Continuation Analysis of Surf-riding and Periodic Responses of a Ship in Steep Quartering Seas

Ioannis Tigkas, *Thenamaris Ships Management Inc.(ex-NTUA)*, i.tigkas@thenamaris.com

Kostas J. Spyrou, *National Technical University of Athens*, k.spyrou@central.ntua.gr

ABSTRACT

Steady-state ship dynamics in steep quartering seas are investigated. Further progress achieved on the implementation of bifurcation analysis for ship motions in regular seas by means of a numerical continuation method is reported. Stationary as well as periodic states are traced and complete bifurcation diagrams, including coexisting ship responses of different type, are presented, considering Froude-Krylov wave excitation. A strong feature of the approach is the inclusion of memory effects within a potential flow framework. The main novelty of the paper lies in the extension of the continuation analysis to 6-D.O.F., for a quartering sea environment. A number of stability diagrams that are produced in an automated and time-efficient manner could be a useful guidance for a Master for avoiding the occurrence of surf-riding and broaching-to.

Keywords: *manoeuvring, surf-riding, broaching-to, bifurcation, homoclinic, continuation, nonlinear dynamics*

1. INTRODUCTION

Instability phenomena in severe astern seas are feared by seafarers from the times when wind was the prime mover (Spyrou, 2010). But it is only since the '40s that they started receiving deeper scientific attention (Davidson, 1948, Rydill, 1959). Shortly it was realised that, some key phenomena responsible for the occurrence of instability in following/ quartering seas have a strongly nonlinear nature (e.g. Weinblum and St. Denis, 1950, Grim 1951 & 1963, Wahab & Swaan, 1964). Simulation, i.e. direct numerical integration of the equations of motion, is a very straight-forward technique for predicting ship responses from some assumed initial conditions. As a matter of fact, it is used nowadays in almost all investigations having practical orientation (Motora *et al.*, 1981; Fuwa *et al.*, 1981, Renilson, 1982, Hamamoto *et al.*, 1988 and 1989, De Kat & Paulling, 1989; Hamamoto *et al.*, 1994). However, ordinary simulation alone is generally ineffective for understanding, in a deeper sense, a dynamical system's behaviour and for identifying its stability limitations. Novel techniques that can target more globally and more directly a system's potential for exhibiting rich and unconventional dynamic behaviour need to be implemented. One technique that can help to unravel behavioural changes as some system parameters are varied is numerical "continuation" (e.g. Krauskopf *et al.*, 2007).

In earlier studies this technique was applied for studying surf-riding and broaching-to on the basis of a 4-D.O.F. model (Spyrou, 1995, 1996a and 1996b). In a more recent paper the authors have performed continuation analysis of periodic motions in exact following seas (surging, heaving and pitching), including potential flow memory effects, for a ship that is on the verge of capture to surf-riding (Spyrou & Tigkas, 2011). This work is further expanded here by performing continuation of stationary and periodic responses for all 6-D.O.F. It is demonstrated that, by using the so-called *codimenion-2* continuation method, stability diagrams of the system are directly produced.

2. MATHEMATICAL MODEL

2.1 Equations of Motions

Long harmonic waves are considered to propagate from following/ quartering direction relatively to the ship, which is moving with forward speed while performing also parasitic motions in all 6 degrees of freedom, due to waves' effect. A non-inertial system rotating like the moving ship (body-fixed system) is used for monitoring ship velocities and accelerations (SNAME, 1952). Furthermore, two inertial systems are used in secondary role: one fixed at a wave trough, and thus moving with the wave celerity; and also an earth-fixed system (see Fig1). Assuming that the ship behaves as a rigid-body, the equations of motions can be expressed with respect to the body-fixed system as follows*:

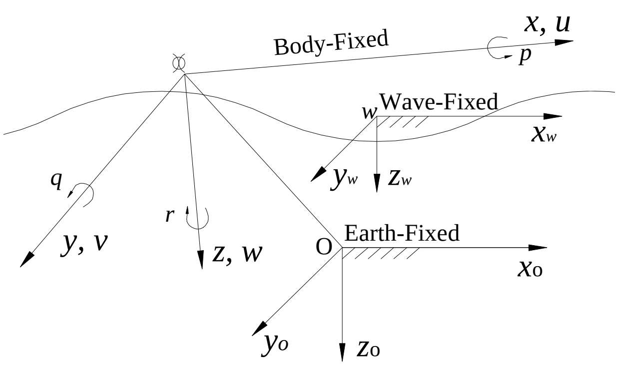


Figure 1: Body-fixed, wave-fixed and earth-fixed coordinate systems.

$\underline{Surge}$:

$$m\left[\dot{u}+qw-rv-x_G(q^2+r^2)+z_G(pr+\dot{q})\right]=X-mg\sin\theta$$

$\underline{Sway}$:

$$m\left[\dot{v}+ru-pw+z_G(qr-\dot{p})+x_G(qp+\dot{r})\right]=Y+mg\sin\varphi\cos\theta$$

$\underline{Heave}$:

$$m\left[\dot{w}+pv-qu-z_G(p^2+q^2)+x_G(rp-\dot{q})\right]=Z+mg\cos\varphi\cos\theta$$

$\underline{Roll}$:

$$I_x\dot{p}-mz_G(\dot{v}+ru-pw)-mx_Gz_G(\dot{r}+pq)=K$$

$\underline{Pitch}$:

$$I_y\dot{q}+(I_x-I_z)rp+mz_G(\dot{u}+qw-rv)-mx_G(\dot{w}+pv-qu)+mx_Gz_G(p^2-r^2)=M-mgx_G\cos\varphi\cos\theta$$

$\underline{Yaw}$:

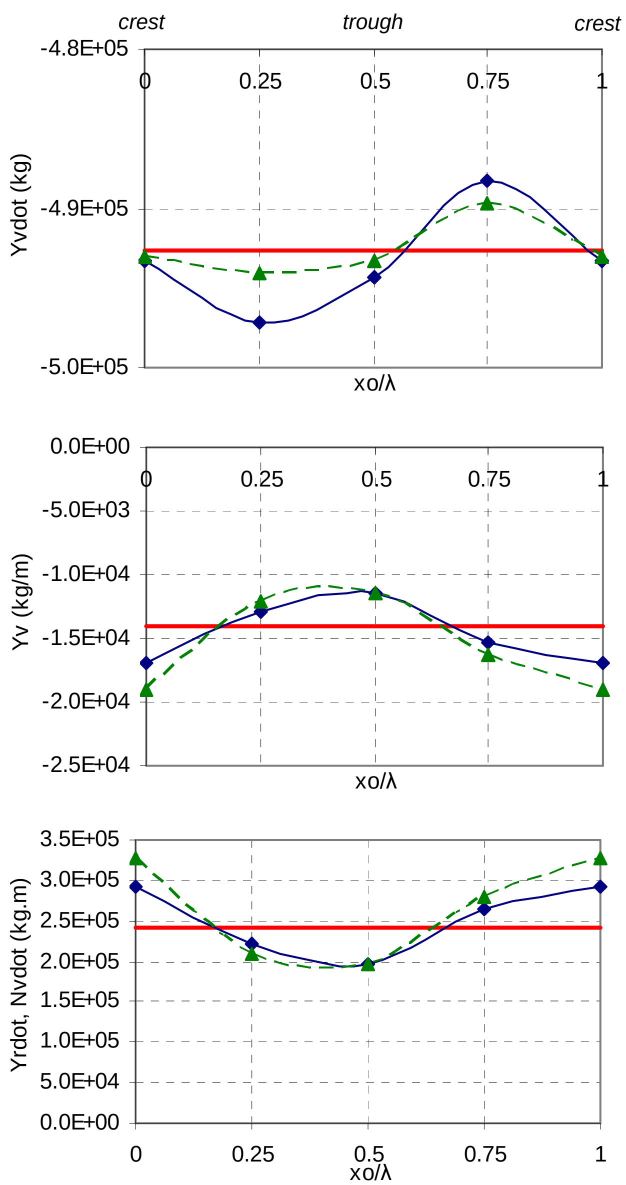
$$I_z\dot{r}+(I_y-I_x)pq+mx_G(\dot{v}+ru-pw)+mz_Gx_G(rq-\dot{p})=N \qquad (1)$$

2.2 Hull Forces and Moments

The external forces and moments acting on the ship are expressed in a modular form as a summation of hull reaction, rudder, propeller and wave excitations. For low frequencies of encounter, added mass and damping coefficients are calculated by the "strip theory" method of Clarke (1972). Whilst "old fashioned", this method is easily integrated in a nonlinear dynamical system continuation analysis. Firstly are computed the potential sway added mass coefficients for a number of (time-varying) hull sections up to the wave surface by multi-parameter conformal mapping. Then by integrating along the length of the hull the forces and moments can be found and expressed as accelerations and velocity derivatives by partial differentiation (Tigkas, 2009). Using pre-processing we obtain the added-mass coefficient of a section as a polynomial function of the instantaneous sectional draught. In this method it is assumed that only the wave profile corresponding to the ship's specific longitudinal position on the wave and the ship's heave and pitch responses, influence the draught of each section and consequently the zero-frequency added mass properties. An example of the obtained results

* All symbols are explained in the Nomenclature at the end of the paper.

is shown in Fig. 2. A universal problem is the luck of accurate calculation of the viscous part of the hydrodynamic derivatives that is usually considerable at the sections near the stern. For this reason, in some occasions we applied an empirical hybrid approach, extracting the viscous part from the semi-empirical zero-frequency derivatives and adding it to the "potential" coefficients obtained by Clarke's method.



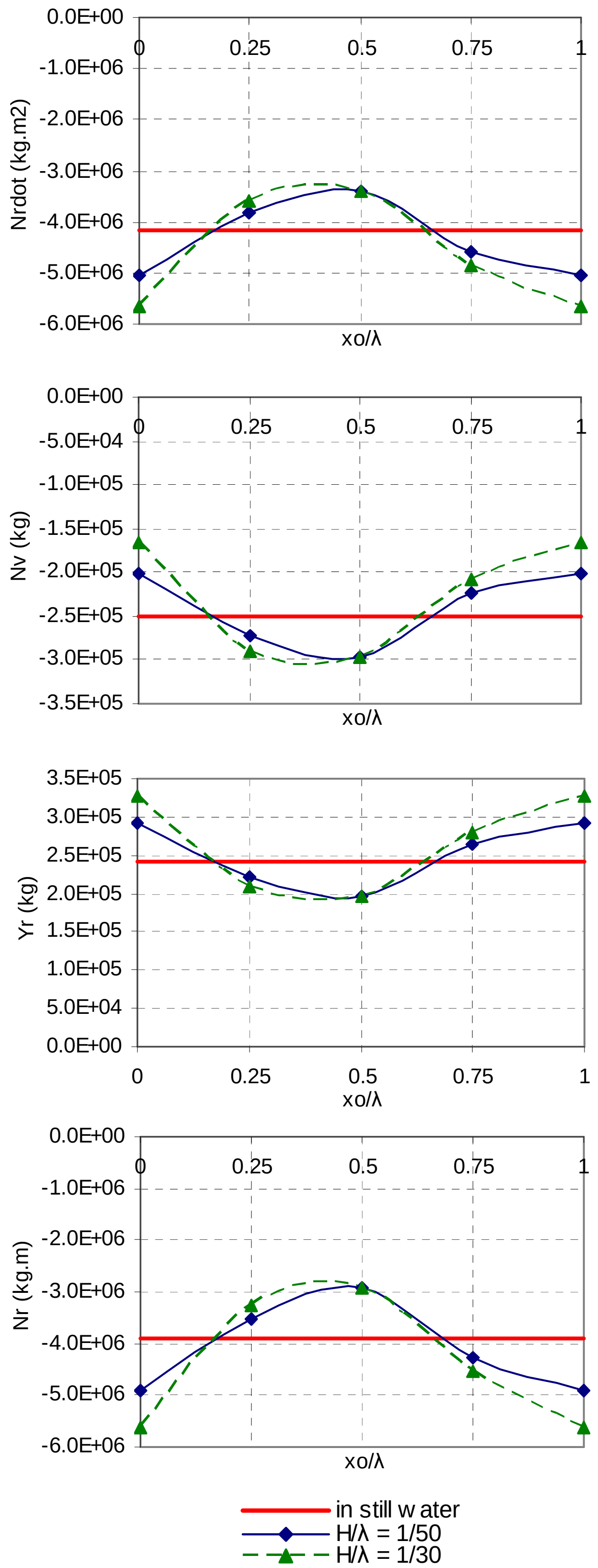


Figure 2: Linear hydrodynamic derivatives for the fishing vessel investigated in Section 4, at different longitudinal positions on a wave starting from a wave crest ($\lambda / L = 2$).

2.3 Memory Effects

To investigate ship motions not only for the stationary condition of surf-riding (zero-frequency of encounter) but for a rather wide range of wave encounter frequencies, “memory effects” should be considered, accounting for fluid’s hydrodynamic radiation loads that are caused in mode *i* due to ship motion in mode *j*. According to Cummins (1962) it can be expressed as:

$$F_{M\,ij}(t) = -A_{ij}(\infty)\dot{v}_j(t) - \underbrace{\int_0^\infty K_{ij}(\tau)v_j(t-\tau)d\tau}_{s_{ij}(t)} \quad (2)$$

The impulse response (or retardation) function K_{ij} in the above equation can be written as follows (Ogilvie, 1964; see however also detailed presentation in Taghipour, 2008):

$$K_{ij}(\tau) = \frac{2}{\pi}\int_0^\infty [B_{ij}(\omega_e) - B_{ij}(\infty)]\cos(\omega_e\tau)d\omega_e \quad (3)$$

The integro-differential equations (2) can be approximated by a system of ordinary differential equations (O.D.E.s) by means of a standard state-space approximation technique (Tick, 1959, Schmiechen, 1975, Jefferys, 1984). As shown by Schmiechen (1975), the following finite set of recursive first-order linear O.D.E.s can replace eq. (2):

$$\begin{aligned}
\dot{s}_{ij}(t) &= s_{ij(1)}(t) - a_{ij(k)}s_{ij(k)}(t) - b_{ij(k)}v_j(t) \\
\dot{s}_{ij(1)}(t) &= s_{ij(2)}(t) - a_{ij(k-1)}s_{ij}(t) - b_{ij(k-1)}v_j(t) \\
\dot{s}_{ij(2)}(t) &= s_{ij(3)}(t) - a_{ij(k-2)}s_{ij}(t) - b_{ij(k-2)}v_j(t) \\
&\cdots \quad \cdots \quad \cdots \quad \cdots \\
&\cdots \quad \cdots \quad \cdots \quad \cdots \\
\dot{s}_{ij(k)}(t) &= -a_{ij(0)}s_{ij}(t) - b_{ij(0)}v_j(t)
\end{aligned} \quad (4)$$

The coefficients $a_{ij(m)}$ and $b_{ij(m)}$ are calculated from curve fitting in frequency domain, on the basis of the values of the added mass and damping terms. However, some attention is required in this identification scheme, for ensuring that the recursive system of eq. (4) is stable (Tigkas, 2009).

It should be noted however that the developed procedure calculates the hydrodynamic memory accounting also zero frequency effect that has already been calculated by the applied strip - theory method. In order to sort out this overlap issue, from each associated linear hydrodynamic derivative we subtract the corresponding potential part at zero-frequency that was calculated by the potential seakeeping code, that has been used in order to extract the added mass and damping coefficients.

2.4 Wave and Hydrostatic Loads

The wave excitation loads of a rigid body in regular harmonic waves, assuming inviscid and irrotational flow, can be linked with the effects of the incident and the diffracted wave potentials. The Froude-Krylov excitation loads (including hydrostatic) are determined by the integration of the fluid pressure of the submerged portion of the hull, up to the free surface of the incident wave. However, the contribution of the diffracted wave excitation loads are not taken into account in this study. Their calculation in a very low to medium encounter frequency range and the efficient integration of such a scheme with continuation analysis is a demanding task on its own that will be resolved in a future research study. Nevertheless, the inclusion of diffraction loads is only expected to alter quantitatively but not qualitatively the results.

2.5 Rudder and Propeller

Kose’s (1982) model is the basis for calculating rudder forces. As for propulsor’s force, a polynomial fit of available propeller performance data is used for approximating the thrust coefficient K_T as function of propeller’s rate of rotation. Wave effects on the rudder and on the propeller are produced by the variation of draught at those locations and by the change in the inflow velocity. Rudder's area and aspect ratio "seen" by the water are obtained from its instantaneous draught. Sometimes the propeller might ventilate or even emerge out of the water with significant efficiency loss (Koushan, 2006, Paik *et al*., 2008). However such losses can vary considerably depending on stern's layout, propeller characteristics and other design characteristics. A simple model calculating the loss of thrust could not be deduced.

When the rudder angle is not taken as fixed, a standard proportional - differential (PD) controller is used, whose equation is expressed in an O.D.E. form as follows:

$$\dot{\delta} = t_r\left[-\delta - a_\psi\left(\psi - \psi_r\right) - a_\psi b_r r\right] \tag{5}$$

3. ADAPTING MATHEMATICAL MODEL TO ENABLE CONTINUATION

To conduct continuation analysis a number of transformations were obliged on the described mathematical model:

(a) The investigated dynamical system needed to come into the generic O.D.E. form $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{a};t)$ where $\mathbf{x}$ is the vector of state variables and $\mathbf{a}$ is the vector of system's parameters. Furthermore, all state variables of the system should be bounded as time progresses. As a matter of fact, continuation analysis could not be attempted before transforming the equations of the system in such a way, so that all state variables take values within bounded limits and, a steady state recognised by the algorithm as such, irrespectively of whether this is stationary or periodic, can be reached.

(b) The last criterion states that the O.D.E.s that constitute the state-space representation should have no explicit dependence upon the time variable. This may include special transformations on the system's form of equations, if the investigated dynamical system is excited by a time-varying force, as for example the wave load.

The first problem is realised when a state variable of the system increases monotonically to infinity as time progresses. The variable that renders impossible for the system to reach a recurring state and is hence imposing non-conformity to the first requirement, is the distance travelled by the ship in the longitudinal direction. In our mathematical model, this is a state variable used for defining the relative position of the ship on the wave and it appears in the module where the Froude-Krylov forces and moments are calculated. To demonstrate the transformations involved, let's consider the surge wave force. By using the relationship $x = x_0 - ct$, the position-dependent part of this force can be written as:

$$X_{FK} = -\rho g A k \cos\psi \int_{-L/2}^{L/2} S\left(x_s, T_s\right) e^{-kT_s\left(x_s, x_0, t, z, \theta\right)/2} \sin k\left(x_s \cos\psi + x\right) dx_s \tag{6}$$

After trigonometric expansion it is written as follows:

$$X_{FK} = -\rho g A k \cos\psi \int_{-L/2}^{L/2} S\left(x_s, T_s\right) e^{-kT_s\left(x_s, x_0, t, z, \theta\right)/2} \left[\left(\cos kx \cdot \sin kx_s \cos\psi\right) + \left(\sin kx \cdot \cos kx_s \cos\psi\right)\right] dx_s \tag{7}$$

Two dummy variables $a = \sin kx$ and $b = \cos kx$ are introduced, replacing the cyclic functions of x in eq. (7). The periodic nature of a and b means that these variables are inherently bounded, unlike x which is monotonically increasing. For consistency an extra pair of O.D.E.s needs however to be added (see also Doedel *et al*., 1997):

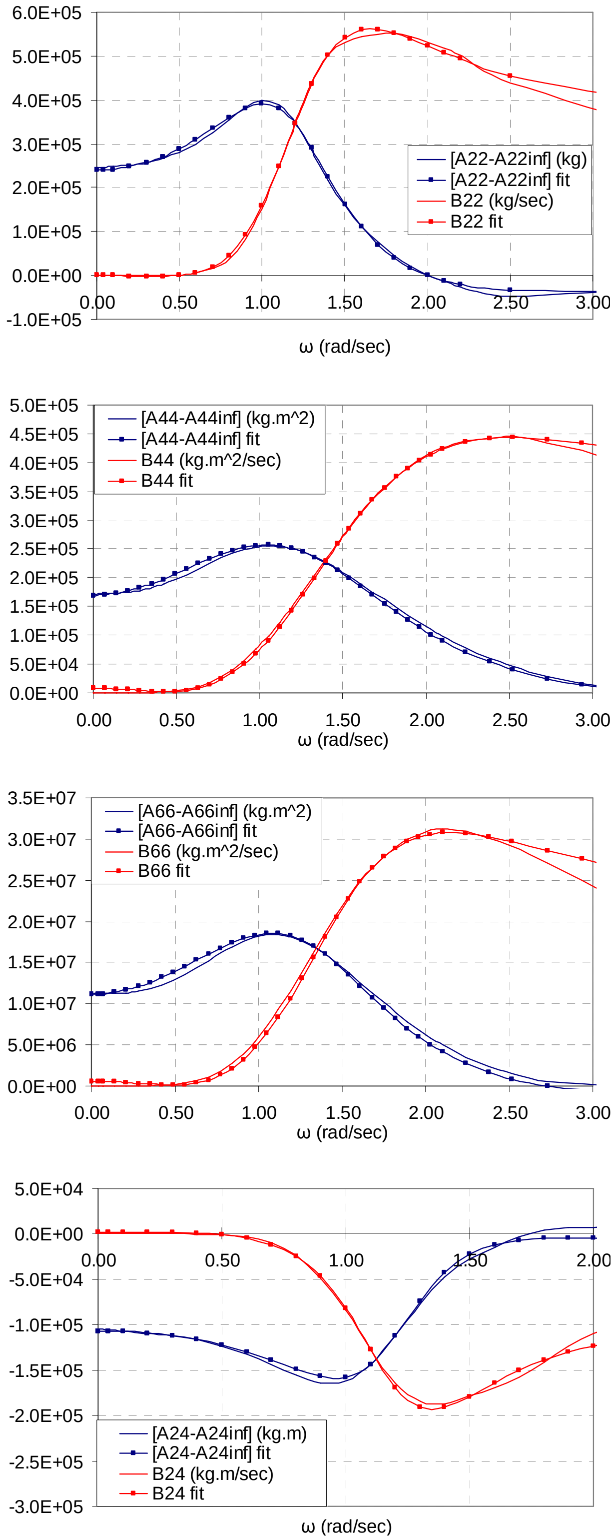
$$\begin{aligned} \dot{a} &= a - \omega_e b - a\left(a^2 + b^2\right) \\ \dot{b} &= \omega_e a + b - b\left(a^2 + b^2\right) \end{aligned} \tag{8}$$

The introduced pair stands basically for a harmonic oscillator that, despite of increasing by two the number of variables of the system, at steady-state it bears no effect on the behaviour relatively to the original system (Spyrou & Tigkas, 2011). This method is applied for all position-dependent forces and moments in all directions of ship motion.

With the above transformations the model can be interfaced with the continuation algorithm where in our case such is MATCONT (Dhooge *et al.*, 2003). The mathematical principles and the main capabilities of this algorithm for investigating nonlinear dynamical systems have been discussed in our earlier works and need not to be repeated (see for example, Spyrou *et al.*, 2007, Spyrou & Tigkas, 2007 and 2011).

4. INVESTIGATION RESULTS

The ship investigated is the well known 34.5m long Japanese fishing vessel (‘purse-seiner’) that has also been examined several times in the past in free running and captive model tests, numerical analysis of dynamics, and benchmarking evaluations (see for example Umeda *et al.*, 1995, ITTC, 2005). Her added mass and potential damping coefficients at each encounter frequency were identified by the commercial code Trident Waveload (2006). Integrations concerning hull geometry were carried out on the basis of 20 transverse stations along the hull length. For the memory fits discussed earlier, the number of the required linear first-order O.D.E.s of the filter has been investigated and a few values of the order k (see section 2.3) were tested, in each case evaluating the quality of the fit produced (Fig3). The order $k = 3$ seems to be a popular choice in relevant work, e.g. see Holappa & Falzarano (1999). Here it was judged that it provides satisfactory accuracy (see also Spyrou & Tigkas (2011) for more detailed explanation).



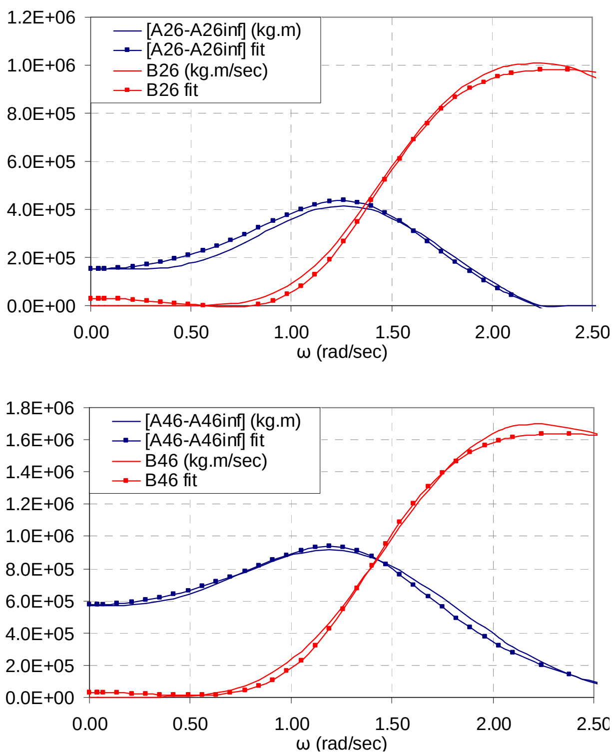


Figure 3: Examples of fits for added mass and damping coefficients for the purse seiner. (Fits for heave, pitch and cross terms of pitch with surge can be found in Spyrou & Tigkas, 2011)

The standard state-space form of the mathematical model after implementing the above described actions is consisted of 77 O.D.E.s. This is considered as a very high number of equations for a continuation study.

4.1 Stationary Responses

In Figure 4 below are shown the obtained “equilibrium headings” for the entire range of rudder angles and for several nominal Froude numbers. These equilibrium headings are unstable unless rudder control exists, in which case the parts of the curve nearer to the trough are stabilised. But for the basic case of a ship without active control, *saddle-type* instabilities are formed between LP1 and LP1’ and between LP2 and LP2’. Equilibrium diagrams for the other modes of motion are also shown in Figure 5.

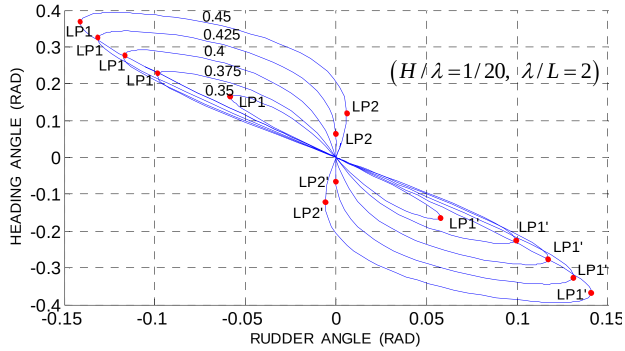


Figure 4: Equilibrium headings in correspondence to rudder angles, based on the 6-D.O.F. model.

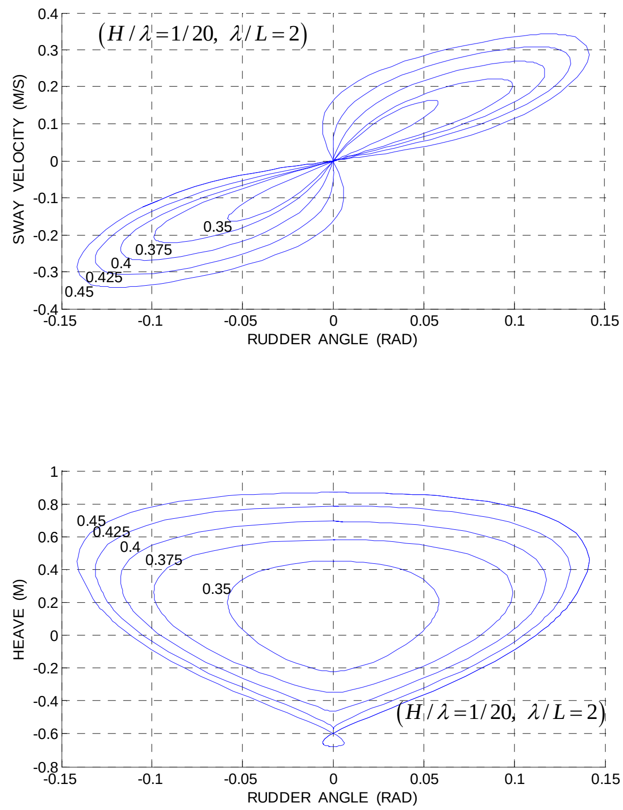


Figure 5: Equilibrium sway velocity (upper) and heave displacement (lower) corresponding to rudder angles for a range of nominal Froude numbers.

Since LP1 points determine the range of potentially stabilisation by rudder control state laws, their locus defines in fact the domain where surf-riding can be experienced in practice. Such a curve can be obtained by *codimension-2* continuation of the *fold* LP1, varying simultaneously Fn and δ. The result is shown in Figure 6. The branch of LP2 has also been included in this diagram. Even though these latter points do not receive an immediate practical interpretation, their "behaviour" is interesting: at the zero rudder angle and for $Fn = 0.42$, a *cusp* point is formed by the tangential contact of the *saddle* - *node* branches corresponding to LP2's evolution.

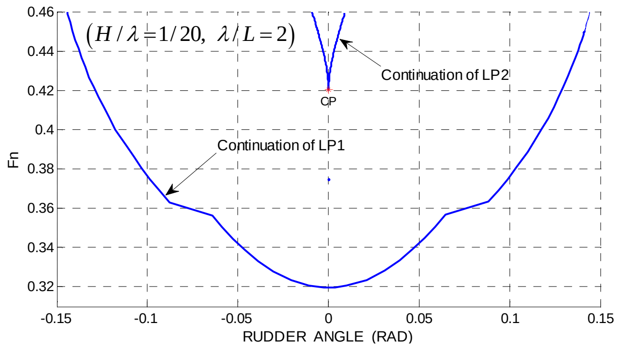


Figure 6: Stability diagram in following/ quartering seas, showing the range of rudder angles where surf-riding is sustainable.

A diagram such as the one of Figure 6 could make a ship Master aware of the range of "equilibrium rudder angles" which could support surf-riding in following/ quartering seas. Of course the accuracy of the input that can become available to him is critical, especially regarding wave height and period.

The influence of proportional gain on the realized heading in the condition of surf -riding in quartering seas is illustrated in Figure 7. Accordingly, the upper parts of the curves in Figure 7 from 0 to LP are always stable, where the lower parts are occupied by saddles. Again, LP points indicate the transition of stability and are occurring this time at the maximum possible commanded heading angles for each selected proportional gain. As one expects, larger commanded angles of heading will not correspond to a reduced heading error (commanded heading minus actual heading), but rather practically mean the ship to start turning. It is also evident that at low values of proportional gain, the heading error especially in relatively large commanded heading angles is also large and thus the controller's function is problematic. The locus of limit points (LP) is finally obtained by *codimension-2* continuation when varying simultaneously the commanded heading and the proportional gain.

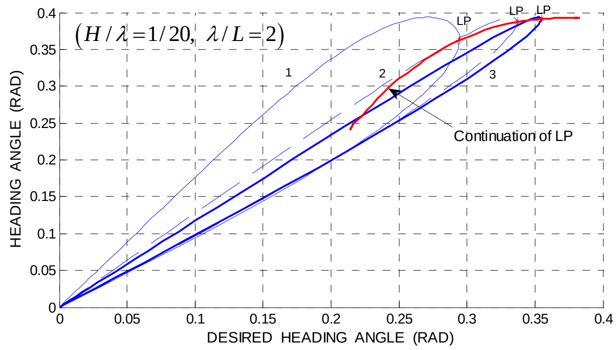


Figure 7: Difference between commanded and actual headings for three different proportional gains (1, 2 and 3), $Fn = 0.45$ and differential gain equal to 1.

Another useful investigation is to determine the locus of lowest nominal Froude numbers for which surf-riding can be realised in quartering waves. This can also be obtained by *codimension-2* continuation, varying *Fn* and ψ_r. In the scenario of Figure 8, the control settings were: $t_r = 3$, $a_\psi = 3$, and $b_r = 1$ Several diagrams of this kind for different heights and lengths can also assist on-board decision-making.

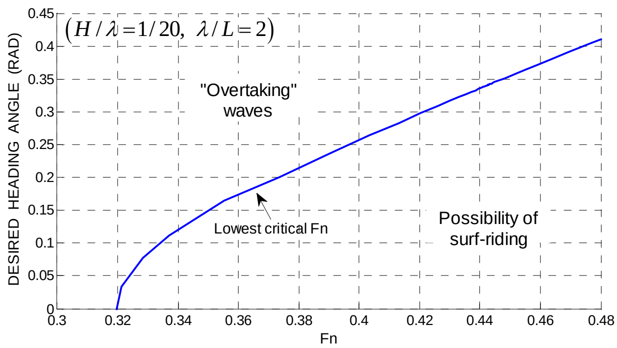


Figure 8: *Codimension-2* bifurcation diagram produced by continuation and showing the lowest nominal Froude number values for which surf-riding can be realised, for a wide range of commanded heading angles.

4.2 Periodic Responses with Active Steering Control

The key element of the current effort targets the periodic responses in quartering waves and furthermore, the conditions that are eligible for broaching-to behaviour not only as a natural consequence of surf-riding but also by a more direct escape mechanism (Spyrou, 1996b and 1997). However controller's settings influence the amplitude of yaw oscillation as well as the maximum commanded heading angle up to which the ship remains controllable.

To run effectively the continuation of these periodic orbits for the complete model in 6-D.O.F. we needed to reduce the system of equations to 42 O.D.E.s by neglecting several memory terms that have a lesser effect on the outcome. Thus, the most influential memory loads kept in this simplified system comprise; s_{22}, s_{26}, s_{33}, s_{35}, s_{44}, s_{55}, s_{53}, s_{66} and s_{62}. Below only two characteristic examples of the several obtained results will be discussed.

Consider firstly the evolution of periodic motions for a fixed commanded heading angle of 15°. For low nominal Froude numbers, ship motion is basically linear but as the speed is increased, it becomes increasingly asymmetrical. As well known, at some stage the periodic behaviour abruptly stops due a *homoclinic connection*. In Figure 9 is shown the contact between the stationary (surf-riding) and the periodic motions that is responsible for this phenomenon. Continuation produces a unique picture of this spectacular encounter between different ship states.

The sequence of phenomena is qualitatively similar to what happens in an exactly following sea environment (see for comparison Spyrou & Tigkas, 2011). However, the critical Froude numbers are moved to slightly higher values since the experienced surge wave force in quartering waves is reduced. Control settings were selected like in section 4.1, sufficient for keeping the ship on a mean heading very close to the commanded one.

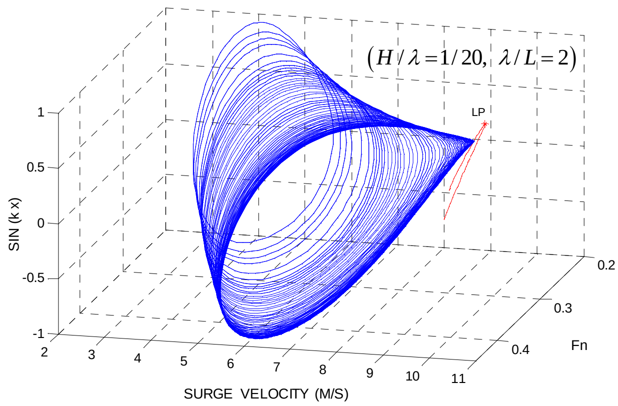


Figure 9: 3-D view of the evolution of limit cycles collapsing onto the branch of saddles, for a fixed 15° desired heading angle.

In the second scenario presented the interest is on the evolution of the periodic motion amplitudes for a fixed nominal Froude number, as the commanded heading angle is increased to values beyond the surf-riding range. As observed in Figures 10 and 11, the amplitudes of both yaw and surge oscillations increase when the commanded heading angle is also increased. Whilst this behaviour continues for the lower range of commanded heading angles, at a critical value of the commanded heading angle a *limit point of cycles* (*fold of cycles*) is encountered and the stable limit cycles are turned unstable. Thus a sufficient condition for a discontinuous jump to a distant state is established. The created unstable limit cycles continue their evolution "backwards" with increasing amplitude. This phenomenon indicates a different broaching-to scenario at *LPC*, which occurs not due to surf-riding, but directly from periodic oscillations by an over-increased heading angle.

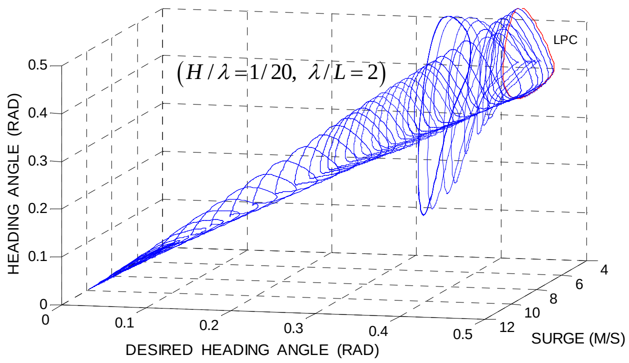


Figure 10: 3-D view of the evolution of limit cycles by increasing the desired heading angle at nominal *Fn* = 0.4

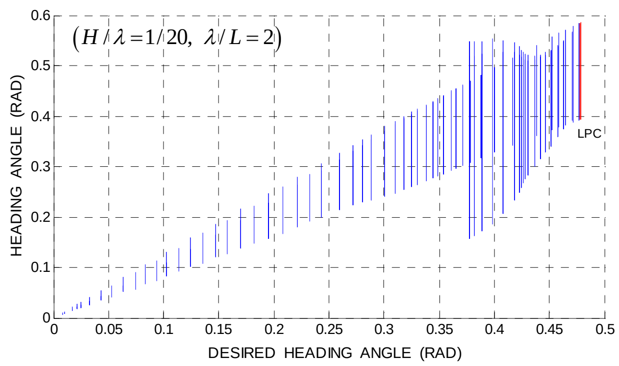


Figure 11: Bifurcation diagram showing the increase of yaw amplitude as the commanded heading angle is increased, at nominal *Fn* = 0.4

5. CONCLUSIONS

A mathematical model in 6-D.O.F. that contains modules for the effect of waves, hull reaction including hydrodynamic memory, propeller and rudder has been developed. The mathematical model is the offspring of an earlier manoeuvring-type model with 4 D.O.F. and it has been brought into such a form that it can be utilised for bifurcation analysis. A continuation analysis algorithm was interfaced with the mathematical model in order to fully capture the stationary and periodic ship motions, with emphasis given at their interactions. Loci in parameters' space of system's bifurcation points have also been traced. The latter can be particularly useful as some of the system's stability boundaries can be automatically produced in this manner. Design diagrams related with controller tuning and ultimately a complete booklet offering operational guidance for averting broaching-to phenomena in steep quartering seas can also be produced.

In parallel to the achieved progress, several areas requiring further work have emerged. For example, the nonlinear mathematical model developed can be further improved; especially the part related to the calculation of the manoeuvring derivatives and the diffraction wave loads within a user-friendly framework for applying continuation analysis and other nonlinear dynamics techniques. The combined effects of wind and waves can also be studied. Despite that this appears to be straightforward given the current development and previous work studies by the authors (e.g. Spyrou *et al.*, 2007), the wealth of dynamical phenomena that may arise from a combination of excitations, merits in our view a dedicated study. Another, more ambitious, direction of research is the extension of the presented methods of analysis for a probabilistic environment.

6. ACKNOWLEDGMENTS

Ioannis Tigkas would like to thank "Alexander S. Onassis" Public Benefit Foundation for his PhD scholarship at the time of his research studies at NTUA.

7. REFERENCES

Clarke, D., 1972, "A Two-Dimensional Strip Method for Surface Ship Hull Derivatives: Comparison of Theory with Experiments on a Segmented Tanker Model", Journal of Mechanical Engineering Science, Vol. 14, No. 7, pp. 53-61.

Cummins, W.E., 1962, "The Impulse-Response Function and Ship Motions", Schiffstechnik, Vol. 9, No. 47, pp. 101-109.

Davidson, K.S.M., 1948, "A Note on the Steering Of Ships in Following Seas", Proceedings of the 7th International Congress of Applied Mechanics, London.

De Kat J.O. and Paulling J.R., 1989, "The Simulation of Ship Motions and Capsizing in Severe Seas", SNAME Annual Meeting, Presentation, No. 5.

Dhooge A., Govaerts W., Kuznetsov Yu.A., Mestrom W., Riet A.M. and Sautois B., 2003, "MATCONT and CL_MATCONT: Continuation Toolboxes for MATLAB". Report of Gent and Utrecht Universities.

Doedel E.J., Champneys A.R., Fairgrieve T.F., Kuznetsov Yu.A., Sandstede B. and Wang X.J., 1997, "AUTO97-00: Continuation and Bifurcation Software for Ordinary Differential Equations (with HomCont)", User's Guide, Concordia University, Montreal, Canada.

Fuwa T., Sugai K., Yoshino T. and Yamamoto T., 1981, "An Experimental Study on broaching of Small High Speed Boat", Journal of the SNA of Japan, Vol. 150.

Grim O., 1951, "Das Schiff in von Achtern Auflaufender See", Jahrbuch S.T.G., Bd. 45.

Grim O., 1963, "Surging Motion and Broaching Tendencies in a Severe Irregular Sea", Deutsche Hydrographische Zeitschrift, Bd. 16.

Hamamoto M., 1988, "Study on Ship Motions and Capsizing in Following Seas: 1st Report - Equations of Motion for Numerical Simulation", Journal of the SNA of Japan, Vol. 163, pp. 173-180.

Hamamoto M., 1989, "Study on Ship Motions and Capsizing in Following Seas: 2nd Report - Simulation of Capsizing", Journal of the SNA of Japan, Vol. 163, pp. 123-130.

Hamamoto M., Fujino M. and Kim Y.S., 1994, "Dynamic Stability of a Ship in Quartering Seas", Proceedings of STAB 94, Melbourne, Florida.

Holappa K.W. and Falzarano, J.M., 1999, "Application of Extended State-space to Non-linear Ship Rolling", Journal of Ocean Engineering, Vol. 26, p. 227-240.

ITTC, 2005, "Report of Specialist Committee on Ship Stability in Waves", Proceedings of the ITTC Conference, Edinburgh.

Jefferys, E.R., 1984, "Simulation of Wave Power Devices", Journal of Applied Ocean Research, No. 6, Vol. 1, pp. 31-39.

Kose K., 1982, "On a New Mathematical Model of Manoeuvring Motions of A Ship and its Applications", Journal of International Shipbuilding Progress, Vol. 29, No. 336.

Koushan, K., 2006, "Dynamics of Ventilated Propeller Blade Loading on Thrusters due to Forced Sinusoidal Heave Motion", Proceedings of the 26th Symposium on Naval Hydrodynamics, Rome, Italy.

Krauskopf B., Osinga H. G. and Galan-Vioque J. (editors), 2007, "Numerical Continuation Methods for Dynamical Systems", Springer Publications.

Motora S., Fujino M., Koyanagi M., Ishida S., Shimada K. and Maki T., 1981, "A Consideration on the Mechanism of Occurrence of Broaching-to Phenomena", Journal of the SNA of Japan, Vol. 150, pp. 84-97.

Ogilvie T.F., 1964, "Recent Progress towards the Understanding and Prediction of Ship Motions", Proceedings of the 5th Symposium on Naval Hydrodynamics, Bergen, Norway, pp. 3-79.

Paik B.G., Lee J.Y. and Lee S.J., 2008, "Effect of Propeller Immersion Depth on the Flow around a Marine Propeller", Journal of Ship Research, Vol. 52, No. 2, pp. 102-113.

Renilson M., 1982, "An Investigation Into the Factors Affecting the Likelihood of Broaching-to in Following Seas", Proceedings of the STAB 82, Tokyo, pp. 17-28.

Rydill L.J., 1959, "A Linear Theory for the Steered Motion of Ships in Waves", Transactions of RINA, pp. 81-112.

Schmiechen M., 1975, "Equations for Non-Quasi-Steady Ship Motions", Seakeeping Committee Report, Proceedings of the 14th ITTC Conference, Ottawa, Canada.

SNAME, 1952, "Nomenclature for Treating the Motion of a Submerged Body Through Fluid", Technical and Research Bulletin, No. 1-5.

Spyrou K.J., 1995, "Surf-Riding Yaw Instability and Large Heeling of Ships in Following/ Quartering Waves", Journal of Ship Technology Research, Vol. 42, pp. 103-112.

Spyrou K.J., 1996a, "Dynamic Instability in Quartering Seas: The Behaviour of a Ship During Broaching", Journal of Ship Research, Vol. 40, No. 1, pp. 46-59.

Spyrou K.J., 1996b, "Homoclinic Connections and Period Doublings of a Ship Advancing in Quartering Waves", Journal of Chaos, Vol. 6, pp. 209-218.

Spyrou K.J., 1997, "Dynamic Instability in Quartering Seas – Part III: Nonlinear Effects on Periodic Motions", Journal of Ship Research, Vol. 41, No. 3, pp. 210-223.

Spyrou, K.J., 2010, "Historical Trails of Ship Broaching-to", Transactions of RINA, Vol. 152, Part A4, pp. 163-173.

Spyrou K.J., Tigkas I. and Chatzis A., 2007, "Dynamics of a Ship Steering in Wind Revisited", Journal of Ship Research, Vol. 51, No. 2.

Spyrou K. and Tigkas I.G., 2007, "Principles and Application of Continuation Methods for Ship Design and Operability Analysis", Proceedings of 10th PRADS Symposium, Houston, Texas, USA.

Spyrou K.J. and Tigkas I.G., 2011, "Nonlinear Surge Dynamics of a Ship in Astern Seas: "Continuation Analysis" of Periodic States with Hydrodynamic Memory", Journal of Ship Research, Vol. 55, No. 1, pp. 19-28.

Taghipour R., Perez T. and Moan T., 2008, "Hybrid Frequency–Time Domain Models for Dynamic Response Analysis of Marine Structures", Journal of Ocean Engineering, 35, pp. 685-705.

Tick L.J., 1959, "Differential Equations with Frequency-Dependent Coefficients", Journal of Ship Research, Vol. 3, No. 2, pp. 45-46.

Tigkas I.G., 2009, "Nonlinear Dynamics Analysis of the Directional Instabilities of Ships in Wind and Waves", PhD Thesis, National Technical University of Athens, Greece.

Trident F-D Waveload, 2006, "User's Guide and Theory Manual", Martec Ltd, Halifax, Nova Scotia, Canada.

Umeda N., Hamamoto M., Takaishi Y., Chiba Y., Matsuda A., Sera W., Suzuki S., Spyrou K.J. and Watanabe K., 1995, "Model Experiments of Ship Capsize in Astern Seas", Journal of the SNA of Japan, Vol. 177, pp. 207-218.

Wahab R. and Swaan W.A., 1964, "Course-keeping and Broaching of Ships in Following Seas", Journal of International Shipbuilding Progress, Vol. 7, No. 4, pp. 293-301.

Weinblum G. and St. Denis M., 1950, "On the Motions of Ships at Sea", Transactions of SNAME, Vol. 58, pp. 184-248.

NOMENCLATURE

A : wave amplitude

$A_{ij}(\omega)$: added mass coefficient

A_R : rudder area

a_ψ, a_r : proportional, differential gain

$B_{ij}(\omega)$: damping coefficient

c : wave celerity

F_N : rudder normal force

Fn : Froude number

H : wave height

H/λ : wave steepness

I_x, I_y, I_z : roll, pitch and yaw ship mass moment of inertia

K, M, N : moments in roll, pitch and yaw respectively

$K_{ij}(\tau)$: impulse response function

K_T : propeller thrust coefficient

k : wave number ($k = 2\pi/\lambda$)

λ : wave length

L : ship length

m : ship mass

q, p, r : pitch, roll and yaw angular velocity in a body-fixed system, respectively

$S(x_s, T_s)$: vertical hull sectional area below instantaneous waterline

t : time

t_p : thrust deduction coefficient

t_r : rudder's time constant

$T_s(x_s x_0, t, z, \theta)$: draught of ship at vertical section S

u, v, w, : surge, sway and heave velocity in a body-fixed system, respectively

U_R : inflow velocity at rudder

X, Y, Z : forces in surge, sway and heave respectively

x : longitudinal distance travelled by the ship, with respect to a system fixed at a wave trough.

x_o : longitudinal distance travelled by the ship in an earth-fixed system

x_s : longitudinal distance of a vertical ship section S in the body-fixed system

x_G, z_G : longitudinal distance from amidships and vertical distance from keel of ship's centre of gravity, respectively.

Greek letters

δ : rudder angle

Λ : rudder aspect ratio

θ : pitch angle

ρ : water density

φ : roll angle

ψ : heading angle

ψ_r : desired heading angle

ω_e : encounter frequency

ω : wave frequency