Requirements for Computational Methods to be Used for the IMO Second Generation Intact Stability Criteria

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ABSTRACT

Practical implementation of the second generation of IMO intact stability criteria is not possible without formulation of clear requirements for numerical or other computational methods. Probably, the highest priority should be given to the second-level vulnerability criteria. The first-level is simple enough and, as such, requirements may not be needed or will be obvious based on standard naval architectural practices. While scientifically mature, the application of numerical methods in the second-level may be more difficult as not all of these methods are familiar to practicing naval architects including those employed with administrations and classification societies. This paper focuses on requirements for use of the numerical method for the second-level vulnerability criteria for the parametric roll stability failure mode. Criteria for other modes of stability failure may have similar concerns. Use of a numerical solution of differential equations may be a good way to compute nonlinear ship motions. However, to ensure consistency of its application (i.e, results are reliably repeatable for the same ship in the same condition), all necessary parameters (such as the time increment, the number of steps, the initial conditions, etc.) must be explicitly defined. Further, special attention needs to be given to a ship response on very large waves, for which special procedures may be needed. Since the differential equation is nonlinear, the response to a very large excitation may be chaotic. Also, if capsized equilibrium is not modeled, special measures must be taken to prevent run-time related to a very large, unrealistic roll response.

Key Words: *IMO Second Generation Intact Stability Criteria, Parametric Roll*

1. INTRODUCTION

The development of the second generation IMO intact stability criteria has been an intense multi-year effort. Recognizing the fact that stability failure may be caused by different physical mechanisms, different modes of stability failure are explicitly considered in the new criteria:

- Restoring arm variation problems, such as parametric excitation and pure loss of stability;
- Stability under dead ship condition, as defined by SOLAS regulation II-1/3-8; and
- Maneuvering related problems in waves, such as broaching-to;
- Excessive accelerations (SLF 53/19, paragraph 3.28).

This development was partially motivated by the appearance of novel hull forms that renewed interest in dynamic stability, (see e.g. France, *et al*. 2003). As a result, the emphasis was made on adequate reflection of physics, making new criteria based on performance (Belenky, *et al* 2008). This means that the assessment is based on hull geometry and physics of stability failure rather than past experience with similar ships.

The multi-tiered structure of new criteria addresses the potential complexity of the application of the new criteria. The first-level vulnerability check is very simple and quick, but conservative. If vulnerability to a particular stability failure mode is determined not to occur, no further assessments are needed. If not, then a more detailed, but less conservative analysis follows, which is the second-level vulnerability assessment.

The IMO Sub-committee on Ship Design and Construction, at its 2nd Session, finalized the first three elements of the criteria:

- Draft Amendments to Part B of The 2008 IS Code with Regard to Vulnerability Criteria of Levels 1 And 2 for the Pure Loss of Stability Failure Mode (Annex 1 of SDC 2/WP.4);
- Draft Amendments to Part B of The 2008 IS Code with Regard to Vulnerability Criteria of Levels 1 And 2 for the Parametric Rolling Failure Mode (Annex 2 of SDC 2/WP.4);
- Draft Amendments to Part B of The 2008 IS Code with Regard to Vulnerability Criteria of Levels 1 And 2 for the Surf-Riding / Broaching Failure Mode (Annex 3 of SDC 2/WP.4).

These documents describe the criteria, standards and contain general requirements for the calculation methods. The explanatory notes are expected to be developed to ensure uniform interpretations and application of the new criteria. The technical background of these criteria is described in Peters, *et. al.* (2011). A significant amount of information is being prepared for the explanatory notes, see SLF 53/3/3, Annexes 17, 19, 33, 34 of SDC 2/INF.10, Sections 2.1, 3.1 and 4.1 of Belenky, *et al*. (2011) and Peters, *et al*. (2014). The particular objective of this paper is to contribute towards the explanatory notes for second-level vulnerability assessment of the parametric roll stability failure mode.

2. MAXIMUM ROLL ANGLE

The second check for the second-level vulnerability criteria requires calculation of the maximum roll angle resulting from parametric roll. This calculation, while not too complex, is beyond the scope of traditional naval architectural calculations; why?

The conventional way to evaluate ship motions is with the use of Response Amplitude Operators (RAO). The RAO expresses dynamic properties of a ship. Its values are the characteristics of motions, multiplied by the values of sea spectrum and summed up to yield the characteristics of motion. RAO is an element or a form of a solution to the linear ship motion equation in waves.

The term “linear ship motion equation” means that the equation assumes that the motions are small and that non-linear parts of the full ship motion equation can be ignored because their effects are negligible (often because the waves are significantly longer than the ship). In particular, *GM*, which characterizes transverse stability, is used to represent roll stiffness. Indeed, stability at large roll angles cannot be characterized with *GM* alone.

The maximum angle of parametric roll also cannot be found just with *GM* even if its variation in waves is known. However, the responsibility for progressively growing roll angles, i.e. parametric roll, is associated with these *GM* variations together with a frequency ratio in which the encounter frequency is close to twice that of natural frequency (see e.g. SLF 54/3/3).

Once parametric roll motion starts, it grows to a certain maximum angle and the motion repeats (i.e., it remains stable). This occurs because the *GZ* curve is not a straight line over the range of roll motion. As a result, the natural roll frequency changes with the increase of the roll angle (the instantaneous *GM* value also changes). Changing the roll frequency sooner or later will break the parametric roll condition because the supply of energy into roll motion will be stopped. The maximum roll angle is achieved during steady state parametric roll.

Thus, a large portion of the *GZ* curve is needed to find the maximum roll angle. While the GZ curve is known, the motion equation is no longer linear if *GZ* is included and a RAO-type of solution is no longer possible.

3. EQUATION OF MOTION

3.1 Overview of Forces Acting on a Ship

The equation of motion takes into account forces acting on the ship. The simplest mathematical model that is capable of evaluating the maximum roll angle includes four moments:

- Inertia, including added inertia (or added mass) as a part of hydrodynamic forces;
- Roll damping, which expresses energy loss from roll motions in creating waves, vortexes and skin friction;
- Roll restoring (stiffness) is modeled with the calm water *GZ* curve; the variation of stability in waves is included by *GM* represented with a sine function.
- Transverse wave forces are absent for a ship in exact following or head long-crested seas

3.2 Roll Inertia

The roll inertia of a ship as a solid body is measured by the transversal moment of inertia. In absence of ship specific data, it is recommended to assume the radius of gyration r_x as 40% of the molded breadth, *B*:

$$r_x = 0.4B \qquad (1)$$

Then, the moment of inertia, I_x, is calculated as:

$$I_x = \rho \nabla r_x^2 \qquad (2)$$

where ρ is the mass density of salt water; ∇ is the volume of displacement. Use of other approximation formulae may be helpful but only if the limits of their applicability are well known.

Inertial forces are proportional to accelerations. There are also hydrodynamic forces acting on a ship subject to accelerated motion that are also proportional to the accelerations. These hydrodynamic forces are usually expressed as an additional mass or a moment of inertia and referred as "added mass". Again, in the absence of ship specific data, one can assume that the added mass in roll, A_{44}, as:

$$A_{44} = 0.25 I_x \qquad (4)$$

Finally, the roll inertia is expressed as:

$$M_{IN} = (I_x + A_{44}) \cdot W_\phi \qquad (5)$$

where W_ϕ is the angular acceleration in roll.

3.3 Roll Damping

Damping of roll motions is essentially a transfer of kinetic energy of a moving ship to the environment. It is a complex process, because this energy transfer occurs through different physical phenomena. Skin friction causes the layers of water nearest to the hull to move. The moving surface of the hull leads to formation of vortexes; the kinetic energy of the water moving in those vortexes is taken from the ship. Due to its motion, the ship also makes waves on the surface that also dissipate energy. The complexity of these physical phenomena is the reason why a model test is the most reliable source of information on roll damping. However, recent developments in computational fluid dynamics (CFD) holds good promise for the availability of this computational method in the future.

In the absence of ship-specific or prototype data, the simplified Ikeda method can be recommended (Annex 3, SDC 1/INF.8). A moment of roll damping is presented in the following form:

$$M_D = \left(I_X + A_{44}\right)\cdot\left(\delta_1 V_\phi + \delta_3 V_\phi^3\right) \quad (6)$$

where δ_1 and δ_3 are coefficients computed with simplified Ikeda method and V_ϕ is the angular velocity of roll motions.

The simplified Ikeda method contains some empirical elements and, for this reason, the range of its applicability should be observed.

3.4 Roll Restoring

A proper representation of roll restoring is very important for the correct representation of parametric roll. The variation of stability in waves is a primary mechanism of development of parametric roll (an explanation is provide in SLF 54/3/3). The calculation of the instantaneous roll restoring, while straight forward, may be too complex for the level-two vulnerability check. (See the description of one of the simplest algorithms of direct calculation in Weems and Belenky, 2015). Hence, a quasi-static approach can be used instead.

The quasi-static approach means that the *GZ* curve for the ship on a wave is calculated using the "conventional" static algorithm (in which forces and moments are balanced in heave and pitch as required in Annex 2 of SDC 2/ WP.2), but the waterplane is not flat – it is determined from the intersection of a wave and the hull surface. Known also as "wave-pass" calculations, the capability for this calculation is provided by a number of commercially available hydrostatic software packages (see Figures 1 and 2). For the assessment of parametric roll, calculation of the *GZ* curve up to 180 degrees is recommended; it sets a natural maximum and prevents the numerical solution from growing too large and cause a numerical error.

Figures 1 shows the *GZ* variation in waves as a series of curves. Each curve is calculated for a particular position of the wave crest relative to the midship which results in a surface shown in Figure 2. For the intermediate values of heel angle and of the wave crest position, a bilinear or bi-cubic spline interpolation can be used. The definition of wave crest position is illustrated in Figure 3.

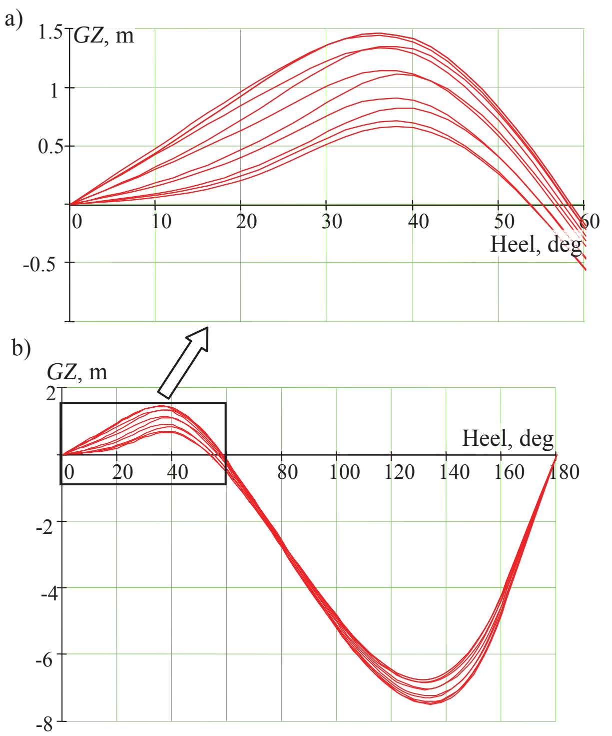

Figure 1: The *GZ* curve in waves (steepness 0.02, C11 class containership, full load) (a) positive range, (b) full range

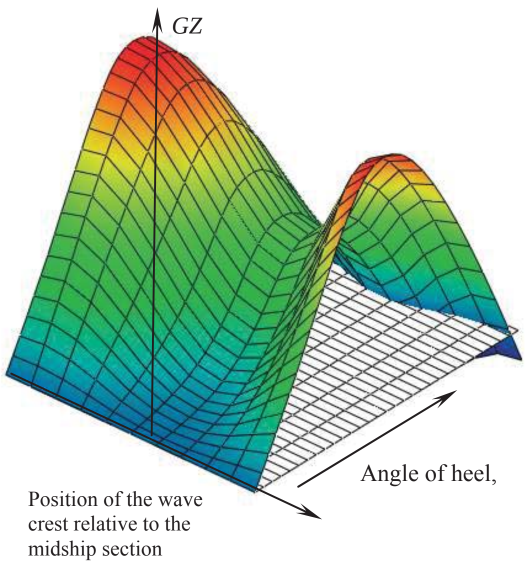

Figure 2: The *GZ* curve in waves as a surface (steepness 0.02, C11 class containership, full load)

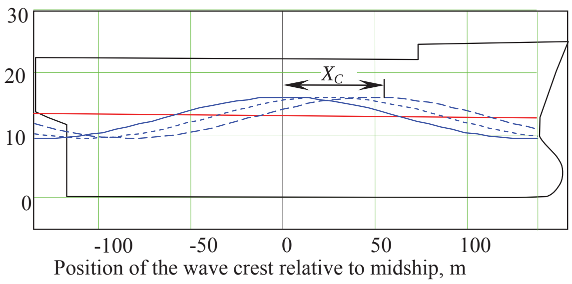

Figure 3: Definition of the position of the wave crest relative to the midship section

The position of the wave crest is a function of time:

$$X_C(t) = 0.5\lambda \sin(\omega_e t) \qquad (7)$$

where λ is the length of the wave and ω_e is the wave frequency of encounter:

$$\omega_e = \omega - \frac{\omega^2}{g} V_S \cos\beta \qquad (8)$$

where g is the gravity acceleration, β is the relative wave heading (0 degrees – following waves, 180 deg – head waves), and V_S is the forward speed in m/s. Thus, the value of the *GZ* curve in waves can be presented as a function of time and angle of heel, ϕ:

$$GZ = GZ(t, \phi) \qquad (9)$$

If, for some reason, the calculation software is not available, the *GZ* curve in a wave can be approximated using only the *GM* value that may be already available from the Level 1 vulnerability check. Indeed, as required by Annex 2 SDC 2/WP.2, the calculation of *GM* must be done with forces and moments balanced in heave and pitch. An example of the *GM* variation is shown in Figure 4:

Then, the *GZ* in waves may be approximated by the calm-water *GZ* "modulated" by the *GM* in waves

$$GZ(t, \phi) = \frac{GM(t)}{GM_0} GZ_0(\phi) \qquad (10)$$

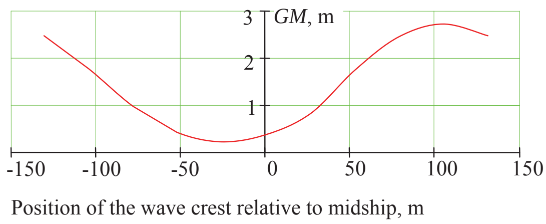

Figure 4: The *GM* value in waves as a function of wave crest position relative to midship (wave steepness 0.02, C11 class containership, full load)

Assuming that the *GZ* curve is symmetric, the total restoring moment is expressed as:

$$M_R = \text{sign}(\phi) \cdot \rho \nabla g \cdot GZ(t, |\phi|)$$
$$\text{sign}(\phi) = \begin{cases} 1 & \phi \geq 0 \\ -1 & \phi < 0 \end{cases} \qquad (11)$$

3.5 Equation of Motion and Its Solution

Following Newton's second law, the equation of roll motion is expressed as the inertial force equal to the sum of all other forces. Since the ship is in longitudinal waves, there is negligible or no direct forcing that comes from the waves:

$$M_{IN} = -M_D - M_R \qquad (12)$$

In equation (12), the negative sign is inserted because both damping and restoring forces are directed against the roll motion or the rate of motion. The equation of roll motion can be re-written with each force as a function of motion parameters or time:

$$M_{IN}(W_\phi) + M_D(V_\phi) + M_R(t,\phi) = 0 \qquad (13)$$

Equation (13) relates the roll motion with the roll rate and the angular roll acceleration. These quantities are related through differentiation: the angular velocity is a derivative of roll and the angular acceleration is a derivative of angular velocity. Thus, equation (13) is a differential equation.

The solution of a differential equation (13) is a time history of roll motions, similar to that shown in Figure 5.

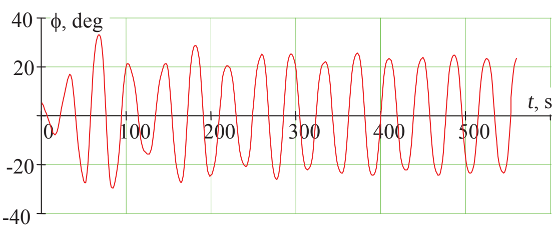

Figure 5 Time history of parametric roll

Figure 5, indeed, shows parametric roll. As the ship is sailing in longitudinal waves, there is no forcing in the transversal plane, so the observed rolling motion is a result of parametric resonance.

The equation (13) is solved by a standard program available from most numerical or engineering software packages. Numerical solvers of differential equation also are available in MS Excel in Visual Basic. To use the solver, the equation (13) must be presented in a form of a vector-valued function:

$$F\left(\begin{pmatrix}\phi \\ V_\phi\end{pmatrix}, t\right) = \frac{1}{I_X + A_{44}}\begin{pmatrix} V_\phi \\ -M_D(V_\phi) - M_R(t,\phi)\end{pmatrix} \qquad (14)$$

Besides the vector-valued function (14), the solver requires initial conditions, *i.e.* values of roll angle and roll rate at the beginning (or at time step $t = 0$) of the calculations. The solution, as illustrated in Figure 5, was computed with assumed initial conditions (ϕ = 5 deg and V_ϕ = 0 deg/s). While the calculation can assume zero for both ϕ and V_ϕ, the development of parametric roll may not occur until a much longer duration is calculated.

To complete the inputs necessary for the calculation, two more parameters are needed: the time increment Δt and the total number of points N. These parameters can be related to the natural frequency of roll, ω_0, in calm water because a steady state parametric roll motion in longitudinal waves occurs with this frequency:

$$\omega_0 = \sqrt{\frac{\rho \nabla g \cdot GM_0}{I_X + A_{44}}} \qquad (15)$$

Then, the period of the roll motion in calm water is expressed as:

$$T_0 = \frac{2\pi}{\omega_0} \qquad (16)$$

The time increment Δt can be expressed in terms of the number of points per period N_{ppp}:

$$\Delta t = \frac{T_0}{N_{ppp}} \qquad (17)$$

Thus, the number of points depends on the number of periods N_{per} to be reproduced:

$$N = N_{ppp} N_{per} \qquad (18)$$

Practical experience recommends use of the following values:

$$N_{ppp} = 30\,; \quad N_{per} = 15$$

3.6 Calculation of Maximum Roll Angle

The parametric roll response has a transition from the state where the initial conditions still have an influence to the steady state where the amplitudes are similar or close to each other.

Different criteria for "closeness" can be used: relative (the difference is less than 3 - 5%) or absolute (say, less than one degree). Following this criteria, the steady state portion of the response can be extracted (see Figure 6) and the resultant maximum roll angle can be found as an average of steady state roll amplitudes.

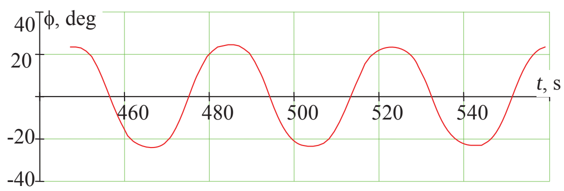

Figure 6 Steady-state portion of the roll motion in parametric resonance conditions

The steady state parametric roll is not the only possible type. If parametric roll is not possible for the given wave conditions, the response is represented by decaying roll oscillations – as shown in Figure 7. Indeed, the maximum roll angle here is the initial roll angle of 5 degrees. The response is not expected to look like a decaying sine function because of both the parametric excitation and nonlinearity of the equation (13).

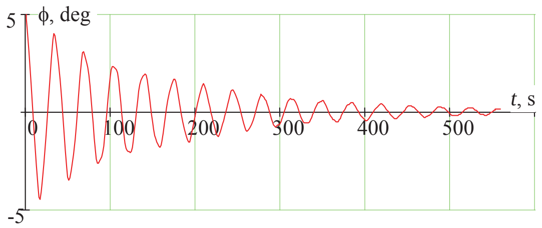

Figure 7 Roll response in absence of parametric roll

Another possible response may include "capsizing" (see Figure 8) if the *GZ* curve was computed for the entire range of 180 degrees (like in Figure 1). If the *GZ* curve is computed only for the positive stability range ($GZ > 0$), the calculation must be explicitly stopped once the roll angle exceeds the angle of vanishing stability.

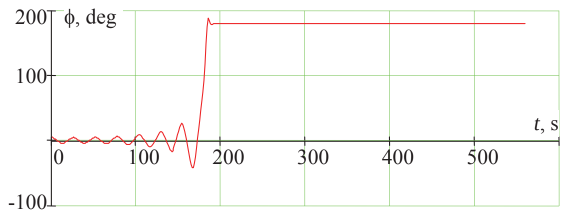

Figure 8 Roll response with parametric roll and capsizing

The mathematical model (13) is, probably, too simple to model actual capsizing, but the response, similar to that shown in Figure 8, indeed indicates a condition of strong parametric roll in which the maximum roll angle exceeds the standard level of 25 degrees as stipulated in Annex 2 of SDC 2/WP.4.

In rare cases, the user may observe response that does not stabilize. The roll amplitude may grow steadily or look like roll in irregular waves. These responses are not the result of an error, but of a known type of nonlinear behavior. In this case, the maximum achieved roll angle during N_{per} periods is used.

4. CALCULATION OF PARAMETRIC ROLL AMPLITUDE FOR A POST-PANAMAX CONTAINER SHIP

An example of a calculation of parametric roll amplitude, in compliance with the Level 2 criteria, is presented below. The investigated ship is a baby post-Panamax container ship with the characteristics as shown in Table 1:

Table 1: Main ship characteristics

Length L_{BP} (m)	238.35

Beam (m)	37.3
Depth (m)	19.6
Mean Draught (m)	11.5
Block Coefficient	0.657
GM (m)	0.84

The steady amplitude of parametric roll is calculated by using the following four methods:

a) a direct numerical solution of the non-linear differential equation of roll that is included in SDC 2/INF.10, Annex 17;
b) a numerical solution of the algebraic equation derived after applying the analytical method of averaging on the previous non-linear differential equation. (This algebraic equation is proposed in SDC 2/INF.10, Annex 17 to be used for obtaining the steady amplitude of parametric roll.)
c) a numerical solution of the non-linear roll equation used by Spyrou (2005); and
d) an analytical, closed-form, formula obtained by the method of harmonic balance, predicting the steady amplitude of parametric roll at principal resonance condition (Spyrou 2005).

The moment of inertia, I_x, and natural roll period T_0 are calculated through the roll radius of gyration by using Kato's formula, as proposed in SDC 2/INF.10-Annex 11. For the loading condition under investigation, $T_0 = 39.3$ s is assumed.

The linear damping coefficient is calculated by using Ikeda's method as proposed in the above IMO document, including the bilge keel component. While acknowledging that the criterion requires both linear and non-linear damping, at this stage, the comparison involves only linear damping.

The four methods use the same inertia and damping terms. Their main differences lie in the representation of the restoring terms. The SDC model (methods a and b above) is in the following form:

$$I_X\ddot{\phi} + B_{44}\dot{\phi} + mgGZ = 0 \qquad (19)$$

$$GZ = GM_0\phi + l_3\phi^3 + l_5\phi^5 + GZ_W \qquad (20)$$

$$GZ_W = GM_{mean}\phi + GM_{amp}\cos\omega_e t\left\{1-\left(\frac{\phi}{\pi}\right)^2\right\}\phi$$

where I_x is roll moment of inertia including added moment of inertia; B_{44} is linear damping coefficient; m is the ship's displacement; g is gravitational acceleration; l_3, l_5 are third and fifth order coefficients of GZ curve fit; ω_e is the encounter frequency; GM_{amp} is half the difference between the maximum and minimum value of GM on the span of a wave; GM_0 is the initial metacentric height in calm water. GM_{mean} is the mean of metacentric height variation on the span of the wave which, given the expressions in equation (20), and is interpreted to be the difference between the mean value of the GM in waves and the GM in calm water.

On the other hand, methods c) and d) (above) from Spyrou (2005) model parametric roll as follows:

$$\begin{aligned}\ddot{\phi} + 2\zeta\omega_0\dot{\phi} + \omega_0^2[1 - h\cos(\omega_e t)]\phi \\ - c_3\omega_0^2\phi^3 - c_5\omega_0^2\phi^5 = 0\end{aligned} \qquad (21)$$

where ζ is the damping ratio, ω_0 is roll natural frequency, c_3, c_5 are third and fifth order restoring coefficients and $h = GM_{amp}/GM_{mean}$.

The two differential equations for parametric roll, equations (19) and (21), are not identical and, therefore, the solutions are not expected to replicate on each other completely.

Roll amplitude is calculated for ten different cases where the ship is under the effect of following waves with $\lambda = L_{BP}$ and ten different heights with $H_j = 0.01jL$, where $j = 1, 2, ..., 10$, as requested in SDC 2/INF.10, Annex 17. This leads to waves some of which are extremely steep and with extremely low probabilities of encounter. For each wave height, hydrostatic calculations of GM_{mean} and GM_{amp} are carried out by using the well-known commercial software MAXSURF. The ratio of the calculated GM_{amp} to GM_{mean}, together with the corresponding wave heights, are shown in Table 2.

The encounter frequency for the ship when sailing in following waves of length equal to the ship length and with the design speed of 21 knots is 0.224 rad/s. This leads to a frequency index $a = 4\omega_0^2/\omega_e^2 = 2.04$, which is far to the right of the principal resonance value $a = 1$. The analytical manipulations that have been applied in the context of SDC and related literature on the parametric roll differential equation assume a condition very near to exact principal resonance. This may lead sometimes to questionable results if the detuning is large. Because the wave length is fixed to ship length, this discrepancy (i.e., a large difference between the frequency index and the principal resonance value) is quite likely to be present whenever a large ship is tested.

Table 2: Wave Height, Probability of Occurrence, and Ratio of GM_{amp} to GM_{mean}

Wave Length λ=238.35m			
N	Wave Height H(m)	$\frac{GM_{amp}}{GM_{mean}}$	Probability W
1	2.384	0.703	0.2367
2	4.767	1.155	0.1196
3	7.151	1.422	0.0336
4	9.534	1.571	0.006146
5	11.918	1.624	0.0009333
6	14.3	1.632	0.0001025
7	16.685	1.656	0
8	19.068	1.673	0
9	21.452	1.737	0
10	23.835	1.815	0

As said in SDC 2/INF.10, Annex 11, the roll amplitude is calculated by a numerical solution of an algebraic equation deduced through the averaging method. This equation is repeated below for linear damping only:

$$\left(\frac{8\pi^2\omega_e\alpha}{(2\pi^2-A^2)\omega_0^2}\right)^2+\left(\frac{6A^2-8\pi^2}{4(\pi^2-A^2)}\frac{GM_{mean}}{GM_0}+\frac{8\pi^2-5\pi^2A^4l_5-6\pi^2A^2l_3}{4(\pi^2-A^2)}+\frac{8\pi^2\omega_e^2}{4(\pi^2-A^2)\omega_0^2}\right)^2=\left(\frac{GM_{amp}}{GM_0}\right)^2 \quad (22)$$

where A is the roll amplitude and α is the linear damping term. Because the analytical solution of equation (22) is not provided in SDC 2/INF.10, Annex 17, the implementation of a numerical scheme to determine the solution cannot be avoided. However, since (22) is nonlinear with respect to amplitude A, more than one solution can exist. Therefore, guidance is required on the process of how to ensure that a solution identified is the correct one for use in the criterion. In general, a numerical calculation performed directly on the differential equation (19), which produces automatically a stable solution, is in many respects preferable to a calculation performed on the averaged form of equation (22), which produces also unstable solutions.

For completeness, the steady roll amplitude of the analytical solution of Spyrou (2005) is given in equation (23) also:

$$A^2=-\frac{3c_3}{5c_5}\pm\left(\left(\frac{3c_3}{5c_5}\right)^2-\frac{8}{5c_5}\left(\frac{1}{\alpha}-1\pm\sqrt{\frac{h^2}{4}-\frac{4k^2}{\omega_0^2\alpha}}\right)\right)^{0.5} \quad (23)$$

where k is a linear damping coefficient and $\alpha = 4\omega_0^2/\omega_e^2$

In Figure 9, the results obtained by each method, for the ten different wave heights discussed earlier, are shown. As each wave height corresponds to a specific value of GM_{amp}/GM_{mean}, this ratio is selected for the horizontal axis. The numerical simulations are initiated from an assumed roll angle of 0.01 rad (0.57 degrees).

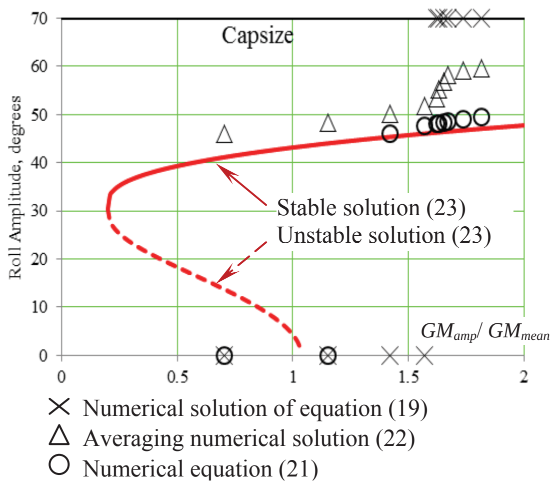

Figure 9: Parametric Rolling Amplitude for ten different wave heights

According to the analytical solution curve appearing in Figure 9, there are two possible responses by the ship: either a stable parametric resonance (continuous curve), similar to what was shown in Figure 6; or a decaying rolling that eventually leads to the upright position (as shown in Figure 7). Unstable solutions represented by the dashed curve cannot be physically realized. Nevertheless, they play the role of establishing a boundary between the coexisting solutions of zero and finite amplitude.

Figure 9 shows that all methods calculate a roll amplitude either close to the curve of parametric resonance or to the x-axis of decaying rolling. The numerical solutions of equation (19) for roll amplitude grow to infinity for the greater values of wave height which can be interpreted as a capsize event in mathematical terms.

When the solutions of parametric resonance and decaying roll coexist, the SDC method that uses equation (19) gives conflicting results. Also, the value of the parameter GM_{amp}/GM_{mean}, after which only parametric resonance occurs, is different for each method. These inconsistencies may lead to important differences between the index values of the second-level vulnerability check for parametric roll.

For greater values of wave height (and subsequently of the parameter GM_{amp}/GM_{mean}), the response becomes highly non-linear. One such example is shown in Figure 10 which corresponds to the numerical solution of equation (21) for $H = 21.45$m.

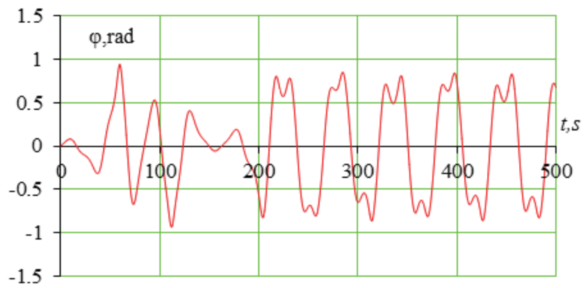

Figure 10: Highly non-linear parametric roll response

According to Figure 10, steady parametric rolling with very large amplitude (about 50 degrees) occurs. This essentially means that capsize is highly likely although the solution remains theoretically bounded. On the other hand, equation (19) for the same wave height detects capsizing, as can be seen from Figure 11. While the standard level of 25 degrees as stipulated in Annex 2 of SDC 2/WP.4 is exceeded in both cases, same order roll equations with similar terms show different dynamic characteristics for large waves.

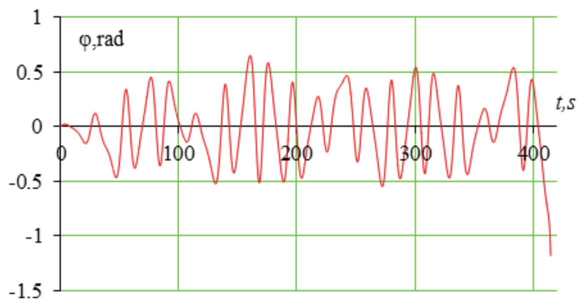

Figure 11: Non-linear parametric rolling that leads to capsize (19)

5. CONCLUSIONS

The second generation intact stability criteria presents new approaches for the assessment of ship stability failure. To perform these assessments, calculation methods are used that are not commonly used by practicing naval architects.

The equation of roll motion for the second check in the second-level of vulnerability criteria for parametric rolling is a differential equation. While the form of this equation may not be the same (see equations (19) and (21) above), a reliable solution of each requires a process to be followed if the solutions are to be replicated. The results show that such reliable solutions can be determined provided that the boundaries of application are respected.

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