaching case has been simulated in Fig. 11 for a fishing vessel that has been examined in detail both experimentally and theoretically in Japan by the author as well as by other researchers. The closed orbit is the initial periodic motion as the waves overtake the ship. The periodic motion passes from the vicinity of the unstable surf-riding point (near the crest) and for this reason its shape is distorted. As the ship is set on the verge of the homoclinic connection any slight increase of the propeller rate by an unaware Master, or a slight variation of the wave, would be sufficient to instigate broaching (unbounded, upwards moving orbit of Fig. 11).

## CONCLUDING REMARKS

Nonlinearities play an important role in ship dynamics, especially under extreme weather conditions. Our long term research goal is to develop a rigorous technical basis for the assessment of ship stability in a seaway with full account of the large-amplitude ship dynamics. This could lead to the development of a more rational maritime legislation, improved ship design and a better guidance for Masters on how to cope with adverse weather.

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