A NEW METHOD TO ANALYSE ESCAPE PHENOMENA IN MULTI-DEGREE SHIP DYNAMICS, APPLIED TO THE BROACHING PROBLEM

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ABSTRACT

A method to analyse the transients arising in broaching is presented. Currently, a proper framework for the systematic study of transient responses of multidimensional systems seems still to be lacking. This is even more true in the marine field where transient coupled motions are very rarely studied. The specific transition that is presently under study is the one triggered by a sudden control parameter variation effected in the vicinity of the threshold of surf-riding. This can lead to surf-riding, periodic motion, broaching or capsize. Each one of these types of behaviour is associated with a specific domain in the plane of actual or desired heading and nominal Froude number. The organisation on this plane is presented. This leads to the establishment of a simple procedure for quantifying the tendency of a ship towards broaching.

1. OVERVIEW

Recently there is increasing awareness about the effect of nonlinearity on large-amplitude ship motions and, in particular, about the critical role that it can play for safety. The phenomena of ship capsize and broaching, both of an *escape* nature, are two characteristic examples of nonlinear behaviour that have been recently the subjects of in-depth studies, [1], [2].

The best known feature of nonlinearity is that it often allows the existence of multiple solutions, 'born' at *bifurcation* points. Bifurcations are basically smooth or discontinuous changes in the character of the response and they can be local or global. In their more critical versions, they are associated with sudden changes of response amplitude or with jumps towards remote and usually undesirable destinations. A very useful summary and classification of the known types of bifurcation for energy-dissipating dynamical systems can be found in [3]. Quite often however, the knowledge of bifurcations, especially of the local ones, does not suffice in order to assess the safety margin of an engineering system. Given for example an initial state and a certain excitation level, one cannot say in general whether there will be an escape towards types of behaviour that are regarded as unacceptable. This represents a problem of *transient dynamics* which should be considered in parallel with the study of bifurcations of steady-state responses.

The possibility to predict ship motions beyond the realm of linear theory is obviously highly appealing, this however is confronted with shortcomings in two key areas: In solving the hydrodynamic problem which would allow calculation of the external loads acting on the ship; and in eliciting the variety of response patterns, the corresponding *manifolds* and the subsequent *state-space* organization for a dynamical system which, in general, is multi-degree.

It is well known that the accurate calculation of the forces acting on ships moving at high speed in large waves represents a very difficult problem for which practical, 'universally' accepted solutions will take some time to be produced. From the perspective of dynamical analysis, the standard method to get round this is through a judicious combination of ship theory, experiment and intuition with main objective the derivation of relatively simple mathematical models that, on the basis of the concept of *universality* of dynamical systems, present potential to capture the key features of system response.

Nonlinear ship motion analysis is 'traditionally' carried out in respect to the roll problem in beam seas. This corresponds to the single-degree forced oscillator which has been under intensive investigation with analytical and numerical techniques for a number of years [4], [5], [6]. The study of other ship motions is however often severely restricted by the necessity to account for multidimensional dynamics in a global sense. By-and-large, this is still an unresolved matter. In the marine field even the most ambitious studies of multi-degree systems do not go much further than the stability analysis of steady-states and the occasional simulation, [7], [8], [9]. These approaches alone can offer however only limited insights into the nature of phenomena, broaching being here one typical example, where transient dynamics seem to play an important role.

An effort to develop a suitable framework for the study of broaching that would include steady state as well as transient analysis has been presented recently, [2], [10],[11], [12]. In the present paper we set in focus the transient motions that are connected with phenomena of capture to- and escape from surf-riding that have been shown earlier to lead to broaching, [2]. Some preliminary studies about the effect of autopilot gains on the boundary of the broaching and capsize domains on a suitable plane of control parameters' are also reported.

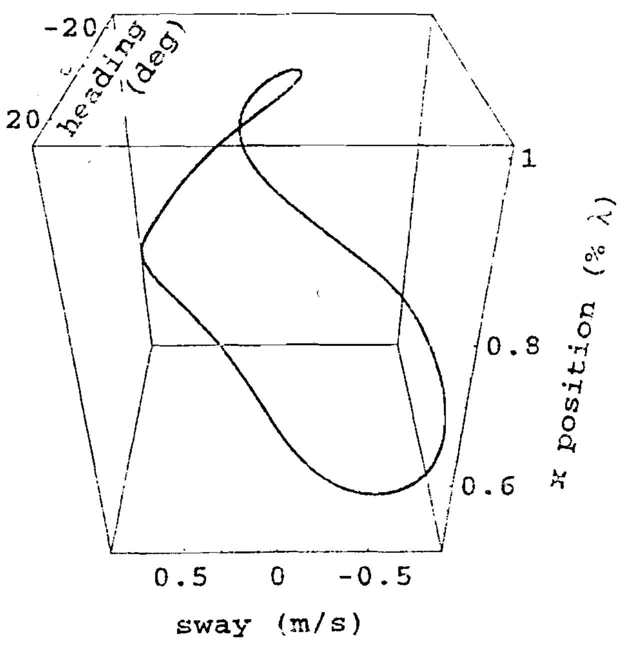


Fig. 1 : A 3-d projection of the stationary states of surf-riding

2. KEY ELEMENTS OF THE APPROACH

It is well established that in broaching motions in at least four different directions participate in the dynamics (surge, sway, yaw and roll, including also rudder control and assuming quasi-static equilibrium in the heave and pitch directions). This leads to an 8-dimensional, or higher, state-space which, obviously, one can visualize only through its 2-d or 3-d projections. It is reminded that by state-space we mean the 'enlarged' physical space that includes velocities in addition to displacements. The usual compact-form representation for an autonomous dynamical system is $d\mathbf{z}/dt = \mathbf{f}(\mathbf{z};\ \mathbf{a})$ where $\mathbf{z}$, $\mathbf{a}$ are re-

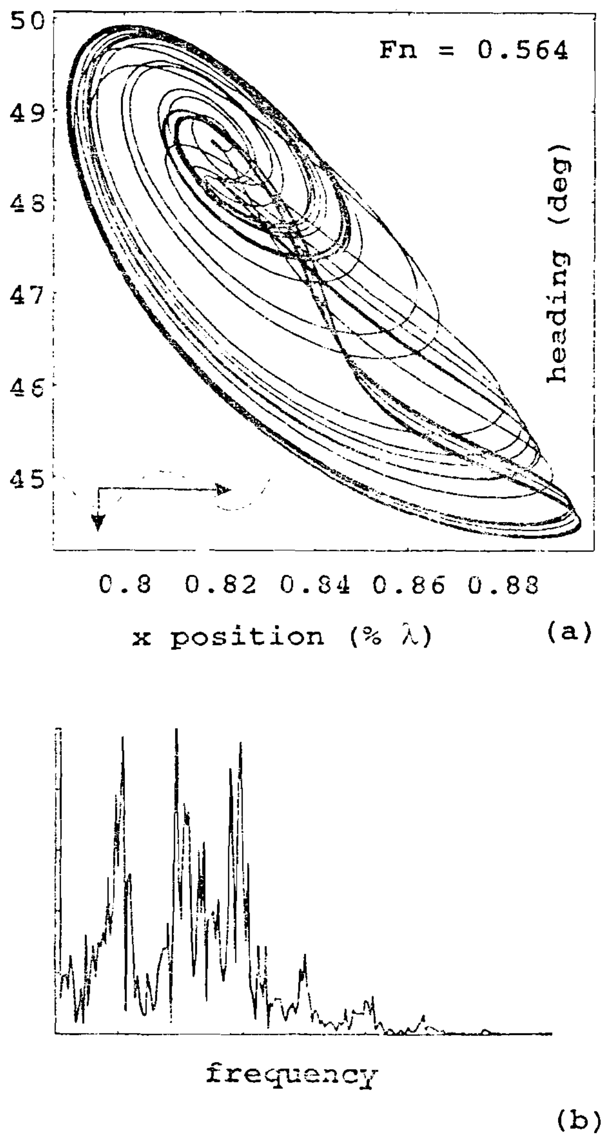


Fig. 2 : Chaotic surf-riding, [18]: (a) Phase-plot, and (b) power spectrum. The corresponding Lyapounov exponent is + 0.01.

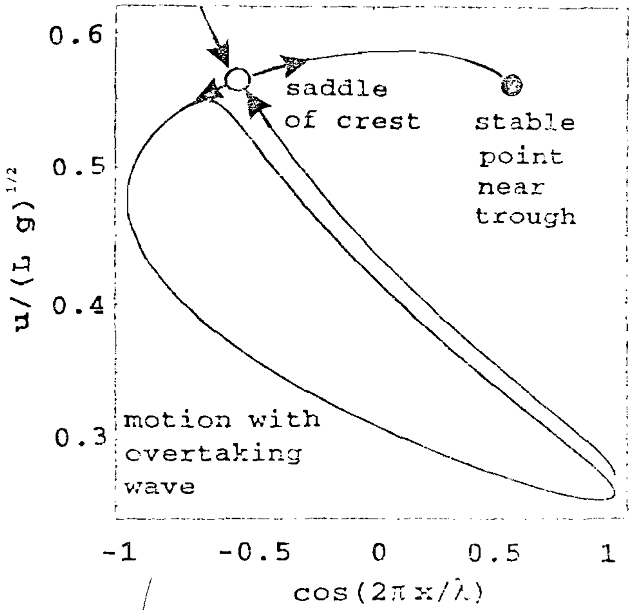


Fig. 3: Inset and outset of the saddle of crest that 'control' capture in surf-riding

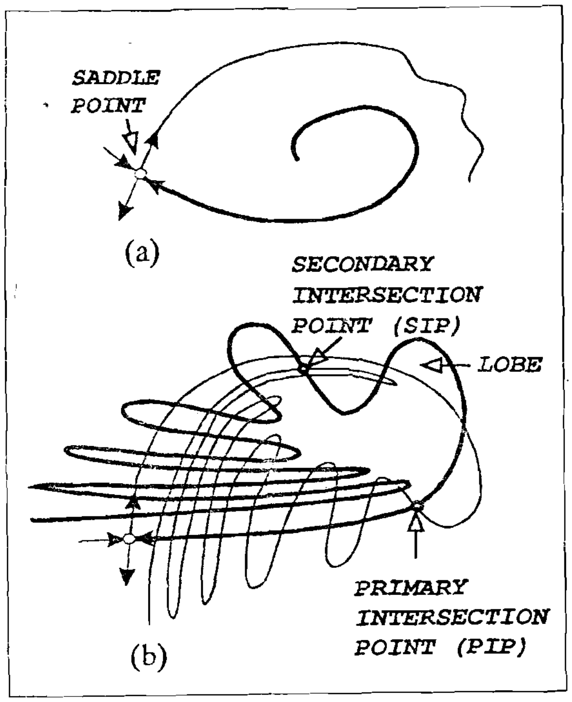


Fig. 4: Typical crossing of manifolds for a single saddle (homoclinic) on the Poincare' map for a 2-d system : (a) before and (b) after tangency of inset/ outset

spectively the state and control vectors; t is time; and **f** is a function that generates the *flow* (the geometric equivalent of the entirety of solutions of the vector differential equation) in the state-space. The detailed form of the mathematical model (in other words of the function **f**) has been presented earlier and will not be repeated here, [2], [11]. It should only be mentioned that, in order to bring the equations into the autonomous form one has to present the wave loadings as dependent on the relative position of the ship on the considered regular wave rather than as functions of time. This can be done by using a system of coordinates moving with the wave celerity.

(a) Analysis of steady-states

Usually nonlinear analysis begins by locating the steady-states corresponding to the given vector equation. Very helpful tools for this are *continuation* (or *path following*) programs that can trace the dependence of steady states on one or more control parameters, see for example [13], [14], [15], [16]. It is usual to couple such algorithms with simultaneous eigenvalue analysis in order to know also the stability of each state identified. One step further, it is possible to follow also the evolution of bifurcation points, leading to the generation of bifurcation diagrams. Tracing equilibria, such the surf-riding states presented in Fig. 1, is the simplest possible application of continuation. The types of bifurcation associated with stationary surf-riding are, *saddle nodes* and *supercritical Hopf bifurcations.* Details about them are given in [2]. To analyse possible periodic, quasi-periodic and chaotic responses one needs to employ additional techniques such as *Poincare' maps, power spectrum analysis*, calculation of *Lyapounov exponents* and others, Fig.2, [17], [18].

(b) Transient responses

Transient analysis will tell us what type of behaviour a ship will tend to adopt, when presently lying at an initial condition determined by the vector $\mathbf{z}_0 = (u_0, v_0, r_0, p_0, \phi_0, \psi_0, x_0, \delta_0)^T$; u , v, are surge and sway velocities; r, p are yaw rate and roll velocity; ϕ, ψ, δ are angles of heel, heading and rudder; x is the position of the ship on a considered regular wave measured from a trough; the subscript $_0$ indicates the values of variables at $t = 0$. It must be reminded here that, knowing the existing stable steady-states means only knowing what are the 'candidate' forms of long-term behaviour for the ship. Yet this information cannot be of any help towards predicting where exactly the ship will settle in response to a change in the setting of some control parameter. This can be extracted only by locating the *insets* of any existing saddle points (a pair for each saddle) in state- space, [19]. The insets define the boundaries of the basins of the attracting states. Equally important are also the *outsets* of the saddles that define the direction of the flow. The insets and outsets (or *invariant manifolds*) are rather unique objects that tend asymptotically towards the saddle as $t \rightarrow +\infty$ or $-\infty$ respectively [for the 2-d system that corresponds to a simple, second order differential equation of a single variable, say y, they are orbits on the (y, dy/dt) plane, Fig.3]. However for a usual saddle of index-m (that means only m positive eigenvalue-real-parts and all other negative) 'living' in a n-dimensional state-space, the inset would constitute a (n - m) - dimensional hypersurface and the outset a m - dimensional one. Even without considering the very intriguing phenomena in which the manifolds are often engaged (*homoclinic tangles* generating *chaotic transients* and *fractal boundaries*, see Fig. 4), [19], [20], [21], the difficulties involved in calculating their deployment in state-space are quite obvious. A sensible alternative is to proceed with a so-called *transient map*, that features repetitive integration from a "dense-enough" grid of initial conditions that span the whole state-space; or perhaps with a *cell-map* which is a refined version of the transient map with the addition that, one makes sure that the vicinity of any point ("cell") in state-space is not visited twice. However the number of initial conditions that need to be considered is overwhelming. An additional difficulty is that it is not exactly obvious what is the best presentation method.

In [10] it was advocated that 2-d intersections of state-space near its "interesting areas" can provide useful insights, particularly when there is no desire for restricting the range of possible initial conditions of the ship. Often however it is reasonable to assume that the change of state that the ship underwent due to a sudden variation of some control parameter (ship-based or exogenous) was effected upon a *nearly steady* initial motion pattern. Imagine for example a ship in steady periodic motion overtaken by following waves and operating 'unconsciously' near to the threshold of surf-riding; and then a group of steeper waves approaching it from behind ; or the propeller rate to be set, for some reason, suddenly higher (in a Heav side function fashion); or finally, the desired heading to be suddenly modified. From a dynamical analysis point of view the assumption of steady state for the initial motion makes the difference between an unmanageable and a manageable problem.

The steady-states that we are interested about are fixed points and limit-cycles. In an extended state-control space and under the effect of some control parameter variation they will appear respectively as lines or as cylindrical surfaces. Both are relatively easily located from the steady-state analysis that precedes the investigation of transients. In [10] we assumed the initial conditions lying on stable equilibria. This was done basically for the sake of simplicity and in order to make the first demonstration of the method easier. Such setup could be physically realized only if the ship had been captured in surf-riding. Now will be shown how the more general problem can be tackled, with the initial conditions lying either on stationary or, as is the far more usual, on periodic states.

3. THE MULTIPLE-EFFECT PROBLEM ARISING AT THE THRESHOLD OF SURF-RIDING

Let's consider once more a steered ship in stable periodic motion, as is overtaken from behind by sinusoidal waves of significant height. As stated earlier, the key assumption of the proposed approach is that the ship had been operating at steady-state at the moment when the selected control parameter was varied. This can be practically interpreted in two possible ways: The more obvious possibility is that the ship was in steady, overtaking-wave periodic motion. Then, the control parameter change could cause:

(a_1) a transition towards another *periodic* state,
(a_2) turning motion that cannot be checked (*broaching*),
(a_3) *capsize*,
(a_4) capture in the stationary condition of *surf-riding*.

The second scenario is that the ship had already been in surf-riding. Then the control change could potentially lead to:

(b_1) another stable *surf-riding* state,
(b_2) escape from surf-riding and return to *periodic* motion,
(b_3) escape from surf-riding followed by *broaching*,
(b_4) *capsize* en route to any of the above three destinations.

In either case, it is quite obvious that we are dealing with a rather unique for ship studies multiple-effect problem, resulting from the consideration of multi-degree dynamics. The primary effects may either be felt in surge (surf-riding), in yaw (broaching) or in roll (capsize). The determining factor is the initial state and the magnitude of change of the control parameter.

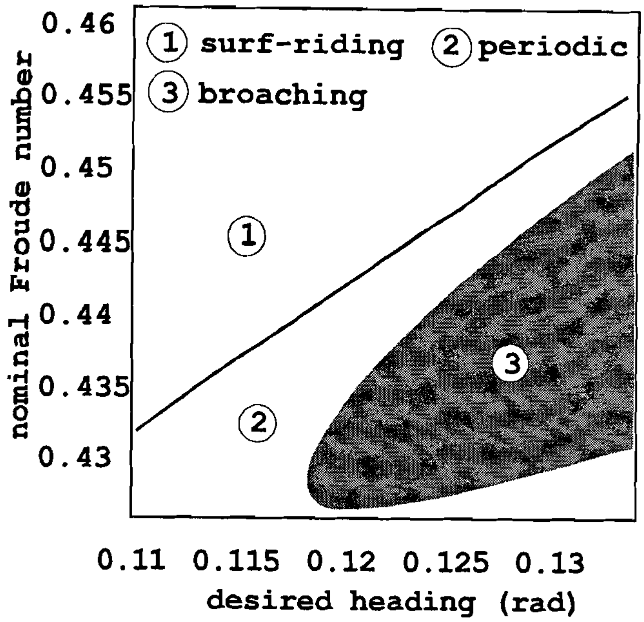


Fig. 5 : Organization of the domains of periodic,motion, surf-riding and broaching. The initial motion type was periodic.

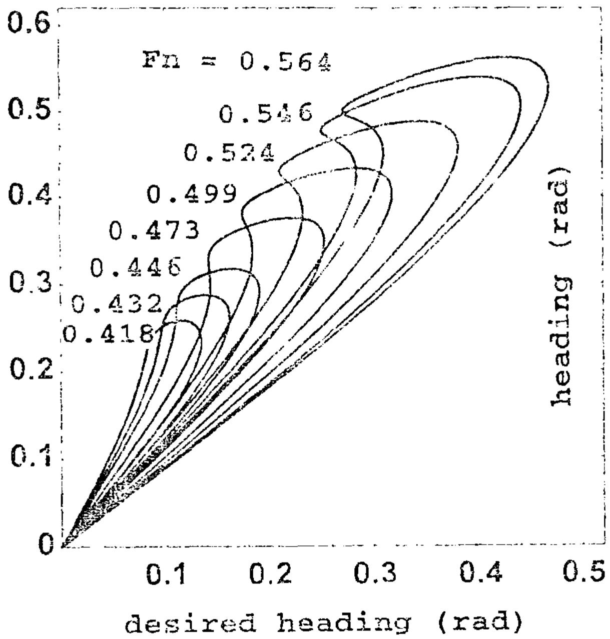


Fig. 6 : The evolution of the stationary surf-riding states with increasing *Fn*

4. PERIODIC INITIAL STATE

Consider now the ship sailing with nonzero encounter frequency and angle, and speed that brings it very near to the higher threshold of surf-riding. It has been pointed out that this threshold is basically a classic *homoclinic connection* where a limit cycle comes nearer and

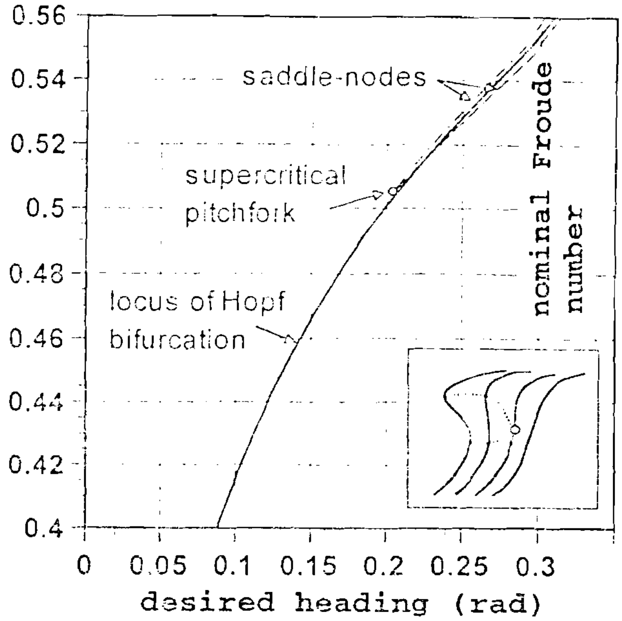
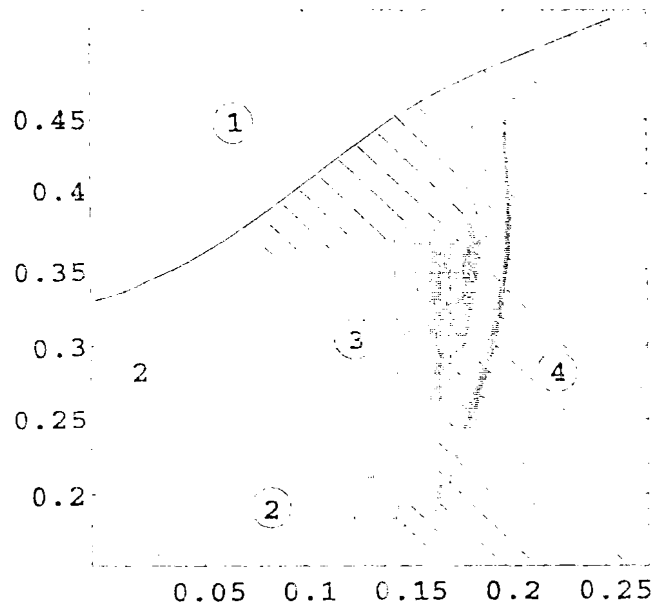


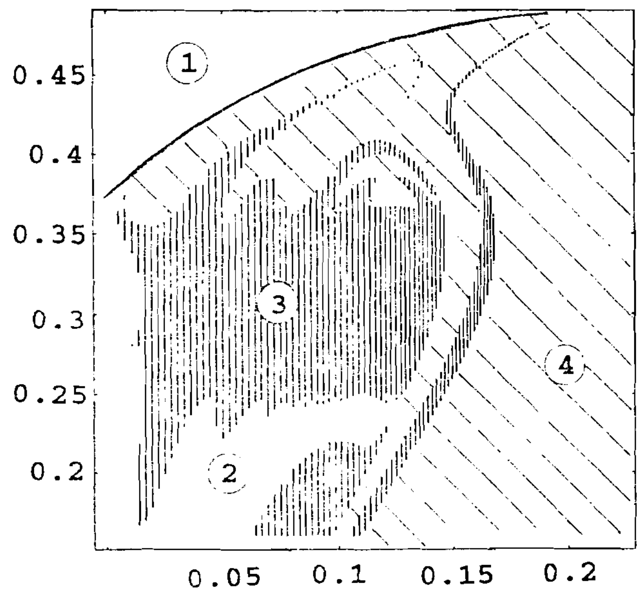
Fig. 7 : Loci of bifurcation points

nearer to a saddle point in state-space until the two collide, [2], [22]. A slight further increase of wave steepness is likely to cause the crossing of the inset of the saddle of crest, Fig.3, which will generate in turn the usual jump associated with surf-riding. Although in a physical sense this is something relatively easy to imagine, a [illegible] and hydrodynamically feasible method for modelling such change of the wave characteristics is not immediately obvious and this is a matter that is currently under investigation. However a qualitatively similar effect will be invoked if the propeller rate is suddenly stepped up and, at this stage, it is much simpler to let the propeller rate play the role of the varied control parameter. Here one must specify however with what phase, relatively to the periodic motion, the change of the control parameter setting is effected.

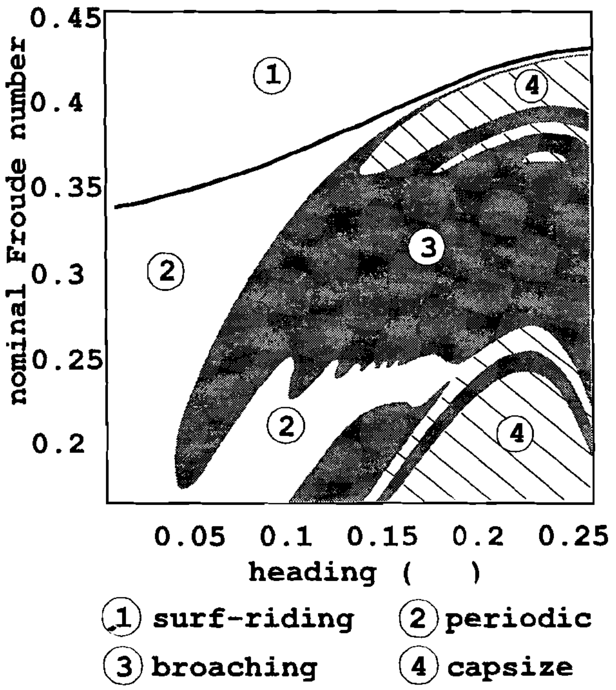
Ideally, one must find out at which point of the cycle the distance from the inset of the saddle of crest is minimum, Fig. 3. A rigorous mathematical solution to this problem will be discussed in another publication. Practically speaking however, it is known that it is more likely to be captured in surf-riding if the propeller thrust is increased when the ship centre lies in the vicinity of the trough, [2]. Also, as the higher threshold of surf-riding is approached (in general we shall name as *lower threshold* the nominal speed or Froude number at which equilibrium states come into existence; and as *higher threshold* the encounter of the homoclinic connection), the periodic motion tends to aline itself with the inset of the saddle, Fig 3. In this case the distance may not be very sensitive to the phase.



(a)



(c)



(b)

Fig. 8 : Escape from surf-riding :
(a) $a_\psi = 3,\ b_r = 1,\ t_\delta = 3$
(b) $a_\psi = 3,\ b_r = 3,\ t_\delta = 3$
(c) $a_\psi = 2,\ b_r = 1,\ t_\delta = 3$

The new method will be demonstrated through application to the purse-seiner that has been used extensively in our earlier studies ([2], [10]). The first step is to select a range for the desired heading, ψ_r, and nominal Froude number, *Fn*, and draw a grid on the plane of these two parameters. Then the nodes of this grid are considered sequentially for simulation : The first node is selected and simulation is carried out from a reference initial state until the ship settles into the steady periodic motion that corresponds to its desired heading and nominal Froude number. Then, as the first trough is encountered, the propeller rate is increased leading to a higher nominal Froude number. Depending on the outcome the node under consideration is stored in the file of surf-riding, periodic motion, broaching or capsize. Then the numerical experiment is repeated for the next node until all the nodes have been examined.

The transition that is triggered by the change in the nominal Froude number is dynamically interesting for a number of reasons; not the least being that the periodic (overtaking wave) and the stationary (surf-riding) responses that correspond to the same desired heading have considerably different stability characteristics, even for the same or almost the same nominal Froude numbers. The autopilot gain values that guarantee stability for the one cannot necessarily do the same for the other, [12]. But even if the surf-riding point is stable in a steady-state sense, still, this cannot guarantee attraction there because there exists also the possibility of being engaged in turning.

The type of reference wave considered is simple sinusoidal of considerable steepness (the results presented in this paper are based on $H/\lambda = 1/20$, $\lambda/L=2.0$) where H, λ are respectively, wave height and length; and L represents the ship length. In Fig.5 is shown how the domains of broaching, periodic motion and surf-riding are arranged on the plane (ψ_r, Fn), having deliberately set the metacentric height very high in order to avoid, at this stage, capsize. The domain of broaching is very clearly defined. In addition, in Fig. 6 we present the Fn-family of steady-state curves that are relevant to Fig. 5. We use projections on the plane (ψ_r, ψ). The loci of the local bifurcation phenomena that would create instability for the steady-state responses can be seen in Fig. 7.

5. ESCAPE FROM SURF-RIDING

Here we consider, in a sense, the reverse scenario: The ship is assumed operating with nominal speed equal with the wave celerity c. If $\lambda / L = 2.0$ the corresponding Froude number of wave celerity is

$$Fn = \frac{c}{\sqrt{gL}} = \frac{\sqrt{(g\lambda / 2\pi)}}{\sqrt{gL}} = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\lambda}{L}} = 0.564$$

As usual, g is the acceleration of gravity. For this Fn we derive through stationary-states continuation, with free parameter the desired heading, the curve of surf-riding equilibrium states. We store in a file those states near to the trough which are stable. It is reminded that a critical role in determining the length of the stable region in the vicinity of the trough is played by the autopilot gains, [2] : The proportional one defines the location of saddle-nodes; both however the proportional and differential gains have an effect on where the Hopf bifurcations may arise.

Each state stored in the file is represented by the corresponding desired heading ψ_r in the following way: Suppose that the autopilot equation is given by :

$$\frac{d\delta}{dt} = t_\delta \left[-\delta - a_\psi (\psi - \psi_r) - a_\psi b_r \; r \right]$$

where a_ψ, $a_\psi b_r$ are respectively proportional and differential gains; and t_δ is the inverse time constant of the steering engine. At equilibrium the heading and rudder rates are zero and therefore the above equation reduces to a heading-error relation :

$$\psi - \psi_r = -\frac{\delta}{a_\psi}$$

The relation between ψ and δ at equilibrium requires solution of an algebraic system of equations which is done automatically in the continuation process. Since ψ_r is uniquely defined from the above equation, it can play the role of the representative of the equilibrium state. It should be remarked that the presence of large gain a_ψ will tend to minimize the heading error. Important is also the required rudder angle for achieving equilibrium, with small δ leading to lower error.

Having defined the range of ψ_r to be between 0 deg and the largest desired heading where stable surf-riding is possible (we have assumed of course the rudder equally effective to port or starboard deflections), a similar range must be specified also for the second control parameter, that is the final nominal Froude number. This range should be as wide as possible and for our current studies that are based on $\lambda / L = 2.0$ we adopted the Fn range [0.164, 0.564].

In Figs. 8 (a), (b) and (c) we show the results of the investigation for three different pairs of autopilot gains. The four types of behaviour, surf-riding, periodic motion, broaching and capsize occupy respective domains of the (ψ_r, Fn) plane. Each one of these domains presents its own interesting structure. There is an intrusion of capsize into the broaching domain from larger headings. The boundary between periodic motion and broaching brings to mind a periodically forced Duffing-type oscillator, [10].

It is rather clear from Fig. 8 that either of the gains can affect considerably the locations of the boundaries. Yet, their qualitative characteristics do not seem to be seriously affected with the exception perhaps of a part of the broaching/periodic motion boundary (at low headings) which is under further investigation.

6. CONCLUDING REMARKS

A method to analyse the global dynamics of the transition between periodic motion and surf-riding has been put forward. The specific structure of the broaching and capsize domains, as they appear in the control parameters' plane (ψ_r, Fn), has been shown for the first time. The proposed method uses the assumption of steady initial motion pattern for coping with the multi-dimensional character of the problem. On the basis of the current results, the method seems to provide a simple and effective means for quantifying the tendency of a ship for broaching. An additional advantage is that it can be used at the design stage.

In this paper we have focused on the link between surf-riding and broaching, the existence

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of which was established theoretically in a previous publication, [2]. A classification of the hitherto understood broaching mechanisms is discussed in [2] and [22].

In general broaching does not necessarily need to involve surf-riding. It may also take place directly from the overtaking-wave periodic motion at relatively higher frequencies of encounter. This route seems to require however higher wave loadings and it may be more relevant to vessels of relatively large size since for these surf-riding should represent a very low probability event. Detailed analysis of the interesting dynamics underlying the loss of stability of the overtaking-wave periodic motions in a lateral sense is presented in [12].

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