# Evaluation of the Probability of Surf-Riding in Irregular Waves with the Time-Split Method

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**ABSTRACT**

Surf-riding is an important phenomenon for the evaluation of ship dynamic stability, as it is related to one of the principal mechanisms of broaching-to, and the evaluation of the probability of surf-riding in irregular waves is a necessary step toward determining the probability of broaching-to after surf-riding. The concept of wave celerity in irregular waves introduces the possibility of developing a probabilistic description of surf-riding. The phenomenon of surf-riding can then be treated as a problem of the exceedance (or upcrossing) of the wave celerity by the instantaneous surging velocity.

To facilitate the probabilistic study of surf-riding, a simple model of surging and surf-riding in irregular waves of variable bandwidth is introduced. This model can be used to identify patterns of surf-riding in irregular waves and to determine the relationship between surf-riding and the appearances and disappearances of the surf-riding equilibria.

**Keywords:** *Surf-riding, Broaching-to, Split-time method*

## 1. INTRODUCTION

The paper focuses on the basic study of surf-riding in irregular waves, and continues from the work presented at the most recent International Ship Stability Workshop (Belenky *et al.*, 2011). The study is oriented toward building a description of surf-riding in irregular waves through which the probabilistic assessment methods discussed above could be effectively applied. A key element of this description is the definition of wave celerity in irregular waves, which is detailed in a separate paper submitted to this conference (Spyrou *et al.*, 2012). The overall objective of this work is to develop a procedure for probabilistic assessment of surf-riding and broaching using the split-time method.

The split-time method has been developed to provide a practical method for assessing the probability of capsizing for an intact ship, and was originally implemented for capsizing in beam seas (Belenky *et al.*, 2008). To handle the extreme rarity of capsizing, the problem was separated into two problems. The first problem ("non-rare") is formulated as an assessment of upcrossing of some intermediate level usually associated with the angle of maximum of the roll restoring (GZ) curve. The second problem ("rare") is the determination of conditions that lead to capsizing given that an upcrossing has occurred, and is associated with exceeding some critical roll rate at the instant of upcrossing. The method is being extended for stern quartering seas where the variation of stability in waves is significant and can lead to a "pure loss of stability" event (Belenky *et al.*, 2010). A key advantage of the split-time method over more customary probabilistic methods is that it can be effectively applied to a finite volume of time-domain data, which allows it to be efficiently used with advanced hydrodynamic simulation codes such as LAMP, the Large Amplitude Motion Program (Shin *et al.*, 2003, Spyrou, *et al.*, 2009). This advantage is shared by other methods that use different logic to separate the dynamic problem into parts, such as the "critical wave groups" method (Themelis and Spyrou, 2007). The strength of the wave group method is its flexibility in terms of the selected level of rigor, thus allowing simplicity. The split-time method, however, may be more robust in terms of reflecting the statistics of initial conditions. Since the methods share the same philosophy of handling rarity (Belenky *et al.*, 2012), they have a potential to complement each other.

Capsizing or large roll angle in stern quartering and following seas may also be caused by broaching-to, which is defined as a violent uncontrollable turn in spite of maximum steering effort applied. One of the possible scenarios of broaching-to is related to surf-riding and realized as a directional instability of one of the surf-riding equilibria (Spyrou, 1996). The application of the "critical wave groups" method would entail finding a set or sets of wave (or wave group) characteristics that lead to broaching-to and then finding the probability of encountering such a wave or wave group.

## 2. SURGING AND SURF-RIDING IN IRREGULAR FOLLOWING WAVES

### 2.1 Equation of Motion

The simplest mathematical model for surging and surf-riding in following irregular waves is a single degree of freedom equation of motion along the x-axis:

$$\begin{aligned}(M + A_{11})\ddot{\xi}_G + R(\dot{\xi}_G) \\ - T(\dot{\xi}_G, n) + F_X(t, \xi_G) = 0\end{aligned} \quad (1)$$

Here $M$ is the ship mass, $A_{11}$ is longitudinal added mass, $R$ is resistance in calm water, $T$ is the thrust in calm water, $n$ is the number of propeller revolutions, $F_X$ is the Froude-Krylov incident wave force, and $\xi_G$ is longitudinal position of the ship's center of gravity in an Earth-fixed coordinate system. The dot above the position indicates a temporal derivative.

For compatibility with Spyrou (2006), polynomial presentations are used for the resistance and thrust in calm water:

$$\begin{aligned}R(U) &= r_1 U + r_2 U^2 + r_3 U^3 \\ T(U, n) &= \tau_1 n^2 + \tau_2 nU + \tau_3 U^2\end{aligned} \quad (2)$$

Since the Earth-fixed coordinate system is used, the irregular waves are presented as a spatial–temporal stochastic process using the standard Longuet-Higgins model:

$$\zeta_W(t, \xi) = \sum_{i=1}^{N} a_i \cos(k_i \xi - \omega_i t + \varphi_i) \quad (3)$$

Here $a_i$ is the amplitude, $k_i$ is the wave number, $\omega_i$ is the frequency, and $\varphi_i$ is a random, uniformly distributed phase shift of the wave component $i$.

As the model is meant at this stage to be qualitative, a linear wave-body formulation is appropriate. Therefore:

$$F_X(t, \xi_G) = \sum_{i=1}^{N} A_{Xi} \cos(k_i \xi - \omega_i t + \varphi_i + \gamma_i) \quad (4)$$

Here $A_{Xi}$ is the amplitude of the component of the surging force, while $\gamma_i$ is the phase shift between the wave and the force components. Details of the surging force calculation can be found in Belenky *et al.* (2011).

## 2.2 Surf-Riding

The physical mechanism of surf-riding includes the appearance of dynamical equilibria and a ship's attraction to the stable equilibrium. The equilibria appear when the wave surging force becomes large enough to offset the difference between the ship's thrust and its resistance at wave celerity. The equilibrium points are the positions of the ship on the wave where the forces balance exactly.

To illustrate this, consider surf-riding in regular waves and plot the variation in the wave-induced surging force as a function of the ship's position on the waves; see Figure 1. In this plot, the horizontal axis is the position of the ship's center of gravity ahead of the wave crest, the dashed blue line is the wave profile, and the red line is the wave surging force, with a negative value indicating a forward (accelerating) force. The largest forward surging force (most negative on this plot) occurs when the ship is running down the wave face, while the magnitude of surging force is a function of wave amplitude.

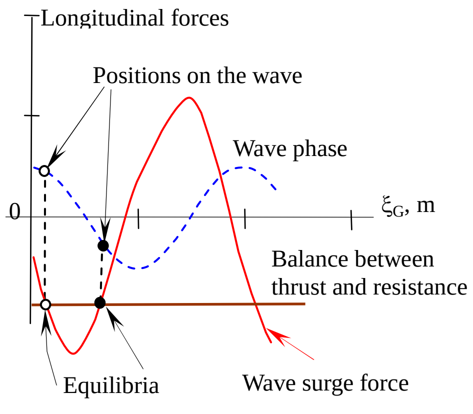



Figure 1: On appearance of dynamic equilibria

Since the commanded speed is insufficient to propel the ship with wave celerity in calm water, additional wave force is necessary to drive the ship at wave celerity. If the amplitude of the wave surging force exceeds the absolute value of the balance between thrust and resistance, two intersection points appear, as shown in Figure 1. Those will be called "surf-riding equilibria" (knowing that this is not an exact condition of equilibrium); one shows stability features (black point, located around the wave trough) and the other behaves as unstable (empty point, located around wave crest).

While these considerations are well-established in the field, they were repeated here to highlight the difference between the regular and irregular waves. If Figure 1 is considered as a snapshot an irregular wave, all the elements of the surf-riding problem can be readily transferred from regular waves into irregular, except for wave celerity. How can celerity be defined in irregular waves?

## 2.3 Celerity of Irregular Waves

The very definition of celerity in irregular waves is actually a very deep problem and is given full consideration in Spyrou *et al.*, (2012), which discusses the formulation of a practical definition of wave celerity and the implementation of schemes for evaluating it for theoretical and numerical analysis. An extremely simplified version of the approach is used here, in which the local celerity is defined by identifying the three profile zero-crossing points that are closest to the ship and tracking their movement from time step to time step, as illustrated in Figure 2.

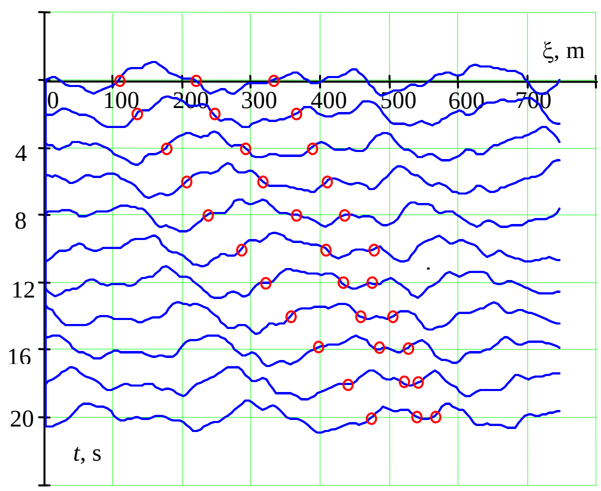



Figure 2: Zero-crossing points in space and time

Such a definition attempts to provide the speed associated with a particular wave face. Since the zero crossing points, like other individual waves and wave features, have a finite time of existence, the celerity calculated by this approach is not continuous. Nevertheless, it is sufficient for this paper's objectives.

## 3. NUMERICAL STUDY

### 3.1 Objectives

The previous numerical study (Belenky *et al.*, 2011) has shown some sort of surf-riding behavior in irregular waves, which was most pronounced for cases using "filtered" irregular waves with limited bandwidth. Since the principal objective of this numerical study is to see the relation between the visible surf-riding behavior and appearance of equilibrium, irregular waves with a very limited bandwidth – two and three frequency components – are used.

### 3.2 Two-Component Wave Model

Figure 3 shows a spectrum for the two-component waves used in the study. While these waves look nearly regular to the naked eye (see Figure 4), the duration of surf-riding in these waves is no longer unlimited as for regular waves.

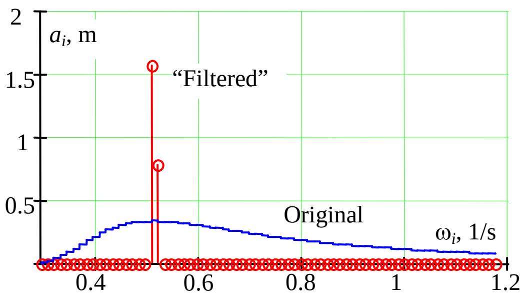



Figure 3: Two-component wave spectrum

As shown in Belenky *et al.*, (2011), a ship "caught" by the wave at certain time is later "released." To see how this is related with the appearance and disappearance of the surf-riding equilibria, the wave celerity has been estimated by tracking the zero crossing points as described above and illustrated in Figure 4.

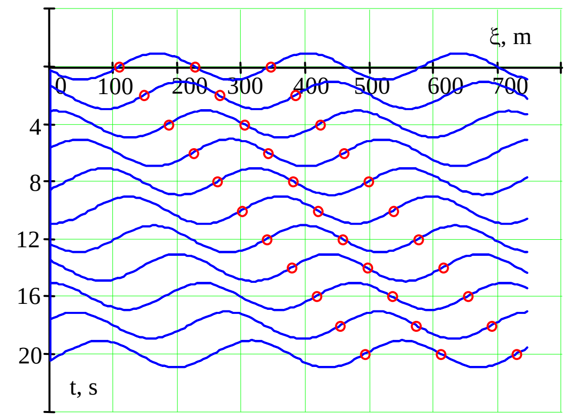



Figure 4: Zero-crossing points of two-component wave

The time history of the celerity of the wave tracked in Figure 4 is shown in Figure 5 along with values calculated for the previous and the next wave. The change of the celerity over time has a magnitude of about 0.6 m/s. The celerities of the previous and next waves experience similar changes, but at different times.

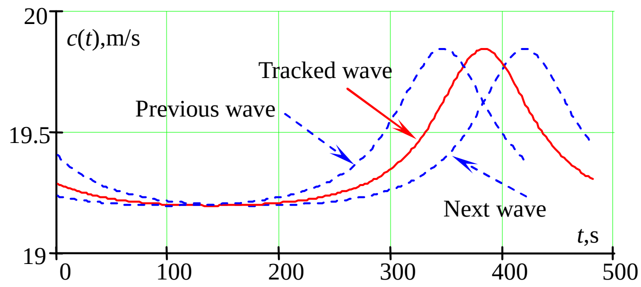



Figure 5: Celerity of the two-component wave tracked in Figure 4, along with the celerities of the previous and next waves

Figure 6c shows a time history of the instantaneous ship speed and the estimated wave celerity for a ship modeled using Equations (1) through (4) and running in this two-component wave. While the thrust is set so that calm water speed is below wave celerity, the initial speed was set to wave celerity, so the ship is immediately captured.

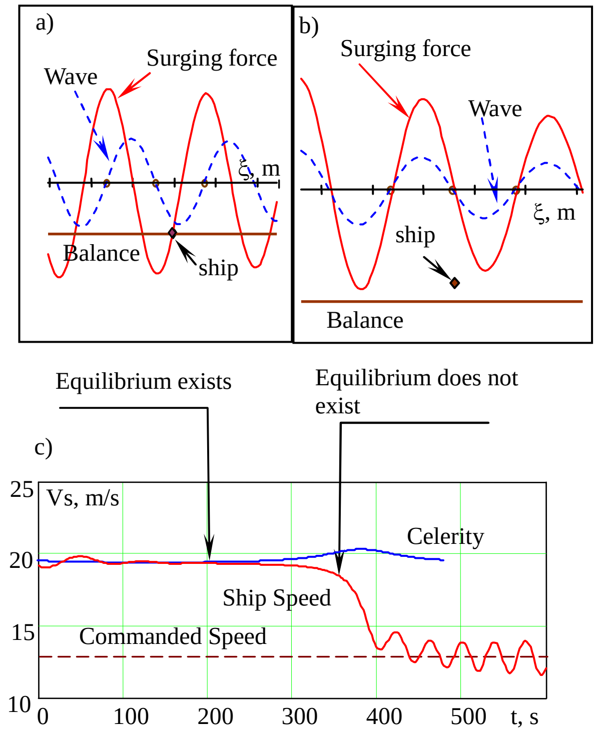



Figure 6: Release from surf-riding in two-component waves:
(a) "spatial snapshot" at 200 s;
(b) "spatial snapshot" at 340 s;
(c) time history

Some decaying oscillations are observed during the first 100 seconds, which is typical for an attraction to the equilibrium. The "spatial snapshot" in Figure 6a shows the existence of the equilibria and the ship located close to one of them. This is, indeed, the stable equilibrium, which is located near the wave trough, as can be seen from the spatial wave profile superimposed on the plot.

At around 250 s, the time histories of the wave celerity and instantaneous ship speed diverge, after which the ship is released from surf-riding and experiences periodic surging. The reason of the release is, most likely, the increase of the wave celerity observed in Figure 5. As seen from the "spatial snapshot" in Figure 6b, the surf-riding equilibria do not exist at this time because the wave can no longer generate a large enough surge force to propel the ship at celerity. The higher wave celerity increased the resistance and the level of balance between the resistance and thrust went down.

Note that the estimated wave celerity is *not* part of the surging / surf-riding calculations. It has been evaluated independently from the same wave field that was used for motion calculations. The indications of the equilibria's appearance and disappearance are therefore independent interpretations of the observed phenomena. At the same time, the interpretation based on the estimate of celerity and observed behavior of the dynamical system are consistent. This consistency suggests that that estimate of the celerity is, to some degree, valid (Spyrou *et al.*, 2012).

### 3.3 Three-Component Wave Model

Figure 7 shows the spectrum for a three-component model of the irregular wave. While still very simplistic, simulations using this wave and the same simplified ship surging model show three transitions – two captures and one release – over the passing of six waves. The time histories of the celerities of these six waves are shown in Figure 8.

Along with larger (about 1 m/s magnitude) changes in wave celerity, there are quite dramatic peaks with three secondary peaks on the top. These secondary peaks may be artifacts of the simplified wave tracking scheme and/or results of waves overtaking one another. Further studies of this and similar effects are described in Spyrou *et al.*, (2012).

Figure 9a shows the time history of the instantaneous ship speed and the wave celerity of the "current" wave, the wave closest to the ship, at any given time. One can see that the celerity curve at Figure 9a is a combination of all six time histories in Figure 8, and can be discontinuous as the current wave changes.

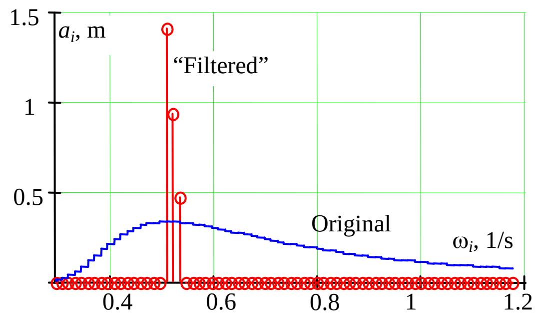



Figure 7: Three-component wave spectrum

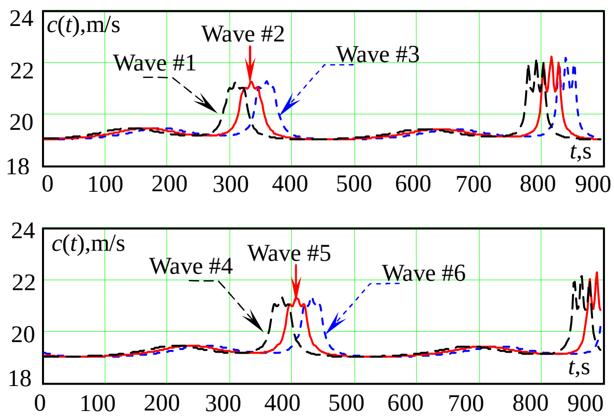



Figure 8: Time histories of wave celerities

Figure 9a also shows the temporal boundaries for waves. During periodic surge motion, they coincide with downcrossing the commanded speed line. Figures 9b through 9r are the "spatial snapshots" corresponding to specific instant of time as noted in the captions. These instances of time are identified in Figure 9a with lettered arrows referring to the respective "spatial snapshot."

On each "spatial snapshot" plot, the blue line shows the wave profile around the ship, with the horizontal position of the diamond marking the ship's position relative to the wave. The circles on the wave profile mark three zero-crossing points that are tracked to estimate wave celerity, of which the outermost circles define the spatial boundary of the "current" wave. The direction of the ship (and wave) motion is to the right.

The vertical position of the diamond indicates the ship speed, while the middle line (0 wave) marks the commanded speed (calm water) and higher speeds are down.

Each special snapshot also contains a plot for surging wave force (red curve) and the balance between the available thrust and resistance at the current wave celerity in the same scale (lower line).

The first spatial snapshot, Figure 9b, corresponds to the initial conditions, with the instantaneous speed equal to the commanded speed. The ship has just encountered wave #1 and is located just within its boundary. The surf-riding equilibria exist, since the surging force crosses the line corresponding to the balance between the thrust and resistance.

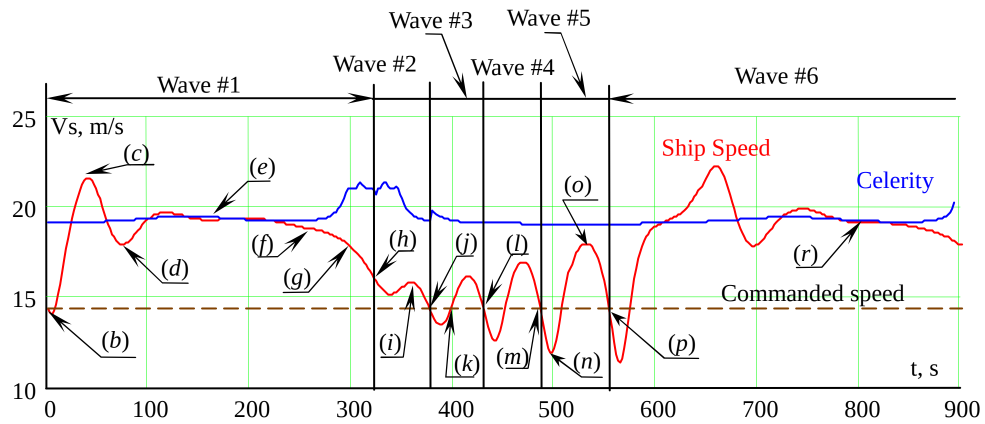
The stable surf-riding equilibrium attracts the dynamical system and one oscillation period is seen in Figure 9a until approximately t=100s. The next two spatial snapshots, Figures 9c and 9d, correspond to the positive and negative peaks during this transition, respectively. The transition is completed and the dynamical system reaches the stable equilibrium at around t=150 s, in Figure 9e.

Looking at Figures 9b through 9e, one can see that the amplitude of the surging force is decreasing due to lower wave amplitude. This tendency leads to the disappearance of the surf-riding equilibria around t=256 s and to the release of the ship from surf-riding (Figure 9f).

The ship slows down (Figure 9g), the wave #1 overtakes her, and wave #2 is encountered at around t=325 s (Figure 9h). The ship experiences the first almost periodic surge with the positive peak corresponding to the spatial snapshot in Figure 9i. As expected, wave #2 overtakes the ship quite quickly and wave #3 is encountered around t=380 s (Figure 9j).

The modulation of wave amplitude and surging force then reverses and they begin to increase. This is may be already seen in Figure 9h, but becomes quite apparent in Figures 9i and 9j. New surf-riding equilibria appear around t=398 s (Figure 9k).

a)

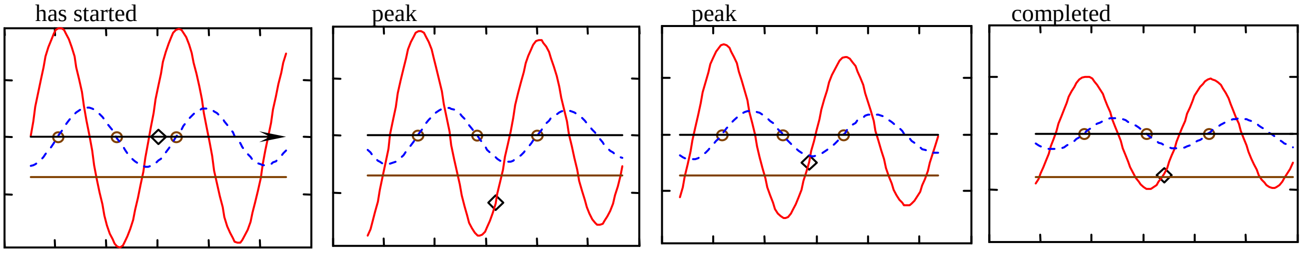



b) t=0 s, wave #1, the equilibrium exist, attraction has started

c) t=40 s, wave #1, transition to the equilibrium, positive peak

d) t=74 s, wave #1, transition to the equilibrium, negative peak

e) t=150 s, wave #1, transition to the equilibrium has been completed

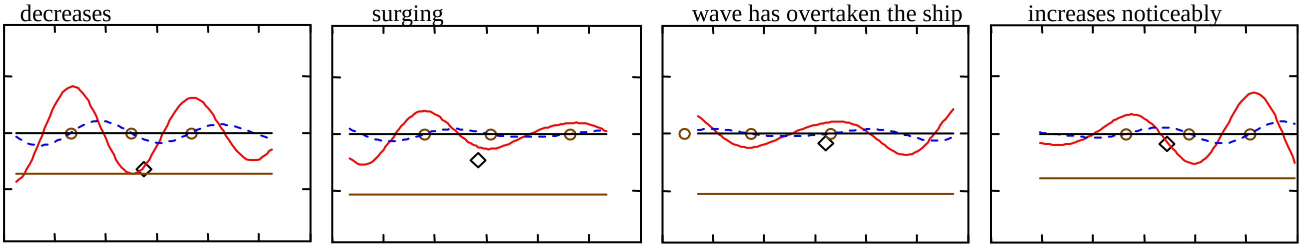


f) t=256 s, wave #1, equilibrium disappears, the surging force decreases

g) t=300 s, wave #1 overtakes the ship, transition to surging

h) t=326 s, wave #2 is encountered, the previous wave has overtaken the ship

i) t=360s, wave #2 overtakes the ship, the surging force increases noticeably

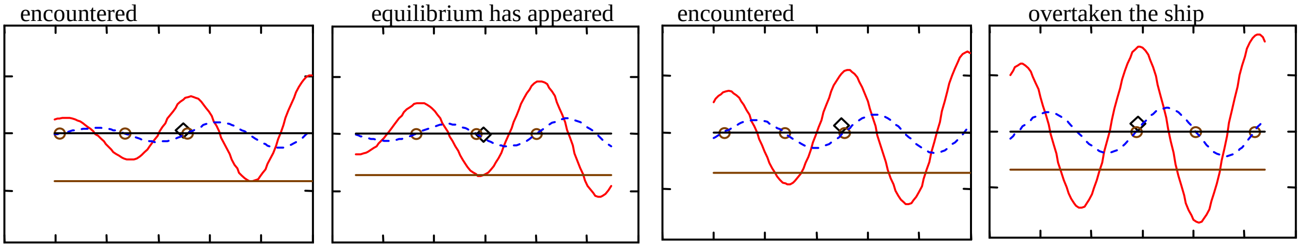


j) t=380 s, wave #3 is encountered

k) t=398 s, wave #3, the equilibrium has appeared

l) t=434 s, wave #4 is encountered

m) t=490 s, wave #4 has overtaken the ship



n) t=496 s, wave #5, negative peak in periodic surging

o) t=530 s, wave #5, passing by the unstable equilibrium

p) t=554 s, wave #5 takes over the ship

r) t=800 s, wave #6, transition to the equilibrium has been completed

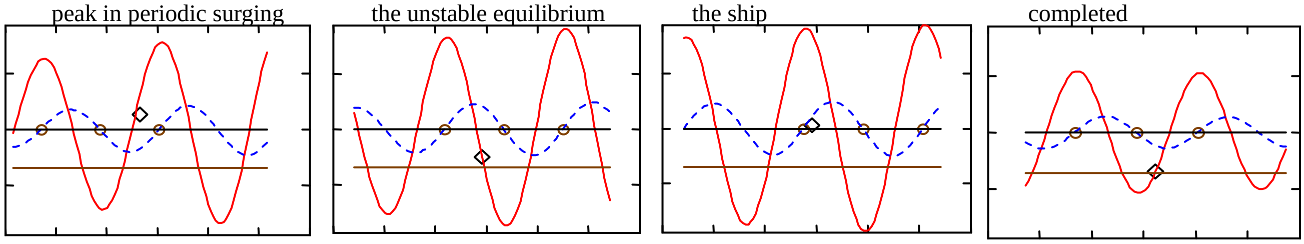


Figure 9: Two captures and one release from surf-riding in a three-component irregular wave; time history (a) and “spatial snapshots” (b-r)

The existence of the surf-riding equilibria has an immediate influence on the surge motions, which become asymmetric with wider positive peaks and sharper negative ones (Spyrou, 2006). Symmetry is observed during the passing of waves #4 and #5, during which the surf-riding equilibria exists continuously (Figures 9l through 9o). Figure 9o shows how the dynamical system passed near the unstable surf-riding equilibrium, but the ship is not yet “caught” and wave #5 takes over (Figure 9p). The ship is finally “caught” by wave #6 and at around t=800 again reaches the stable surf-riding equilibrium (Figure 9r).

### 3.4 Conclusion of Numerical Study

In the analysis of the numerical simulations of surf-riding using a 1 DOF surge equation, an evaluation of wave celerity based on the tracking of wave zero-crossing points was successfully used to reveal the existence or non-existence of surf-riding equilibrium, which was then able to explain the ship’s transition into and out of surf-riding in irregular seas. This ability to characterize the behavior of the dynamical system from these equilibria allows consideration of a probabilistic formulation for surf-riding in irregular waves.

## 4. POSSIBLE FORMULATION FOR THE NON-RARE PROBLEM

This numerical study demonstrated the possibility of quantifying surf-riding equilibria in irregular waves by considering the estimated celerity of a local wave feature and the variation of wave forces for the spatial domain within the boundaries of a “current wave.” The existence of the surf-riding equilibrium can then be formulated as an upcrossing problem of the following process:

$$X(t) = F_X\left(\xi_G(t)\right) - R\big(c(t)\big) \tag{5}$$

The level of upcrossing is defined by the commanded speed defined through the number of propeller revolutions and expressed through the thrust in calm water.

The existence of the surf-riding equilibria is a necessary, but not sufficient condition of surf-riding. Similarly, surf-riding is necessary, but not sufficient condition of the considered type of broaching. Furthermore, the act of broaching-to following surf-riding may or may not induce a dangerous roll angle. It is therefore reasonable to expect that the split-time method may include several rare problems: surf-riding if the equilibria exist, broaching if surf-riding occurs, and then large roll angle or capsizing if broaching occurs. Formulation of these problems belongs to future work.

## 5. CONCLUSIONS

The work presented in this paper is aimed towards developing an understanding of surf-riding behavior in multi-frequency waves, understanding that could enable the application of the split-time and critical wave group methods for evaluating the probability of capsizing (or large roll angle) due to broaching-to following surf-riding.

The attempt to describe surf-riding in a probabilistic framework has led to the necessity of defining wave celerity in irregular waves. The formulation of a viable definition of wave celerity in irregular waves and the development of robust methods for evaluating it in numerical theoretic analysis is a quite substantial task and represents its own area of research. For the present study, a simple definition was adopted, in which the wave celerity was defined as a speed of zero-crossing points of a wave and evaluated using a point tracking scheme.

This approximate definition of wave celerity in irregular waves nevertheless allowed the characterization of the surf-riding equilibria (or the lack thereof) for numerical case studies involving irregular waves with two and three components. These numerical studies showed that an almost complete explanation of the dynamical behavior of surf-riding can be presented in terms of the surf-riding equilibria.

Based on these results and considerations, a non-rare problem has been postulated for the appearance of surf-riding equilibria in irregular waves that can be related to an upcrossing of a process of the difference between wave surging force and the ship resistance at the speed equal to the wave celerity in irregular waves. The non-rare problem can be considered as the first step in the development of a split-time method for the probability of capsizing after broaching-to following surf-riding.

## 6. ACKNOWLEDMENTS

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