# Critical Wave Groups vs. Direct Monte-Carlo Simulations for Typical Stability Failure Modes of a Container Ship

Vladimir Shigunov, *Germanischer Lloyd SE, Hamburg, Germany*, vladimir.shigunov@gl-group.com

Nikos Themelis, *Department of Naval Architecture and Marine Engineering, National Technical University of Athens, Athens, Greece*, nthemelis@naval.ntua.gr

Kostas J. Spyrou, *Department of Naval Architecture and Marine Engineering, National Technical University of Athens, Athens, Greece*, k.spyrou@central.ntua.gr

**ABSTRACT**

In the new generation intact stability criteria currently debated at IMO, the process of direct assessment of stability is interlaced with a requirement of performing several numerical simulations. However, extreme responses that are generally behind stability failures are rare events, usually based on the non-Gaussian roll response stochastic process. A practical approach discussed recently exploits the idea that extreme events occur due to the encountering of extreme wave groups (critical wave episodes). This could alleviate the need for a large number of simulations by focusing on the systematic identification of those deterministic wave sequences that generate unacceptable roll responses. Taking a first step towards a systematic validation process of the wave groups method, the present study compares the exceedance probabilities of 40 deg roll angle and of $g$/2 lateral acceleration, computed by the critical wave groups method for a large containership, with Monte-Carlo simulations. The nonlinear seakeeping code *rolls* is used as mathematical model of ship motion. Typical loading conditions where various stability failure modes can occur are examined.

**Keywords:** *Numerical Simulations; Monte Carlo Method; Critical Wave Groups*

## 1. NOMENCLATURE

| | |
|---|---|
| $B_{wl}$ | waterline breadth |
| $GM$ | initial metacentric height |
| $g$ | acceleration due to gravity |
| $H$ | wave height |
| $\mathbf{H}$ | vector of wave heights in a wave group |
| $H_{cr}$ | threshold wave height |
| $\mathbf{H}_{cr}$ | vector of critical wave heights |
| $h_s$ | significant wave height |
| $KG$ | centre of gravity height above keel |
| $L_{pp}$ | length between perpendiculars |
| $p$ | probability |
| $r$ | exceedance rate |
| $T_m$ | mean draft |
| $T_w$ | wave period |
| $T_z$ | zero upcrossing period of the seaway |
| $T_\varphi$ | linear natural roll period |
| $T_1$ | mean seaway period |
| $v$ | ship speed |
| $\mu$ | mean seaway direction (0 and 180° for following and head seas, respectively) |
| $\omega_\varphi$ | linear natural roll frequency |

## 2. INTRODUCTION

Application of numerical simulations for direct stability assessment is currently discussed at IMO as an important part of the new generation intact stability criteria. This requires both sufficiently accurate mathematical models of nonlinear ship dynamics, and rational probabilistic procedures able of predicting probabilities of rare extreme motion events for non-Gaussian processes.

A practical solution for the latter problem proposed by Themelis & Spyrou (2007) exploits the idea that extreme events occur due to the encountering of extreme wave groups (critical wave episodes). The identification of the critical wave groups in terms of height, period and duration is then possible on the basis of nonlinear deterministic ship motion analysis, while the probability of encountering specific wave groups is based on statistical seaway models.

The method is by its nature approximative in its consideration of the ship-wave encounter, because, at its current stage of development, certain regular wave profiles are examined. On the other hand, the method is very versatile, combining arbitrarily sophisticated nonlinear analysis of ship dynamics (and thus any stability failure mode captured by the method used for motion analysis) with accurate probabilistic analysis of the seaway.

The present study is a step towards the comparison of this method with the direct stability assessment through numerical simulations with the nonlinear seakeeping code *rolls*, for typical stability failure modes of a modern post-panamax container ship in a range of practically relevant initial *GM* values. Short-term average exceedance rates of the maximum (over the ship) lateral acceleration value $g/2$, as well as of roll angle 40 deg, are determined from Monte-Carlo simulations. They are compared against the exceedance rate obtained by using the critical wave groups approach.

## 3. CRITICAL WAVE GROUPS

According to the principle of this method, the ship can be assumed as performing initially ordinary (linear) oscillatory motion of small to moderate amplitude in the considered mode(s). Then extreme behavior is realised due to the encounter of a wave group. Thus the critical wave groups identification process supplies in fact the threshold excitation that generates undesirable ship behavior. Such wave groups are identified for frequencies spanning the usual range of wave frequencies. Each group is characterised by its run length, period and height. However, the choice of fixed height does not imply that groups physically display such a property. Instead, it specifies the critical height above which ship behavior exhibits at least one unacceptable characteristic. As for the discrete period, it should be seen as a representative of the small range of periods around it. It is important to note that, in the implementation of this method, all wave groups that result in undesirable dynamic response are extracted beforehand. Then, probabilities can be determined for encountering conditions "worse" than the critical, for the seaway situations that exist in the considered area of ship operation.

The process of consecutive waves is modeled as a first order "autoregressive model" which is in fact equivalent to assuming the Markov chain property for the waves, a well established characteristic of sea waves. However, it is essential that no similar assumption is necessary concerning roll response. The targeted probability is calculated as the product of (1) the conditional probability $p\left(T \middle| H > H_{\mathrm{cr}}\right)$ of encountering $n$ successive waves with periods $T$, lying in a specific range, and heights above a threshold level $H_{\mathrm{cr}}$, and (2) the probability $p\left(H > H_{\mathrm{cr}}\right)$ that $n$ successive waves have heights exceeding this critical threshold[1]. For

[1] The slightly unorthodox inequality in the above means that each component of vector $\mathbf{H}$ obtains greater value than the corresponding value in vector $\mathbf{H}_{\mathrm{cr}}$. However, in the current implementation, all entries of vector $\mathbf{H}_{\mathrm{cr}}$ receive the same value $H_{\mathrm{cr}}$.

the calculation of the first probability, the conditional multivariate normal probability density function (pdf) is used (Wist et al., 2004), combined with Tayfun (1993) work on the joint probability density function (pdf) of large wave height and its associated period. On the other hand, the calculation of $p(H > H_{cr})$ is based on the bivariate Rayleigh pdf of two successive wave heights, Battjes & Van Vledder (1984), combined with the Markov chain property. This modelling approach can handle efficiently both the period and the height of successive waves and represents a key step beyond the Kimura-type modelling of wave groups that is not sensitive to the period, Kimura (1980).

## 4. SHIP AND LOADING CONDITIONS

A modern 8000 TEU container ship was selected for the study. Such vessels are presently the work horses of east-bound container shipping routes, and many of them might be employed in west-bound routes with more harsh weather conditions after the modernisation of Panama channel. Many years of full-scale measurements onboard several vessels of this size are available, thus their loading and speed profiles are well known. Moreover, roll damping parameters are known from model tests.

The selected vessel has a length between perpendiculars of about 320.0 m, waterline breadth of about 43.0 m and design speed of about 25.0 knots. Vessels of this size operate most frequently in partial loading conditions with *GM* from about 2.5 to about 4.5 m; such loading conditions are relatively safe with respect to both parametric and synchronous resonance. Therefore, a wider range of loading conditions was studied, Table 1, including (1) nearly full load with *GM* of 1.2 m, which might be vulnerable to parametric roll, (2) a "typical" loading condition with *GM* of 3.8 m and (3) ballast loading condition with a very high *GM* of 7.5 m, expected to lead to large lateral accelerations due to synchronous roll.

Table 1: Loading conditions

| | | | |
|---|---|---|---|
| $T_m$ [m] | 14.44 | 12.84 | 11.36 |
| *GM* [m] | 1.2 | 3.8 | 7.5 |
| $T_\varphi$ [s] | 30.1 | 18.5 | 12.6 |

## 5. SELECTION OF SPEEDS AND SEAWAY PARAMETERS

Synchronous roll is most relevant in beam waves, where the added resistance is rather low and thus forward speed can be rather high, see case 01 in Table 2. The corresponding critical seaway parameters for this loading condition are shown with a black point in Figure 1.

Table 2: Selected conditions

| Case | *GM* [m] | *Fr* [-] | $\mu$ [°] | $T_1$ [s] |
|---|---|---|---|---|
| 01 | 7.50 | 0.16 | 98.0 | 11.5 |
| 02 | 1.26 | 0.09 | 50.0 | 10.0 |
| 03 | 1.26 | 0.12 | 49.0 | 9.2 |
| 04 | 1.26 | 0.04 | 180.0 | 13.4 |
| 05 | 3.80 | 0.16 | 60.0 | 12.7 |

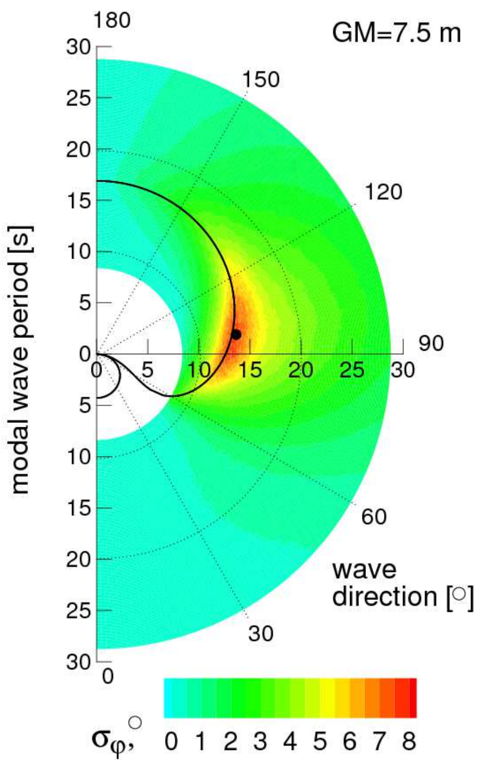


Figure 1: Calculated rms of roll angle for the loading condition with *GM*=7.5 m at *Fr*=0.16 in irregular waves vs. modal wave period and wave direction (radial and circumferential coordinates, respectively). Black line indicates resonance condition.

Principal parametric resonance occurs typically at low *GM* in following waves (at low forward speeds) to quartering waves (at higher forward speeds, which are more realistic in waves from the stern directions), Shigunov et al. (2009). Cases 02 and 03 in Table 2 represent this failure mode, and Figure 2 shows these cases on polar plots; two cases are selected to study the influence of the ship speed.

Principal parametric resonance also occurs at low *GM* in head waves at low forward speeds, which are realistic in high head waves; case 04 in Table 2 and Figure 3 illustrate the selected seaway conditions and the ship forward speed.

Finally, case 05 in Table 2 and Figure 4 represent direct excitation case for a "typical" loading condition.

## 6. MONTE-CARLO SIMULATIONS

To study the influence of the significant wave height on the accuracy of the critical wave groups method, significant wave height was systematically varied with a small step for each of the selected combinations 01 to 05 of wave direction, wave period and ship speed. Simulations were performed in long-crested seaways described by JONSWAP spectrum with the peak parameter $\gamma$=3.3. For each combination of ship speed and seaway parameters, 50 to 500 simulations were carried out with different initial phases of seaway components, until maximum over the ship lateral acceleration exceeded the value $g$/2. In the other series of simulations (denoted as cases 01a to 05a, with the conditions corresponding to those in cases 01 to 05), exceedance of the roll angle of 40 deg was considered as the extreme event. The average exceedance period was calculated by averaging over all exceedance periods. Numerical simulation method *rolls*, proposed by Söding (1982), was used for simulations (see Petey, 1986, for details of the method and Shigunov et al., 2009, for validation examples).

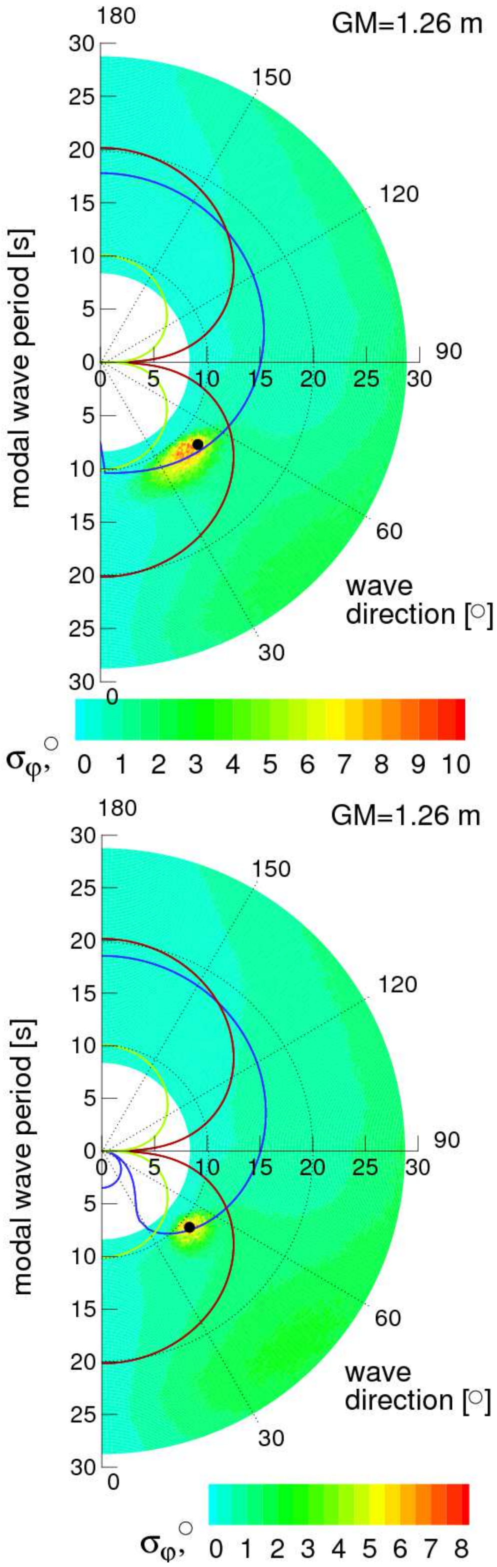


Figure 2: Calculated rms of roll angle for *GM*=1.2 m at *Fr*=0.09 (top) and 0.12 (bottom); the black points indicate representative scenarios for parametric resonance in quartering waves; blue line indicates resonance condition while yellow and green lines limit the area of suitable wave lengths.

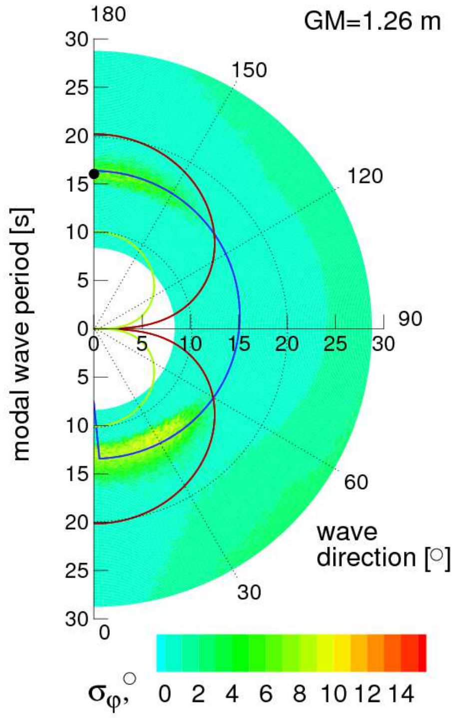


Figure 3: Calculated rms of roll angle for *GM* of 1.2 m and *Fr* of 0.04 in irregular waves. Black point indicates selected case for parametric resonance in bow waves; blue line indicates resonance condition, yellow and green lines limit the area of suitable wave lengths.

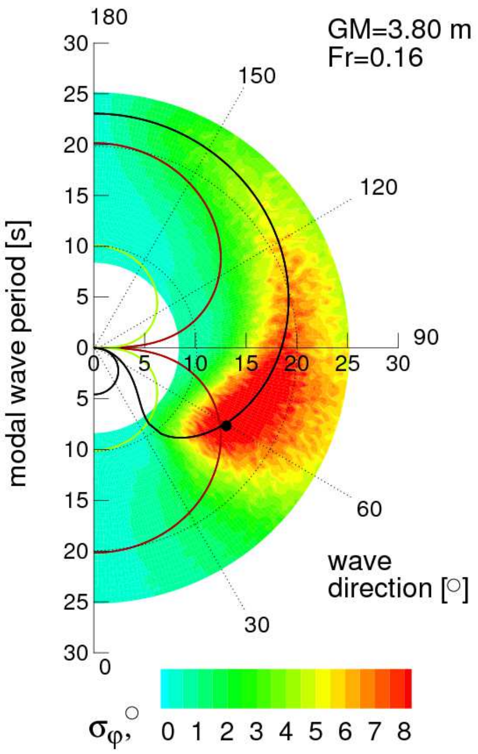


Figure 4: Calculated rms of roll angle for *GM*=3.8 m at *Fr*=0.16 in irregular waves.

## 7. CALCULATION OF HYDRODYNAMIC DATABASE

For each of cases 01 to 05 (and 01a to 05a, respectively), a corresponding hydrodynamic database of roll responses in regular waves was computed. The period of regular waves was varied in a broad range with a step of 0.5 s, and *GM*, forward speed and wave direction were kept constant. The same simulation method *rolls* was used for the computation of the hydrodynamic database. The initial condition was an upright ship with zero roll velocity.

Time histories of responses (maximum over the ship lateral acceleration or roll angle) are processed to obtain response maxima, i.e. $\max(a_n, -a_{n*})$, Figure 5, per cycle of oscillation. Figure 5 explains the definitions used: the reaction period *n* from the start of simulation and $^*$ for negative peaks. Post-processing considered 9 roll cycles. The wave height was varied until the response maximum in each response period exceeded the threshold (*g*/2 for lateral acceleration or 40 deg for roll amplitude). In this way, nine “critical” wave height values $H_{\rm cr}(T,n)$, $n=1,\ldots,9$ were identified, each of which leads to a response amplitude equal to, or greater than, the threshold during the corresponding wave encounter.

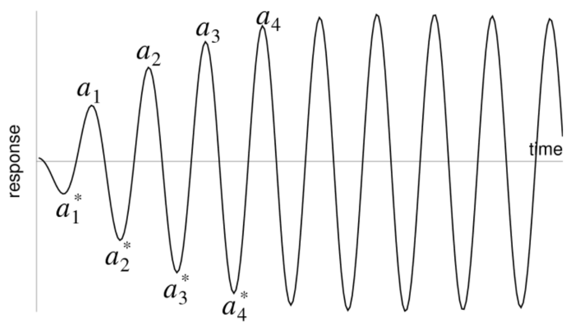


Figure 5: Post-processing of responses in regular waves.

Figures 6 and 7 present examples of the identified critical wave heights. The corresponding scenario and the threshold are also indicated. The shown curves were obtained for the wave group period equal to the mean period of the seaway, Table 2.

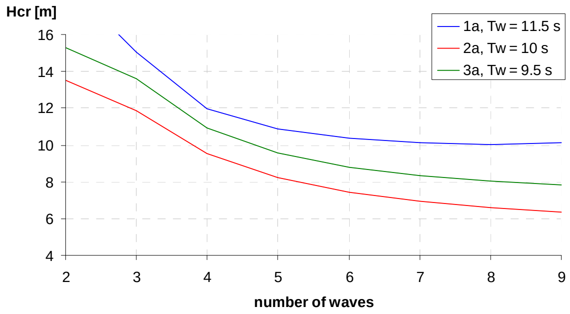


Figure 6: Characteristics of critical wave groups for 40 deg roll angle exceedance.

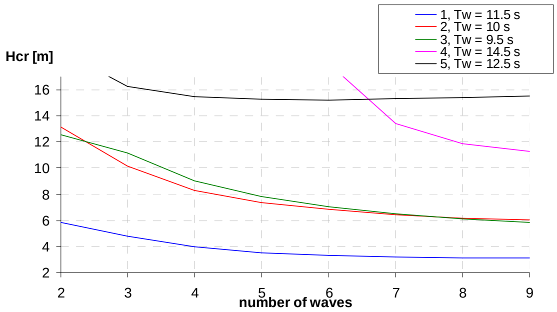


Figure 7: Characteristics of critical wave groups for *g*/2 lateral acceleration exceedance.

In general, exceedance of 40 deg roll angle requires higher waves than those required to exceed *g*/2 lateral acceleration. Figure 6 also indicates that higher waves are required for 40 deg roll angle exceedance in beam seas (case 01a) compared to quartering seas parametric roll (at a low forward speed with *Fr*=0.09 – case 02a, and at a higher forward speed with *Fr*=0.12 – case 03a). Figure 7 indicates that the exceedance of acceleration threshold in beam seas is realised in lower waves (case 01) compared to the two quartering sea parametric roll scenarios 02 and 03.

The critical heights for case 04 (parametric rolling in head seas at low speed) and case 05 (direct excitation in a typical loading condition) are much higher than those in cases 01, 02 and 03. Note that the identified critical wave heights for cases 04a and 05a (these correspond to the 40 deg roll angle threshold) were found to be extremely high even for high run length (they are not shown in Figure 6), and thus their encounter probability should be very low.

## 8. EXCEEDANCE RATE AND HOURLY PROBABILITY OF EXCEEDANCE

To calculate the exceedance rate by the critical wave groups approach, the same JONSWAP spectrum was used as in the Monte-Carlo simulations. Figures 8 and 9 show the calculated exceedance rates per case and threshold (roll angle and lateral acceleration) obtained by the two methodologies, as a function of the inverse significant wave height squared.

For the 40 deg roll angle threshold (Figure 8), case 02a presents the best example of agreement between the two approaches. For case 03a, the results are in a satisfactory

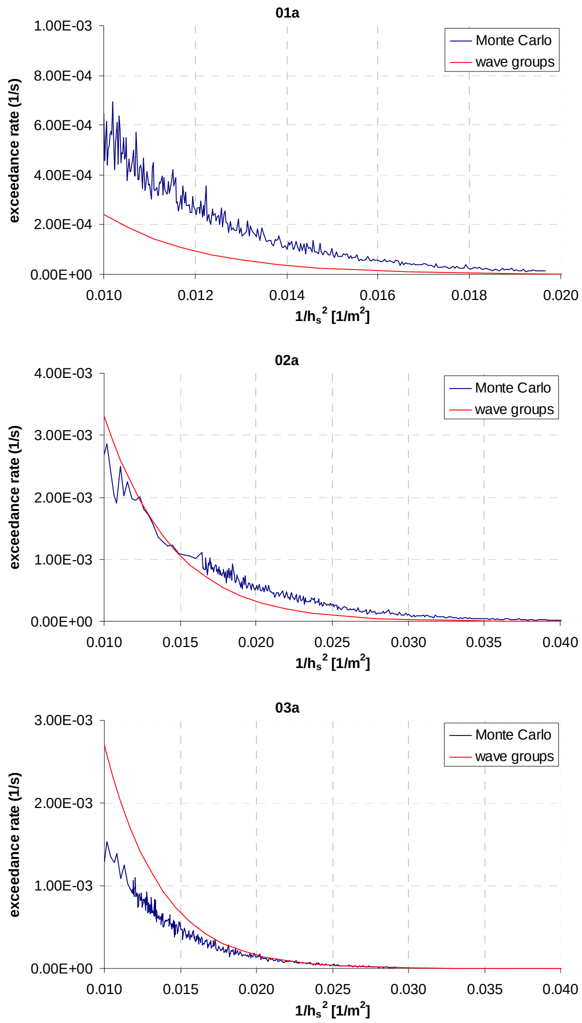


Figure 8: Comparison of exceedance rate of 40 deg roll angle between Monte-Carlo simulations and wave groups approach.

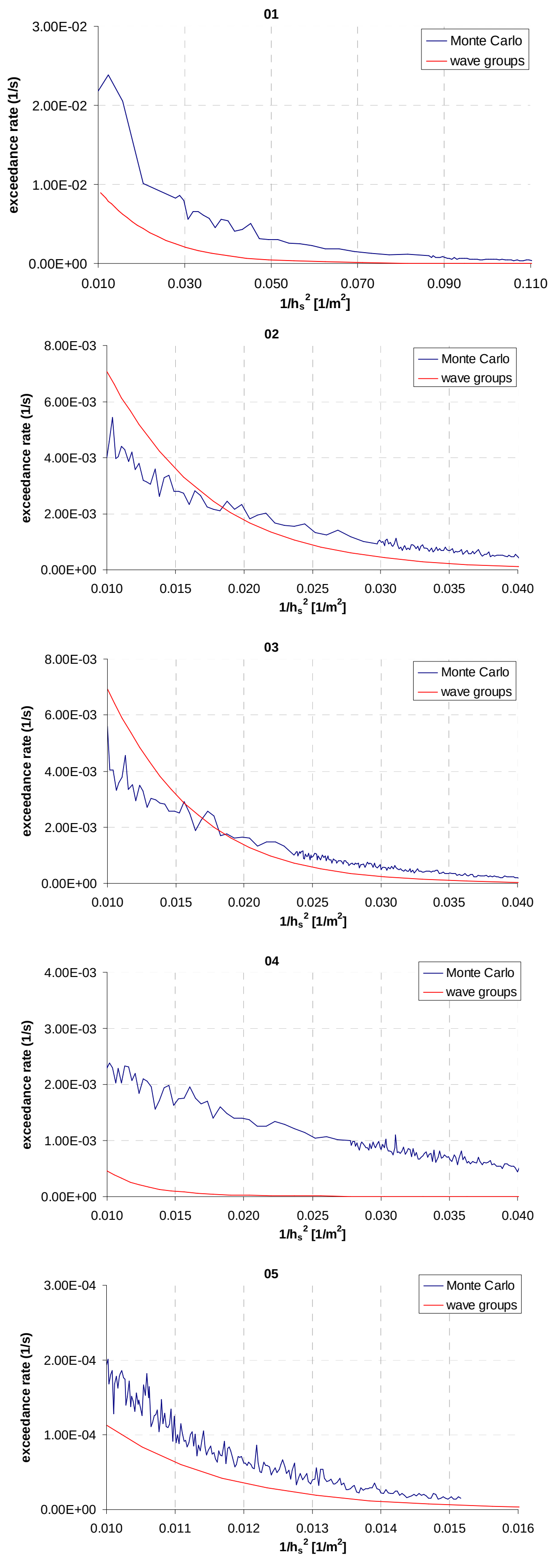


Figure 9: Comparison of exceedance rate of $g/2$ lateral acceleration between Monte-Carlo simulations and critical wave groups approach.

agreement for $1/h_s^2$ larger than 0.015 1/m$^2$, but for the smaller values, the wave groups approach predicts higher rates. On the other hand, the exceedance rates obtained with Monte-Carlo simulations for case 01a are consistently higher than those of wave groups approach.

Figure 9 shows the results related to the acceleration threshold (cases 01 to 05). Case 05 demonstrates the best agreement here, and cases 02 and 03 agree well on average. Moreover it is observed that, for the more severe sea states examined, the “wave groups” approach predicts more frequent exceedances for cases 02 and 03. The largest discrepancies arise for cases 01 and 04, where Monte Carlo simulations predict far more frequent exceedances in the whole range of wave heights.

Knowing the exceedance rate $r$, the probability of exceedance during a given exposure time $T$ (e.g. one hour) can be calculated using Poisson law for the flow of the exceedance events. Both exceedance rate and probability of exceedance per given time have advantages, and offer different comparison viewpoints on the results. The next formula can be used:

$$p(T) = 1 - e^{-rT} \tag{1}$$

The applicability of the Poison flow assumption depends upon the validity of certain assumptions, namely that

- only one event can happen at a given time
- the probability of event happening at a particular time instant is infinitely small
- events are independent of each other.

While the first two conditions are satisfied for roll motion and related processes, the last condition is not, because exceedance events of a certain large roll angle tend to appear in groups. In order to cancel the influence of this strong auto-correlation of roll motion, average estimates of the exceedance time period, derived from multiple realisations of the same seaway, were used in the Monte-Carlo simulations, as proposed by Söding (1987), see Shigunov (2009) for application. Each simulation was continued only until the first exceedance event; then the ship was returned to the upright position, and the simulation was repeated in the same seaway with the new set of random phases, frequencies and directions of seaway components until the next exceedance event. The estimation of the expected exceedance period is found as the average of the exceedance periods obtained in all simulations.

Figure 10 shows the respective results for the 40 deg threshold for each stability failure case. Case 03a seems to show the best agreement between the two approaches in terms of the hourly probability. Larger deviation of the rate occurs at the smaller values of $1/h_s^2$ than 0.015 $1/\mathrm{m}^2$, where the hourly probability is close to 1.0 anyway. Cases 01a and 02a could be also considered as adequately close. The average relative difference between the two methods in terms of $h_s$ corresponding to the same exceedance probability is 9.4% and 7.8% for cases 01a and 02a, respectively.

Figure 11 shows results for the $g/2$ lateral acceleration threshold. Case 05 demonstrates the best agreement between the two methods, similarly to what was observed for the exceedance rate. Relative differences between the two methods in terms of $h_s$ corresponding to the same hourly exceedance probability are 15.6% and 10.45% for cases 02 and 03, respectively.

On the contrary, there is a significant disagreement in the exceedance of acceleration threshold for the beam-sea scenario (case 01) and for the head-sea scenario (case 04). Specifically, for the beam-sea scenario, the 90% exceedance probability within one hour exposure corresponds to significant wave heights 3.31 and 4.78 m for the Monte-Carlo simulations and wave groups method, respectively. For the head-sea scenario, the corresponding wave heights are 5.2 and 10.6 m, respectively. Table 3 shows the significant wave heights corresponding to 90% hourly probability of exceedance for all examined cases.

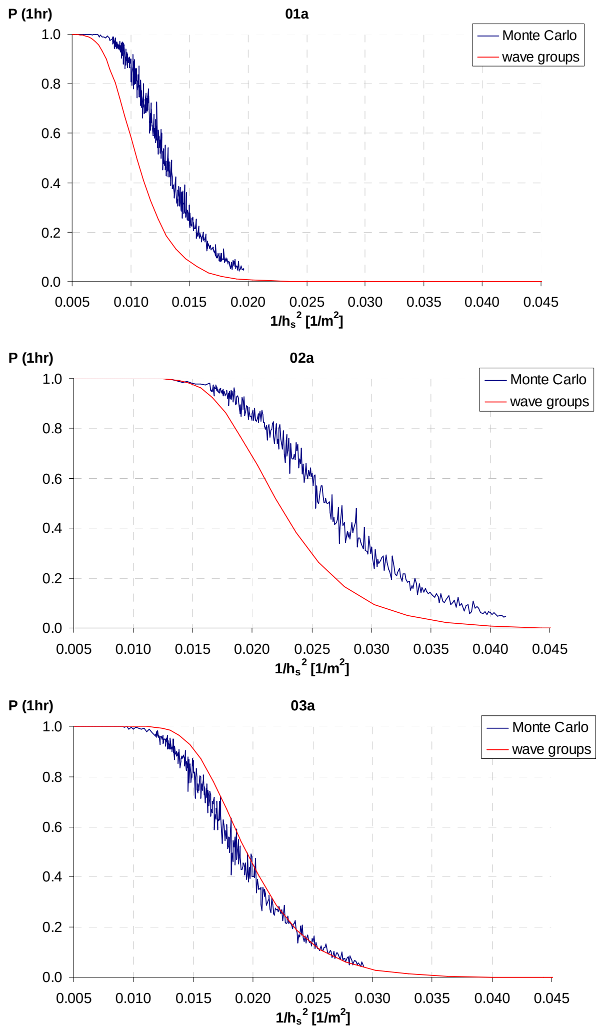


Figure 10: Comparison of exceedance probability per hour of roll angle 40 deg between Monte-Carlo simulations and critical wave groups approach.

The order of the magnitude of probabilities seems to be the same between the two approaches, with the exception, however, of case 04 related to the acceleration threshold in reference to head-sea parametric rolling. Further work will be required in order to identify the reason of the differences in this case.

However it is recalled that a Monte-Carlo method captures the statistic of a process without identifying the phenomenon, whilst the wave groups methods addresses each specific phenomenon individually.

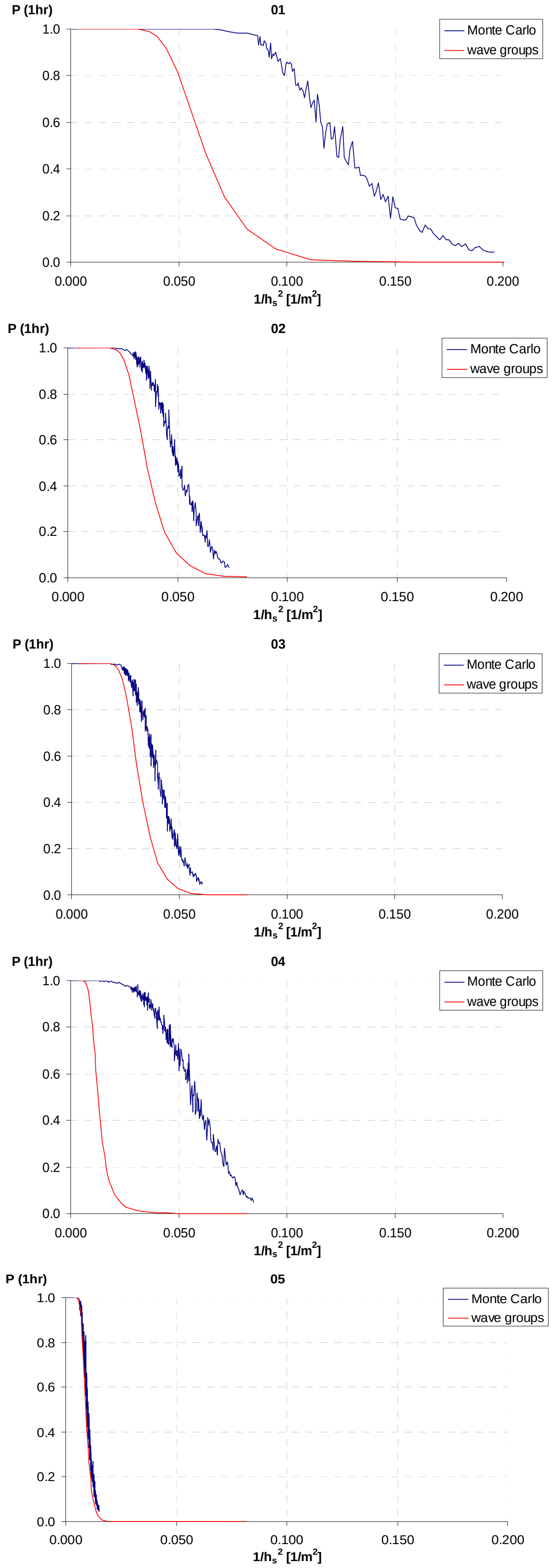


Figure 11: Exceedance probability per hour of *g*/2 lateral acceleration from Monte-Carlo simulations and wave groups approach.

Table 3: Significant wave heights required for 90% hourly probability of exceedance.

| case | Monte-Carlo | wave groups | relative difference |
|---|---|---|---|
| 01a | 10.11 | 11.25 | 10.13% |
| 02a | 7.10 | 7.63 | 6.89% |
| 03a | 8.50 | 8.13 | –4.62% |
| 01 | 3.31 | 4.63 | 28.43% |
| 02 | 5.30 | 6.13 | 13.47% |
| 03 | 5.70 | 6.30 | 9.52% |
| 04 | 5.20 | 10.60 | 50.94% |
| 05 | 11.75 | 11.90 | 1.26% |

For the 40 deg roll angle exceedance criterion, case 02a appears to produce the most frequent exceeedences, while case 01a the least frequent. On the other hand, both approaches indicate that case 01 is the most critical for the acceleration threshold, followed by cases 02 and 03. These trends follow the trend of the required wave heights in Figures 6 and 7.

## 9. CONCLUSIONS

A comparison of exceedance rates and probabilities obtained by Monte-Carlo simulations and the critical wave groups approach has been carried out. Typical stability failure modes have been studied. Thresholds were set in terms of the roll angle and the lateral acceleration. In some cases satisfactory agreement was shown, e.g. for parametric rolling in quartering seas (cases 02 and 03) and direct excitation (case 05), but in some other cases, such as head seas parametric rolling, the difference was not negligible. A factor that can be governing certain discrepancies, especially when these are within an order of magnitude, is the different initial phasing, because in the wave groups method, the ship is assumed initially upright and with a fixed phase with respect to the first wave crest. An assessment of the effect of the initial phase on the results of the wave groups method can be found in Themelis & Spyrou (2008). On the other hand, the average exceedance rate in Monte-Carlo simulations was derived by averaging over a large number of realisations, in each of which the ship was assumed initially upright, but the phases of wave realisations changed randomly.

A source of large quantitative differences, however, can be the fact that Monte-Carlo simulations do not discriminate between phenomena and record threshold exceedance events irrespectively of the underlying causes, which could, in principle, be more than one per realisation. On the other hand, the wave groups method is implemented for each scenario with one specific phenomenon in mind.

A further factor that could produce higher exceedance rate in Monte-Carlo simulations is the possible non-monotonic increase of roll amplitude with time, which can occur due to the passage of a wave group having an intermediate wave with height below the critical one. The critical wave group approach as used in this paper excludes such events, as it sets the same value for the heights of the waves in the group (vector **H** mentioned in section 3). A further study taking into account intermediate variation of the heights of the groups and their associated probabilities of occurrence will consider this effect.

## 10. REFERENCES

Battjes, J. A., and van Vledder, G. Ph., 1984, "Verification of Kimura's theory for wave group statistics", Proceedings of the 10th Int. Conf. on Coastal Engineering, pp. 642-648.

Kimura, A., 1980, "Statistical properties of random wave groups", Proceedings of the 17th Int. Conf. on Coastal Engineering, Sydney, Australia, pp. 2955-73.

Petey, F., 1986, "Forces and moments due to fluid motions in tanks and damaged compartments", Proceedings of the 3rd Int. Conf. on Stability of Ships and Ocean Vehicles STAB'86, Gdansk.

Tayfun, M. A., 1993, "Joint distributions of large wave heights and associated periods", Journal of Waterway, Port, Coastal and Ocean Engineering, Vol. 119, pp. 261-273.

Themelis, N., and Spyrou, K. J, 2007 "Probabilistic assessment of ship stability", SNAME Transactions, Vol. 115, pp. 181-204.

Themelis, N., and Spyrou, K. J., 2008, "Probabilistic assessment of ship stability based on the concept of critical wave groups." Proceedings of the 10th International Ship Stability Workshop, Daejeon, Korea.

Shigunov, V., el Moctar, O., and Rathje, H., 2009, "Conditions of parametric roll", Proceedings of the 10th Int. Conf. on Stability of Ships and Ocean Vehicles STAB 2009, St. Petersburg.

Shigunov, V., 2009, "Operational Guidance for Prevention of Container Loss", Proceedings of the 10th Int. Conf. on Stability of Ships and Ocean Vehicles STAB 2009, St. Petersburg.

Söding, H., 1982, "Leckstabilität im Seegang", Report No. 429, Institut für Schiffbau, Hamburg.

Söding, H., 1987, "Ermittlung der Kentergefahr aus Bewegungssimulationen", Schiffstechnik, Vol. 34, pp. 28-39.

Wist, H., Myrhaug, D., and Rue, H., 2004, "Statistical properties of successive wave heights and successive wave periods", Applied Ocean Research, Vol. 26, pp. 114-136.