A COMPARISON BETWEEN THE YAW AND ROLL DYNAMICS IN ASTERN SEAS AND THE EFFECT OF NONLINEAR SURGE ON CAPSIZE

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ABSTRACT

The paper is consisted of two parts. In the first part we consider in parallel the roll and the yaw dynamics in astern seas and we point out a number of interesting analogies, and also a few differences, between these two modes. In the second part we concentrate more on the roll dynamics presenting some of the results of our latest research into ship capsize due to the parametric and the pure-loss mechanisms. We present for the first time graphs showing the quantitative effect of surging for capsize. Other aspects considered are, the effect of restoring modulation based on two frequencies and the dynamic effect of an initially hardening restoring.

1. INTRODUCTION

As is well-known, fluctuations of the roll righting-arm in large following waves can result in ship roll instability and capsize according to the pure-loss or the parametric mechanism [1]. Fluctuations of a similar nature, concerning the motion's stiffness term, may take also place in yaw, originating from the combined effect of rudder control with the wave induced yaw moment. This could give rise to course instability resulting in deviation from the desired heading and broaching [2].

Consider a ship operating in long following sinusoidal waves. In order to avoid coupling complications let us assume further that, due to high natural frequencies in heave and in pitch compared to the encounter frequency, the ship can maintain a state of quasi-static equilibrium on the vertical plane. If the waves are relatively steep, the geometry of the submerged part of the hull will vary according to the position on the wave. This is likely to be reflected in roll's righting-arm, with a reduced or even negative roll restoring arising when the middle of the ship is near to a wave crest. If roll restoring remains negative for sufficient time, so that heel finds the time to develop unopposed from some small initial angle up to levels well beyond the vanishing angle, then capsize due to the so-called pure-loss of stability mechanism will be realised [3]. In this case the magnitude of roll damping affects little the survivability of the ship. Capsize can occur of course also in a typical parametric resonance fashion and here the magnitude of damping will be a much more critical factor [4]. Practically, the variation of restoring must be however quite intensive, so that the large-amplitude roll motion could build-up within a small number of wave cycles [5].

Considering a similar wave environment, the onset of yaw instability represents a slightly more complex process; because yaw is always coupled with sway and also the control law of the rudder bears a serious influence on the dynamics. Unlike with roll, in the absence of active rudder control no restoring yaw moment can exist in still water. However in waves the movement of the rudder tends to bring the ship back on the correct course. If waves with a length equal to the ship length or longer meet the ship from behind, they will create a yaw moment which will be dependent upon the angle between the direction of wave propagation and the ship's heading. This wave yaw moment works as a positive restoring component when the ship passes from a wave crest (stabilizing effect). The opposite will be happening in the vicinity of a trough because the wave will tend to bring the ship vertical to the direction of wave propagation. It results that the comparative strengths of the rudder and wave yaw moments will give rise to a restoring fluctuation of a certain amplitude which may sometimes have the potential to destabilize the horizontal-plane motion of the ship. The commonality of the underlying dynamics of yaw and roll becomes therefore prevalent.

Our first objective in this paper is to identify the correspondence between yaw and roll parameters from the perspective of these Mathieu-type phenomena. Furthermore, we shall attempt to introduce an approach for assessing a number of additional, and often very influential, effects. Most important of these seems to be the effect of surge motion. As is well known, when the waves are large, the nonlinearity of surge cannot be neglected [6]. A manifestation of this nonlinearity is a virtual rescaling of time as the ship is spending longer time on the crests than on the troughs of the waves. Despite the significance of this mechanism for the yaw and roll motions, there has been no earlier assessment of its effect for ship survivability and safety.

2. A SIMPLE MODEL OF YAW MOTION IN ASTERN SEAS

Consider the linear differential equations of sway and yaw [7], with the addition of wave excitation terms at their right-hand side:

Sway: $$(m' - Y'_{\dot{v}})\dot{v}' - Y'_v v' + (m'x'_G - Y'_{\dot{r}})\dot{r}' + (m' - Y'_r)r' = Y'_\delta \delta + Y'_{(wave)} \quad (1)$$

Yaw: $(m'x'_G - N'_{\dot{v}})\dot{v}' - N'_v v' + (I'_z - N'_{\dot{r}})\dot{r}' + (m'x'_G - N'_r)r' = N'_\delta \delta + N'_{(wave)}$ (2)

In the above v', r' are respectively sway velocity and yaw angular velocity, δ is the rudder angle, m' is ship mass and x_G is the longitudinal position of the centre of gravity; $Y'_{\dot{v}}, Y'_{\dot{r}}, N'_{\dot{v}}, N'_{\dot{r}}$ are acceleration coefficients (added masses/moments of inertia) and Y'_v, Y'_r, N'_v, N'_r, Y_δ are velocity coefficients (hydrodynamic damping terms). The wave's sway force and yaw moment are respectively, $Y'_{(wave)}$, $N'_{(wave)}$. The prime indicates nondimensionalised quantity and the dot differentiation over time. We use ship length L for length scaling, ρL^3 for mass and L/U for time.

At first instance we shall assume that the yaw and sway velocities are restrained from building-up to high values (thus they remain small and the resulting damping forces may be considered linear) through use of appropriate rudder control. Additionally, ship behaviour is examined at "some distance" from the region of surf-riding, so that, for this first part of the paper, surge velocity is nearly constant. Then, we express the wave terms $Y'_{(wave)}$, $N'_{(wave)}$ in respect with the frequency of encounter (rather than as functions of absolute wave frequency and position):

$$Y'_{(wave)} = Y'_w \psi \sin(\omega'_e t') \quad (3)$$
$$N'_{(wave)} = N'_w \psi \cos(\omega'_e t') \quad (4)$$

The following notation is applied: Y'_w, N'_w are wave force/moment coefficients; ψ is the ship's heading relatively to the wave ($\psi = 0$ when the sea is exactly following – generally, ψ is assumed small).

Consider further rudder control with a linear law based on two gains, k_1 and k_2: k_1 multiplies the instantaneous heading deviation from the desired course ψ_r, while k_2 multiplies yaw's angular velocity:

$$\delta = -k_1(\psi - \psi_r) - k'_2 r' \quad (5)$$

Substituting (3), (4) and (5) in (1) and (2), uncoupling yaw from sway and using well known expressions for system gain and time constants, K', T'_1, T'_2, T'_3 [8], the following differential equation of heading angle is obtained:

$$\dddot{\psi}' + b\ddot{\psi}' + p[1 + f\cos(\omega'_e t' - \sigma)]\dot{\psi}' + q^2[1 - h\cos(\omega'_e t' - \rho)]\psi = j \quad (6)$$

The above third-order differential equation has time-dependent coefficients in two places. As is well known however, if T'_1 is much greater than T'_2 and T'_3, we can use the so-called simplified yaw response model of Nomoto [8]. In that case the order of equation (6) is reduced by one:

$$T'\ddot{\psi}' + \dot{\psi}' = K'\delta + A'\psi\cos(\omega_e' t' - a) \quad (7)$$

K', T' are respectively system gain and time constants, ψ is relative heading angle (assumed small), δ is rudder angle, A' is wave excitation amplitude, ω_e' is the encounter frequency and a is a phase angle.
Coupling of (7) with the autopilot (5) and dropping for simplicity of the phase angle a, leads to:

$$\ddot{\psi}' + \gamma\dot{\psi}' + \omega_{0(yaw)}'^{2}[1 - h\cos(\omega_e' t')]\psi = j \quad (8)$$

In the above $\omega'_{0(yaw)} = \sqrt{k_1 K'/T'}$, $\gamma = (1 + k_2' K')/T'$ (damping), $h = A'/k_1 K'$ (amplitude of parametric variation of restoring), $j = k_1 K'\psi_r/T'$. It is easily recognized that (8) is Mathieu's equation with the addition however of bias-like external static forcing term, j.

For stability, positive T' is required as $1/T'$ is the inverse of the damping of the unsteered vessel. However, large positive T' implies slow convergence towards the corresponding steady rate-of-turn which is determined by the value of the static gain K'. A trend exists for large T' to appear in conjunction with large K' which gives a nearly straight-line *spiral curve*. The effect of active control on damping is represented by the quantity $k_2' K'/T'$. It depends thus on the yaw rate ("differential") gain term in the autopilot. If $T' < 0$, suitable choice of k_2' can turn the damping of the system positive since k_2' multiplies the positive quantity K'/T', thereby yielding stability for the steered ship in calm sea. The wave effects are lumped into the restoring and independent-periodic-forcing terms since the quantities K' and T' were assumed to be, at first approximation, unaffected by the wave. If the amplitude of wave excitation A' exceeds $k_1 K'$, then on the basis of (8) negative yaw restoring will arise around the trough. Should the duration of operation under negative restoring be long enough, undesired turning motion will be initiated ("broaching"). From a dynamics perspective there is complete equivalence with a capsize event of the so-called "pure-loss" type. It can be avoided if the proportional gain k_1 is chosen to be always greater than A'/K' even for the most extreme wave environment where the ship will operate (it should be a matter of further investigation to what extend this is technically feasible). A notable difference between the manifestation of this instability in roll and in yaw is that in roll it arises near the crest of the wave, whereas in yaw the ship becomes vulnerable near a trough.

We may rewrite (8) on the basis of heading error $\psi_1 = \psi - \psi_r$ and then apply the transformation $s = \omega'_{0(yaw)} t'$:

$$\frac{d^2\psi_1'}{ds^2} + 2\zeta\frac{d\psi_1'}{ds} + [1 - h\cos(\Omega s)]\psi = f\cos\Omega s \quad (9)$$

where $\Omega = \omega'_e/\omega'_{0(yaw)}$, $h = A'/(k_1 K')$and $f = A'\psi_r/(T'\omega'^{2}_{0(yaw)})$. The damping ratio is given by the expression: $2\zeta = (1+k_2K')/\sqrt{k_1K'/T'}$ (the presence of k_1 should be noted in ζ).

3. COMPARISON WITH THE ROLL EQUATION

It is obvious from (9) that parametric instability of yaw may also arise, very much like that of roll. To establish the analogy we remind that the generic equation of roll in a "following" sea linearised in terms of the roll angle is:

$$\frac{d^2\varphi'}{d\tau^2} + 2\zeta\frac{d\varphi'}{d\tau} + [1 - h\cos(\Omega\tau)]\varphi' = 0 \tag{10}$$

φ' is the scaled roll angle, $\varphi' = \varphi/\varphi_v$, with φ the true roll angle and φ_v the angle of vanishing stability. Also, $\tau = \omega_{0(roll)}t$. The damping ratio is given in this case by, $2\zeta = B\omega_{0(roll)}/(M\,g(GM))$ where B is the dimensional linear damping coefficient, M is ship mass and (GM) is the metacentric height. Roll's natural frequency is $\omega_{0(roll)} = \sqrt{Mg(GM)/(I+\Delta I)}$. With substitution of $\omega_{0(roll)}$ we may obtain further, $2\zeta = B/\sqrt{(I+\Delta I)M\,g(GM)}$. Also, the amplitude of the parametric is $h = \delta(GM)/(GM)$ where $\delta(GM)$ is the difference of metacentric height at the crest from the still water value. This is a common assumption but of course it results in a highly idealised formulation because the average (GM) has no reason to be identical with the still water (GM). In addition, the variation from trough to crest may not be sinusoidal.

Conditions for "exact" resonance

Let us neglect for a while the damping terms of (9) and (10) in order to find out how the Froude number of the vertex of the principal resonance varies between roll and yaw. For overtaking waves the frequency of encounter, ω_e, is positive and the condition of exact resonance is written as: $\omega_e/\omega_0 = 2/n$, $n = 1,2,3,\ldots$ where ω_0 can be the frequency of encounter of either yaw or roll. Thus with increasing n the vertices will accumulate near to zero frequency of encounter. For $\psi_r = 0$ we can write, $\omega_e = (2\pi/\lambda)(c-U)$.

In yaw time is commonly nondimensionalised on the basis of U/L (noted the resulting time dependence). Therefore as far as yaw is concerned, $\omega'_e = 2\pi L(c-U)/(\lambda U)$. Then, with the substitutions $\omega'_e = 2\omega'_{0(yaw)}/n$ and $c/U = Fn_{wave}/Fn$ where Fn_{wave} is the Froude number corresponding to wave celerity, are obtained the parametric equation of the vertices of the corresponding undamped system: $Fn = Fn_{wave}/\left(1+\lambda\omega'_{0(yaw)}/n\pi L\right)$,. Given that $Fn_{wave} = \sqrt{\lambda/(2\pi L)}$ we arrive finally at the expression:

$$Fn = \sqrt{\lambda/(2\pi L)}\Big/\left[1+\omega'_{0(yaw)}\lambda/(n\pi L)\right] \qquad (11a)$$

Considering the domain of variation of the natural frequency of yaw, $\omega'_{0(yaw)} = \sqrt{k_1 K'/T'}$, for conventional ships the ratio K'/T' is usually within the range $[0.3-1.4]$ (see for example [9]). As a matter of fact $\omega'_{0(yaw)}$ should lie in the range $[0.55\sqrt{k_1}$ - $1.18\sqrt{k_1}]$. With a proportional gain k_1 between 1.0 and 2.0, $\omega'_{0(yaw)}$ should be between 0.55 and 1.67. For roll on the other hand, the natural frequency is nondimensionalised on the basis of ship length and acceleration of gravity, $\omega'_0 = \omega_0\sqrt{L/g}$. The different nondimensionalisation results in a different parametric expression of the critical Froude number:

$$Fn = \frac{\sqrt{\frac{\lambda}{L}}}{\sqrt{2\pi}} - \frac{\omega'_{0(roll)}\left(\frac{\lambda}{L}\right)}{\pi n} \qquad (11b)$$

Relative magnitudes and effect of damping

Even when a ship is equipped with bilge keels, the damping ratio is usually quite low. Noted that the roll damping ratio will change if the position of the centre of gravity is varied. As far as yaw is concerned, the damping ratio depends strongly on the autopilot's gains. Common values are in the range $0.8<\zeta<1.0$ [10]. Here lies therefore another significant difference between the roll and yaw equations: The yaw damping ratio is normally very large. As a matter of fact, in order to be placed in one of the regions of resonance, the loss of yaw restoring at the trough should be very considerable. Usually this means that very steep waves will be required. Unfortunately, for such highly damped motion it is not easy to derive simple expressions for the critical parametric amplitude. Even the expression of Gunderson, Rigas & Van Vleck [11], which is applicable also for large damping values, would not work as ζ approaches 1.0:

$$h = \left(1-\zeta^2\right)\tanh\left(\pi 2\zeta\sqrt{\frac{\omega_0^2}{\omega_e^2}}\right) \qquad (12)$$

Nonlinearity

The roll behaviour considering the nonlinearities that in reality exist in the restoring (strong) and in damping (mild) is relatively well understood as it has been studied by a number of investigators, for example [12-15]. For the yaw equation, if the autopilot is relatively effective the nonlinearities will reside mainly in damping.

4. SOME OTHER ASPECTS OF PARAMETRIC ROLLING

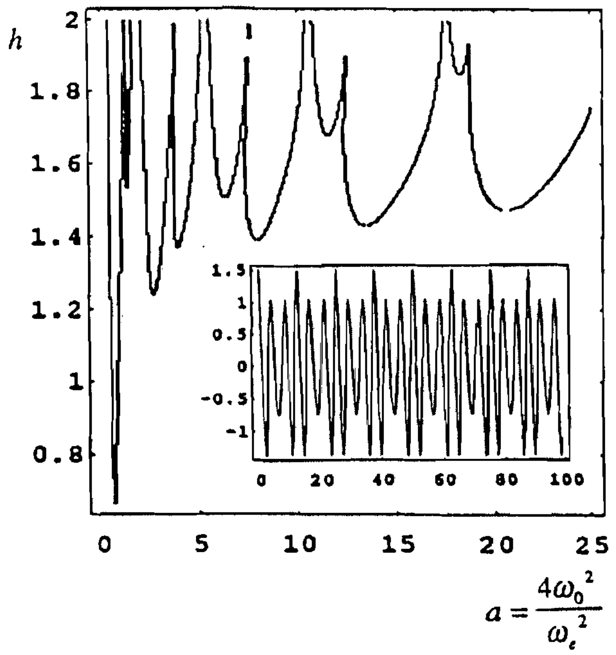

Figure 1: Parametric with two incommensurate frequencies. The inserted graph shows a time realisation of the time-dependent restoring part.

Bi-chromatic waves

Several aspects of parametric rolling have been considered recently [5]. One of these is the effect of having a quasi-periodic variation of (GZ) rather than the ordinary periodic one. The consideration of a Mathieu-type equation with internal forcing based on a single frequency represents a highly idealised scenario. In Fig. 1 are shown the stability transition curves when two independent frequencies are present. This particular graph was drawn with the second frequency being 75% of the first. Also, the amplitude of parametric forcing at the second frequency was 3 times that of the first. It is noticed that secondary "spikes" have grown on each primary resonance region. Further research has shown that their number tends to increase as the second frequency departs from the first.

Hardening restoring

Another matter considered was the effect of an initially hardening restoring on the stability transition curves. We have assumed a quintic restoring curve which can be parameterised on the basis of a single parameter λ. In scaled form the expression of restoring is: $R(\varphi') = \varphi' + \lambda\varphi'^3 - (1+\lambda)\varphi'^5$ where φ' is the scaled roll angle with respect to the vanishing angle. Increase of λ means basically stronger initial hardening. The parametric variation was applied only on the linear term. In Fig 2 are shown the transition curves for restoring which is moderately or strongly hardening. Generally, a process of transformation of the boundary from sharp to 'brittle' is in place as λ is increased. Fractal-like stability boundaries have been presented earlier for a generic cubic restoring [15] and for a more exact restoring curve [16].

The effect of surge

This is a major issue currently treated in detail but here it will be discussed only briefly. We have found what is the quantitative effect of surge on capsize due to the parametric and the pure-loss mechanisms. As is well known, an implicit assumption made in the analysis of these two modes of capsize is that the forward speed may be assumed as constant. Such an assumption is not always valid. Generally, for dangerous dynamic behaviour of roll to arise, steep and long waves are required. Waves of this kind will incur also significant nonlinear effects on surge. The characteristic of large-amplitude surging is that it is asymmetric and the ship stays longer near the crests than near the troughs. This effect is imported into the yaw and roll dynamics through the restoring terms of the corresponding equations. The nonlinearity of surge is detrimental for roll stability because around the crest (where the ship stays longer) restoring capability is reduced. For yaw on the other hand, the effect is opposite. Yaw stability is not worsened because the passage of the ship from the trough is quicker. The danger arises in steeper waves and especially during the process of capture in surf-riding.

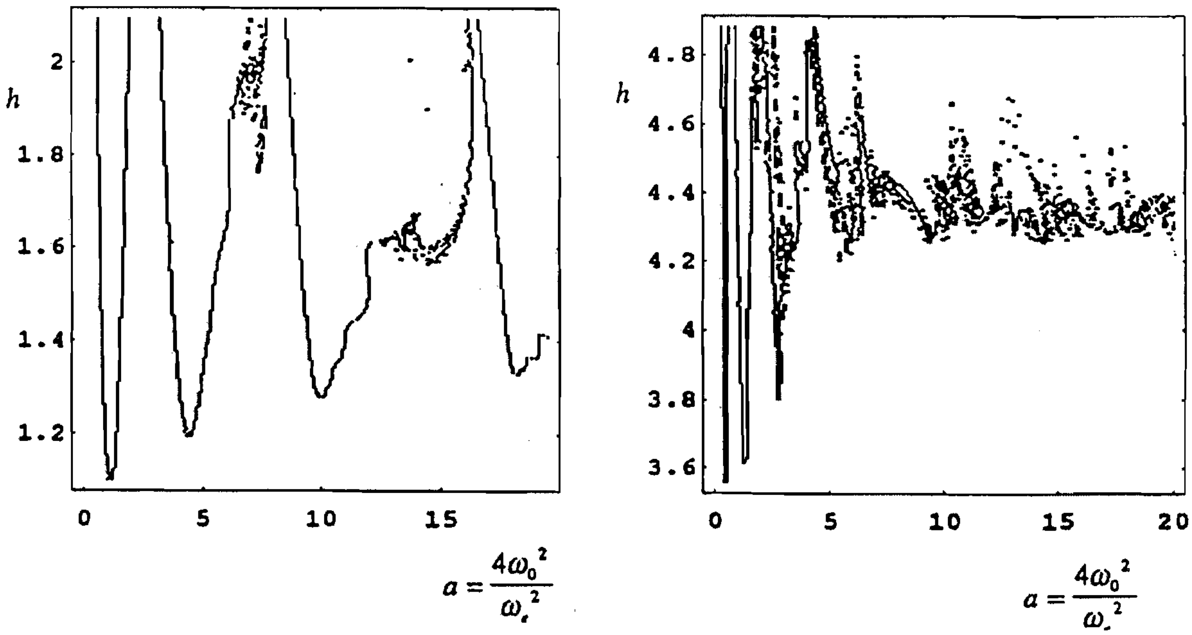

Figure 2: Capsize boundaries for increasingly hardening restoring. The left graph is with moderately hardening restoring ($\lambda = 2$) and the right one is with strongly ($\lambda = 17$).

The three main forces acting in the surge direction are the resistance, the wave and the propulsion force. As is well known these result in a pendulum-like equation for surge the exact form of which may be found for example in [17]. This surge equation should be solved simultaneously with the following equation of roll (or that of yaw for broaching):

$$\ddot{\varphi}' + 2\mu\dot{\varphi}' + \omega_0^2[1 - h\cos(kx)]\varphi' - q\varphi'^3 = 0 \qquad (13)$$

Note that the cosine is written in terms of the position on the wave, x, rather than in terms of time. The coupling to the surge equation exists because of the presence of x in the restoring term of the above roll equation. We can show now how the transition curves are modified when roll is coupled with surge.

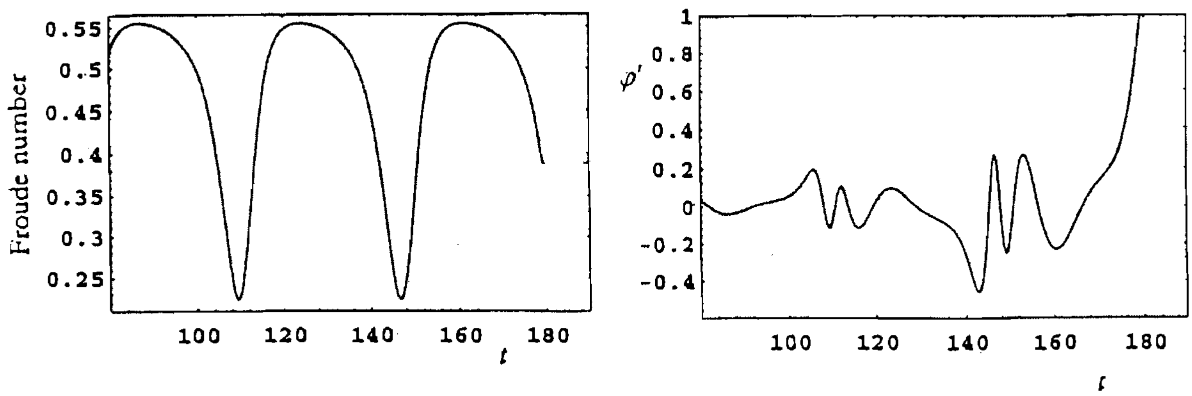

Figure 3: Coupled surging and rolling leading to capsize

The calculations were based on a ship with $\omega_{0(roll)} = 0.84$ ($\omega'_{0(roll)} = 1.577$) and $\mu = 0.0585$. We have examined whether the normalised roll angle φ' exceeds the value of 1 (from an initial perturbation 0.01 and with zero initial velocity) within a specific amount of time ($t = 200\,s$). Fig. 3 gives an illustrative example of time realisation for the coupled surging and rolling leading to capsize when strong nonlinear effects in surge are present. Surge motion can have a very profound effect on the "capsize" domains as realised from the stability diagram of Fig. 4. Rather than plotting the frequency ratio versus the parametric amplitude, we found more relevant to present the information in terms of the Froude number. For the considered ship, the principal resonance cannot be realised in following waves because the required Froude number is negative (the ship should be backing rather than going forward). The lower part of the fundamental is the only place where there is some commonality with the conventional ('damped') Strutt diagram. The upper part of the fundamental tends to become considerably wider. The immediately next resonance occupies an enlarged domain; but the following two seem to degenerate. This may relate with the emergence of a surf-riding domain where the behaviour of the ship is stationary and travels with the wave

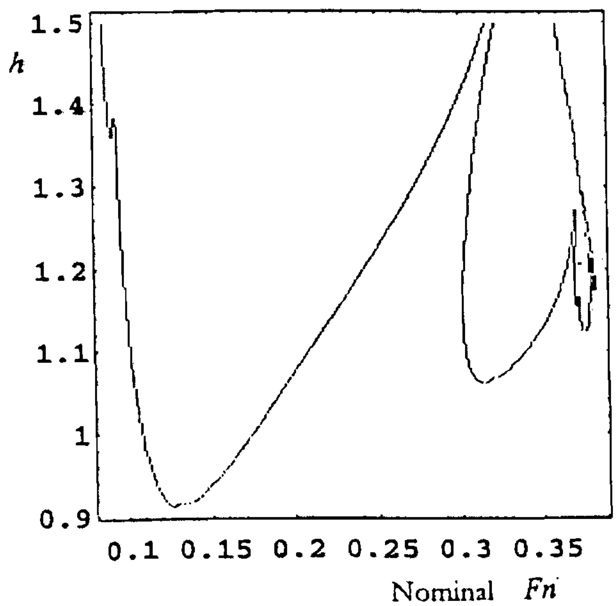

Figure 4: Capsize boundary with surging considered.

5. CONCLUDING REMARKS

When roll stability in a following sea is examined, it is common to distinguish between two mechanisms of capsize: *Pure-loss of stability*, where the ship departs from the state of upright equilibrium due to negative restoring on a wave crest. Then heel increases monotonically until the ship is overturned. In this mode the magnitude of damping plays little role. *Parametric instability*, which is the classical Mathieu-type mechanism where the build-up is oscillatory and the magnitude of damping is very important. Instabilities of a similar nature are possible in yaw, resulting in the behaviour known as broaching: The parallel to the pure-loss mechanism can be termed as *broaching due to surf-riding.* It can happen at Froude numbers near to the wave celerity. The parametric-type mechanism of broaching is relevant for lower Froude numbers but requires higher wave steepness. Extra forcing terms and couplings sometimes alter significantly the domains of instability. Therefore additional studies are needed in order to understand and quantify these effects so that improved design and operational guidance could be developed.

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