ON THE NONLINEAR DYNAMICS OF BROACHING-TO

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SUMMARY

The latest findings of a research project where the aim is the clarification of the dynamics of broaching are presented. A basic novelty of the approach lies in the fact that it unifies contemporary methodologies of ship controllability and transverse stability studies, within the framework of modern dynamical systems' theory. The main sources of nonlinearity are first identified. In the presence of large excitations these nonlinearities can become influential for system dynamics and the links with broaching are presented. Steady-state and transient responses are investigated and is shown how periodic motions, surf-riding, broaching and capsize are organized on a suitable parameters' plane. Different broaching mechanisms are presented concerning frequencies of encounter near to zero, where surf-riding plays the dominant role, as well as frequencies of encounter away from zero, where the instability is an intrinsic feature of yaw response due to the abnormally large waves.

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1. INTRODUCTION

Broaching can be broadly described as the sudden 'loss of heading' by an actively steered ship, that is accompanied by the quick build-up of significant deviation from the desired course. It represents thus primarily a problem of stability on the horizontal plane, although interesting post-critical behaviour is likely to arise also in roll due to energy transfer into this mode through the well known hydrodynamic coupling of sway/yaw with roll. In recent years there has been renewed interest about the broaching problem, largely inspired by some spectacular advances achieved in the analysis of nonlinear engineering systems [1], and the realization that broaching can be formulated as an escape problem of multi-degree dynamics [2], [3].

As is well known, nonlinear systems often admit multiple stable solutions. In that case, a change of value in one or more of the system's parameters, or perhaps the sudden application of a nearly impulsive external load, can create escape from the ordinary type of response and a transition towards some other type whose main characteristics are, in a physical sense, undesirable. This idea seems to be very relevant to the broaching problem; because the sudden encounter of an abnormally large wave, or group of waves, or the excess of a threshold heading angle in relation to the direction of the waves are usually the causes of the abrupt change of state that is commonly perceived as broaching. The first major challenge along this line of thinking is, to discern the system's repertoire of steady dynamic responses from the set of motion equations. It is understood of course that, practically, these equations can only represent a simplified version of the true motion equations. However what is essential here is, to retain in the model the key couplings and nonlinearities.

Furthermore, it is important to be able to predict towards which type of response a certain initial condition will lead, which is to say that the organization of the domains of attraction of these steady responses must be derived. However, unlike linear systems where the dynamics are easily found should the mathematical model be known, for nonlinear systems these tasks are, in general, far from trivial. Extra difficulties arise from the fact that in broaching one has to deal with a system of multi-dimensional nature.

A description and some earlier practical information about broaching are given in [4]. In the traditional methods of analysis one is usually content with a stability examination based on the linear sway/yaw pair [5], [6], [7] (or, surge-sway-yaw [8]) for different positions of the ship on a selected reference wave. There is a number of crucial simplifications that are innate of these approaches such as, their reliance on linearized equations of motion and their exclusive concentration on the static and steady-state problem. The ship is usually assumed to be at quasi-static equilibrium and to travel at exactly zero frequency of encounter (or momentarily 'frozen' at certain positions of the wave). With today's eyes and our easy access to vast computer power such approaches seem of course to present limited potential. Nevertheless, in the early days of ship motions' research they offered truly valuable insights into the effect of following waves on directional stability.

However, the problem with nonlinearity is that it can give rise to responses that are not deducible, even at a qualitative level, from the linear low-amplitude analysis. The significance of this matter for broaching, certainly, is not raised for the first time. In the discussion of the classic 1962 paper of Du Cane & Goodrich [9], Swaan remarked that the terms which could probably explain broaching were dropped from the equations. A similar comment was given by Wainblum in reference to the linear, yet seemingly mathematically consistent, approach of Rydill [10]. These discussions were about a Hill's type equation arising in the controlled ship motion in waves on the basis of the usual sway/yaw pair. Further research on this interesting problem has been presented very recently by the author [11].

Another source of nonlinearity which seems to have received however very limited attention within the broaching framework of analysis relates with the way the nonlinear dynamics of surge influence the other horizontal motions. In large and relatively long waves (particularly for wave length to ship length ratio between 1 and 2 where the tendency for broaching is greater) a ship does not approach 'linearly' the zero frequency of encounter but somehow 'jumps' to it once the threshold of *surf-riding* has been exceeded [12]. Surf-riding is that peculiar type of ship behaviour in long and steep regular waves where the ship is captured and carried forward by a single wave. It has been very recently established theoretically that if the encounter angle between the ship and the wave lies in a certain range (usually between 10 and 30°) this jump is conducive to broaching [2]. This seems however to have been empirically 'known' for long time [13].

In the rest of this paper we shall discuss in further detail the origins of instability and we shall also present some of our latest understandings about the various types of response that can exist in abnormally large, astern seas. It is hoped to make obvious that an approach rooted in nonlinear dynamics offers a powerful and arguably much more scientific framework for explaining broaching, as well as other phenomena of dynamic instability.

2. THE STATIC PROBLEM REVISITED: THE ONSET OF YAW INSTABILITY

In relatively long and steep waves many initially directionally stable ships will experience static instability of yaw when their centres of gravity lie on the down-slope of the wave and towards the trough [5], [6], [7], [8], [9], [10]. The key role is played by the wave yaw moment which in the vicinity of the trough tends to turn the ship towards a position vertical to the direction of wave propagation $\partial N_W(x)/\partial\psi > 0$, where $N_W(x)$ is the wave-excited yaw moment, x is the relative position of the ship in respect to the wave and ψ is the heading angle, Fig. 1). To conceive the onset of this static instability one should think in terms of the well known index of directional stability that normally refers to still-water motion [14]. This index depends on the yaw and sway velocity coefficients. Suppose that a ship is initially stable in which case the index is positive. In waves the index should obviously be modified by introducing the wave force/moment derivatives in respect to the yaw angle of the ship relatively to the wave. These derivatives are of course dependent on the prevailing wave characteristics and, given the length of the wave, a critical steepness exists where in certain regions of the wave this index turns from positive to negative, see for example [6]. For such cases a steering method featuring sufficient proportional-to-heading gain will be required. One should note here however that large gains are not always desirable because they increase rudder activity and they may create rudder saturation. Also for manual steering, the required phase margins may lie beyond the capabilities of the helmsman.

3. NONLINEARITY IN SURGE RESPONSE

The above analysis is underlied by a linearity perception of surge dynamics since it is based on the notion that a ship approaches the condition of zero-encounter-frequency in a continuous and 'smooth' manner when its nominal speed is gradually increased. However, in steep and relatively long following waves this assumption does not hold. This is easily shown if one considers the balance of forces along the horizontal axis pointing in the direction of the ship's forward motion. The thrust generated by the propeller should counteract the ship's inertial and drug force, plus an alternating, position-dependent wave force. The latter pushes forward when the ship's centre is in the down-slope, while it resists the forward motion on the up-slope. The mathematical expression of this relation betrays that the system is of a pendulum-like nature [12] :

$$(m + X_{\dot{u}})\frac{d^2x}{dt^2} + [Res(u) - T(u;n)] + f\sin(kx) = 0 \qquad (1)$$

where m, $X_{\dot{u}}$ are ship mass and added mass in surge respectively; x is the position of a ship on the wave measured from a system of coordinates moving with the wave celerity and fixed at a trough; $u = \dfrac{dx}{dt} + c$ is the speed of the ship measured from an earth-fixed system, where c is the wave celerity that, at first order, equals $\sqrt{\dfrac{g\lambda}{2\pi}}$; n is propeller rate; Res, T

are respectively ship resistance and propeller thrust ; k and f are respectively wave number and amplitude of wave surge force. It is noted that in (1) the wave force in surge appears to be independent of time. Of course in the representation of wave pressure there is dependency on both position and time [15]: $\zeta_p = \zeta_0 e^{-ky}\sin(k\xi - \omega t)$ (ξ is the position of a considered water particle in reference to an earth-fixed system, ω is the wave frequency, y is the distance of the water particle from the stillwater surface; ζ_p and ζ_0 are respectively the instantaneous and the amplitude of wave pressure contour depression).

However, since the position of the ship centre can be expressed also as $\xi = x + ct$, we may write :

$$(k\xi - \omega t) = kx + kct - \omega t = kx + \omega t - \omega t = kx\,.$$

To take things slightly further, let's approximate the resistance curve with a third-order polynomial of velocity and let's write the thrust coefficient K_T as a second-order polynomial of the speed of advance $J(u;n)$) :

$$Res(u) = a_1 u + a_2 u^2 + a_3 u^3 \tag{2}$$

$$\left.\begin{aligned} T(u;n) &= (1-t_p)\rho n^2 D^4 K_T(u;n) \\ K_T(u;n) &= c_1 + c_2 J(u;n) + c_3 J^2(u;n) \\ J(u;n) &= \frac{u(1-w_p)}{nD} \end{aligned}\right\} \longrightarrow T(u;n) = b_1 u^2 + b_2 un + b_3 n^2 \tag{3}$$

where w_p , t_p are respectively the wake fraction and thrust deduction coefficients, D is the propeller diameter and a_i, b_i, c_i are appropriate coefficients. With substitution of (2) and (3) , (1) becomes:

$$\begin{aligned} &(m+X_{\dot u})\frac{d^2x}{dt^2} + \{[3a_3c^2 + 2(a_2-b_1)c + a_1] - b_2 n\}\frac{dx}{dt} + \\ &+[3a_3c + (a_2-b_1)](\frac{dx}{dt})^2 + a_3(\frac{dx}{dt})^3 + f\sin(kx) = \\ &= b_1c^2 + b_2cn + b_3n^2 - (a_1c + a_2c^2 + a_3c^3) \\ &= T(c;n) - Res(c) \end{aligned} \tag{4}$$

If we concentrate for a while on 'locally' linearized damping , we should be able to recognize the equation of a pendulum with constant forcing:

$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + f\sin(kx) = d \tag{5}$$

where the new coefficients that appear in (5) can be found by comparing with (4) (sometimes β is calculated at the wave celerity which however may be well above the average speed of the ship). Unfortunately (4), or even (5), cannot be solved analytically. When the wave steepness is low, surge velocity is described by its mean that is basically the speed in still-water, plus a cyclic term of a certain small amplitude. As the waves become steeper however, surge motion adopts an increasingly asymmetric character, Fig. 2a. The ship remains for relatively long time near the crest but it passes quickly from the trough (*surfing on a crest* [16], or *large amplitude surging* [12]). Such behaviour arises due to the presence of higher-order harmonics *in response* due to the existence of nonlinearity in the term playing the term of restoring. Explanation is possible without consideration of higher-order, non-sinusoidal wave excitation effects (of course the presence of such effects would further complicate dynamics). In Fig. 2b we demonstrate qualitatively how the inclusion of a higher-order harmonic generates the well known features of the large-amplitude surging type of response. If wave steepness is increased even further (or the equivalent, if the propeller rate is stepped up) there will be an interesting coexistence of the oscillatory motion with a pair of stationary fixed points (*surf-riding* condition), the one of which can be stable [17]. On the basis of (5), to achieve equilibrium there should be $f\sin(kx) = d$ (therefore $-1 \le \frac{d}{f} \le 1$) and by solving for x we can obtain pairs of stationary solutions:

$$\begin{aligned} x &= \frac{2\mu\pi}{k} + \frac{1}{k}\arcsin(\frac{d}{f}) \\ x &= \frac{(2\mu+1)\pi}{k} - \frac{1}{k}\arcsin(\frac{d}{f}) \end{aligned} \tag{6}$$

where μ is an integer. The stable point appears nearer to the trough while the unstable one (saddle) nearer to the crest.

By drawing upon the parallel of the pendulum one can understand fairly easily these developments: Depending on the magnitude of the constant external torque and the initial condition, a pendulum can either perform full rotations or it can stay at the asymmetric position of equilibrium characterizing the condition of static balance [18].

Eventually, the oscillatory-type response is likely to disappear altogether due to a phenomenon of *global bifurcation* known in dynamics as *homoclinic saddle connection* [2]. This happens when a limit-cycle 'collides' with a saddle-point in state-space.

To understand the dynamics at a fundamental level, let's write (5) in the following equivalent form :

$$\frac{dx}{dt} = z$$

$$\frac{dz}{dt} = -f\sin(kx) + (d - \beta z) \tag{7}$$

When the damping and forcing are zero (of course in this case the physical relevance of the equation becomes superficial) the system becomes *Hamiltonian* and the solutions on the phase plane are two families of periodic orbits, one corresponding to oscillations and the other to full rotations. The line that separates the two regions is an *heteroclinic orbit*, connecting the two saddles and is given by (see Fig. 3a) :

$$H(x,z) = \frac{z^2}{2} - (\frac{f}{k})\cos(kx) \begin{cases} < \dfrac{f}{k} & \text{(oscillation)} \\ > \dfrac{f}{k} & \text{(rotation)} \end{cases} \tag{8}$$

The introduction of constant external torque d (still without damping) destroys the symmetry in respect to the *vertical* axis and thereafter the two types of motion are separated by the homoclinic orbit of the one saddle [for positive d the right saddle with $x = \frac{\pi}{k} - \frac{1}{k} arcsin(\frac{d}{f})$], Fig. 3b. Finally, if the damping term is included, the orbits in the oscillation domain converge to the fixed point that exists within it, Fig. 3c.

A key characteristic of the homoclinic connection phenomenon is that it occurs at a nominal speed that is considerably lower than the wave celerity. During the transient towards surf-riding ship behaviour is determined to a large extent by the so-called outset of the saddle of crest. This is more conveniently shown on the phase-plane $[\cos(\frac{2\pi x}{\lambda}) , u]$, Fig.3d. The saddle outset usually spirals towards the equilibrium point that exists near to the trough. However, the separatrix of the different types of behaviour is the saddle inset. The transition to surf-riding is an abrupt event featuring a momentary increase of speed before settlement at speed equal with the wave celerity. In stable surf-riding the ship appears to be locked at a specific position in the vicinity of the trough of the wave. It is interesting that stable equilibrium in surge in the vicinity of the crest cannot be achieved because this point is always unstable!

It is logical to expect, on the basis of continuity considerations, that surf-riding will be possible not only in exactly following seas, but also in quartering. On the other hand, since surf-riding at beam sea is not possible due to the immense lateral resistance of the hull at speeds comparable to the wave celerity of long waves, there should be a limiting encounter angle up to which surf-riding is possible. Indeed, through identification of the states of equilibrium for the coupled equations of motion with a *path-following* technique (also known as *continuation*) [2], it has been revealed that surf-riding can occur only within a relatively narrow range of headings [2]. When the sea is on the quarter, the wave force in surge will be reduced and (5) would become:

$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + f\sin(kx)\cos\psi = d \tag{9}$$

where ψ is the relative heading angle between the ship and the wave. This means that, other parameters being the same, higher wave steepness would be required to sustain surf-riding when $\psi \neq 0$. In Fig. 4 we show the 3-d potential energy surface, based on the idea of a ball rolling in tilted potential wells [19].

4. THE CONSEQUENCES OF COUPLING AND SOME OTHER SOURCES OF NON-LINEARITY

Yaw motion dynamics

When the relative heading angle becomes nonzero however, the other two horizontal motions, sway and yaw, begin to participate also in the dynamics. The wave loads in the sway and yaw direction depend, at least, on the position on the wave and the heading of the ship and they are usually calculated in the context of potential flow theory. Consider for example the lowest order expression of the wave yaw moment which results in the familiar Froude-Krylov representation [20] :

$$N_W = \rho g a_0 k \sin\psi \int_L a(\psi, x_S) e^{-kd(x_S)} A(x_S) x_S \sin k(x + x_S \cos\psi) dx_S \tag{10}$$

ρ is water density, g is acceleration of gravity, a_0 is wave amplitude, ψ is the ship-wave encounter angle that can be considered as the heading angle; x_S, $d(x_S)$ and $A(x_S)$ are respectively longitudinal position, local draught and area of a transverse ship section in a ship-fixed system; finally L is ship length and $a(\psi, x_S)$ is a quantity that we assume here equal to 1.0. It is common to calculate (10) assuming that the ship is in static equilibrium in the pitch and heave sense. This of course does not mean that pitch and heave motions are taken into account as has been argued in a recent paper [21]. The assumption of such equilibrium state in the vertical direction is reasonable if the natural frequencies of the ship in heave and pitch are considerably higher than the encounter frequency between the ship and the wave so that the ship finds enough time to adjust itself on the wave. It is obvious that this can easily happen in very low frequencies of encounter but it is questionable if it represents the truth for higher frequencies.

To achieve equilibrium in yaw on the other hand, the rudder yaw moment (increased by the contribution of the rudder-to hull interaction effect) should balance the hydrodynamic hull reaction in yaw as well as the moment due to the wave. From (10) is derived that the wave yaw moment depends nonlinearly on ψ although this effect may be thought as relatively weak. More influential appears to be however the dependence of the moment upon the relative position x of the ship in respect to the wave: The strongly nonlinear effects in the speed regions where large amplitude surging and surf-riding can arise will be "imported" in the lateral motions through their dependence on x. Thus through coupling mechanisms the effects of surge nonlinearity can "spread" into the other modes.

The contribution of diffraction on the total wave force when the frequency of encounter is low can be quite important and is generally twofold: Firstly, it increases the amplitude of wave forcing, although the true magnitude of this effect seems to be somehow controversial [22], [23], [24]. The effect on yaw moment seems to be more pronounced than on sway. Secondly it shifts the peak of the force/moment forward thus creating a phase lead in comparison to the Froude-Krylov excitation. The influence of diffraction on surf-riding and broaching has been discussed in [25].

A number of interesting properties accompany the surf-riding states in quartering waves [2]: The equilibria near trough that appeared stable on the basis of consideration of the uncoupled surge are now unstable in a lateral sense (owing to the fact that, as mentioned in Section 2, near trough the wave has a destabilizing effect). These points are nevertheless potentially stabilizable with active steering. Another interesting property of surf-riding in quartering seas is that, the equilibria that correspond to different settings of the rudder constitute a closed curve in state-space. This means that, the two equilibria that can exist in a directly following sea represent basically the intersection of the closed curve of surf-riding states with the $\psi = 0$ plane. It is interesting that, in addition to the stationary type of surf-riding, it has been shown that oscillatory surf-riding can exist as well [26].

At speeds below the range of surf-riding or the range of large amplitude surging, the effect of surge is equivalent with a time dependence of the yaw moment. It can easily be shown that, away from nonlinearity in terms of x, one can either express the moment as dependent on $\sin(kx)$ or he can opt for exclusive time dependence, $\sin(\omega_e t)$. As usual, by ω_e we mean the encounter frequency between the ship and the wave. In this case however, $\sin\psi$ (or ψ if it is agreed that ψ will be small) that appears outside the integral in (9) will be multiplied by a time-dependent coefficient which can lead to a Mathieu-type of instability. To prove this, let's consider the simplified version of Nomoto's equation extended to include the qualitative effect of the waves :

$$T' \frac{d^2\psi'}{dt^2} + \frac{d\psi'}{dt} = K'\delta + A'\psi \cos(\omega_e' t') \tag{11}$$

where T' and K' are the usual system time and gain constants and A' is a wave excitation amplitude term. We couple (11) with an autopilot equation featuring gains based on heading error and yaw rate, respectively k_1 and k_2:

$$\delta = -k_1 \ (\psi - \psi_r) - k_2 r' \tag{12}$$

where δ, ψ, ψ_r, r are respectively, rudder angle, heading, desired heading, and rate of turn. By combining (11) and (12) we obtain :

$$\frac{d^2\psi'}{dt^2}+\frac{(1+k_2K')}{T'}\frac{d\psi'}{dt}+\frac{k_1K'}{T'}[1-\frac{A'}{k_1K'}\cos(\omega_e' t')]\psi=\frac{k_1K'}{T'}\psi_r \tag{13}$$

With $s=\omega_{0(yaw)}' t'$, where $\omega_{0(yaw)}'=\sqrt{\frac{k_1K'}{T'}}$, (13) becomes :

$$\frac{d^2\psi'}{ds^2}+2\zeta\frac{d\psi'}{ds}+[1-h\cos(\Omega s)]\psi=\frac{j}{\omega_{0(yaw)}'^2} \tag{14}$$

where ζ is the usual damping ratio, $2\zeta=\frac{1+k_2K'}{\sqrt{\frac{k_1K'}{T'}}}$

and also :

$$\Omega=\frac{\omega_e'}{\omega_{0(yaw)}'},\quad h=\frac{A'}{k_1K'},\qquad j=\frac{k_1K'}{T'}\psi_r$$

Equation (14) includes a time-periodic coefficient in the stiffness term and is thus Mathieu-type. A critical parameter that determines to a large extent the behaviour is the damping ratio which for such systems is usually quite high (0.8 or even higher, see for example [27]). The range of variation of ζ is actually very wide as can be confirmed by considering the relation between K' and T' recommended by ITTC, $K'=0.452+0.481T'$ [28]. The usual range for k_1 is between 1 and 3, for k_2 between 0 and 2 and for $\frac{1}{T'}$ between 0 and 2. However when damping is large, the so called instability 'tongues' that are associated with a parametrically excited system almost disappear even around the area of principal resonance (= frequency of encounter two times the natural frequency of the system). Then it is well known that the required amplitude of parametric forcing that can destabilize the system is higher than 1.0. This corresponds basically to having negative restoring at certain locations of the wave.

Nonlinearity may also reside in the hull hydrodynamic reaction terms. Let us consider Nomoto's equation once more, this time though in its full version :

$$\frac{d^2r'}{dt^2}+\frac{T_1'+T_2'}{T_1'T_2'}\frac{dr'}{dt}+\frac{1}{T_1'T_2'}r'+\frac{a'}{T_1'T_2'}r'^3=\frac{K'}{T_1'T_2'}\delta+\frac{K'T_3'}{T_1'T_2'}\frac{d\delta'}{dt} \tag{15}$$

T_1', T_2' and T_3' are the usual time constants with T_1' = $T_1'+T_2'-T_3'$. For fixed positions of the rudder (15) is basically a Duffing-type equation. For a directionally unstable vessel it corresponds to a system with a double-well potential described by the equation:

$$\frac{d^2R}{dt^2}+b\frac{dR}{dt}-R+R^3=F\delta \tag{16}$$

where the following variable and parameter transformations were applied:

$$t'=\sqrt{T_1'T_2'}\,t,\qquad r'=\frac{1}{\sqrt{a'}}R,\qquad b=\frac{(T_1'+T_2')}{\sqrt{T_1'T_2'}},\qquad F=K'\sqrt{a'}$$

On a steep wave the attitude of the ship, and thus the area distribution along the longitudinal axis, will vary considerably depending on the relative position of the ship centre and the relation between frequency of encounter and ship eigencharacteristics. The sensitivity of the directional stability of a ship to trim variations is rather well known [29], [30]. Measurements of the customary manoeuvring derivatives on the basis of captive tests with the ship at various positions of equilibrium of a regular wave have been attempted by a number of researchers, see for example [13]. Theoretical studies of the same effect are however very rare [31]. It is quite obvious that the type of nonlinearity that we discuss here will become important should the rate of turn become large. At first sight, in ordinary controlled yaw motion this should not happen since the build up of yaw is suppressed. In extreme waves however the method of control (read autopilot gains) may be insufficient to cope with the highly demanding environment. Then considerable yaw oscillations are likely to accompany the forward motion of the ship resulting also in serious speed reduction [11]. Under these circumstances this type of non-linearity will become important for the dynamics.

Roll motion

In waves other than longitudinal, there will be direct forcing in roll. When the sea is on the quarter this forcing will be generally low and will be maximized at the beam sea condition. However the roll direction will be subjected to significant excitations due to drift and yaw. Considering for example a transverse section of the ship, say S, lying at a distance x_S from the centre of rotation, the local drift at that position will be $v^{(S)}(x_S) = v + x_S r$, where v and r are the drift and rate of turn in respect to the considered centre of rotation. This local drift produces the well known hydrodynamic reaction force in sway, basically a lateral resistance force. Integrating for the ship's length we obtain the total sway reaction force Y_H which acts in general at a distance z_H off the considered centre of rotation in roll. Therefore the dominant roll excitation components will be:

$$K_{roll} = K_W + K_H \tag{17}$$

with

$$K_W = \rho g a_0 k \sin\psi \int_L a(\psi, x_S) e^{-kd(x_S)} z_W(x_S) A(x_S) \sin k(x + x_S\cos\psi) dx_S \tag{18}$$

and

$$K_H = Y_H z_H \tag{19}$$

For demonstration purposes, in the above expression we kept only the Froude-Krylov component of the wave force; $z_W(x_S)$ is the vertical distance of the centre of pressure acting on a local section from the considered centre of axes (usually from the centre of gravity or from the free surface) when the motion of the ship is neglected. In the roll moment that is due to hydrodynamic reaction we avoided writing the direct contributions from the propeller and the rudder that are sometimes quite influential. The K_H moment depends generally also on the roll angle. Moreover, in steep following or quartering waves it is possible to have significant parametric variation in restoring compared to the stillwater situation. If we consider a simple cubic type restoring and an amplitude of variation due to the parametric effect ε, the dominant terms of the roll equation can be expressed with the following equation:

$$\frac{d^2\Phi}{d\tau^2} + \gamma\frac{d\Phi}{d\tau} + [1 + \varepsilon\cos(\Omega\tau)](\Phi - \Phi^3) = f_p\cos(\Omega\tau) + f_c(v, r) \tag{20}$$

where

- $\Phi = \dfrac{\phi}{\phi_v}$
- ϕ is the true roll angle,
- ϕ_v is the angle of vanishing stability,
- $\Omega = \dfrac{\omega}{\omega_{0(roll)}}$
- ω is the frequency of encounter between the ship and the wave,
- $\omega_{0(roll)}$ is the natural frequency,

$$\omega_{0(roll)} = \sqrt{\frac{W(GM)}{I}},$$

- W is the weight of the ship,
- (GM) is the metacentric height,

- I is the second moment of inertia including the added moment,

- f_p is the nondimensionalized amplitude of external periodic forcing,

$$f_p = \frac{K_w}{(I\omega_{0(roll)}{}^2\phi_v)},$$

- K_W is the amplitude of the true wave excitation,

- $f_c(v,r)$ is the amplitude of the nondimensionalized excitation due to hydrodynamic reaction,

$$f_c(v,r) = \frac{K_H}{(I\omega_{0(roll)}{}^2\phi_v)},$$

- K_H is the amplitude of the true hydrodynamic reaction moment,

γ is the damping coefficient,

$$\gamma = \frac{B}{\sqrt{4W(GM)I}},$$

- B is the true equivalent linear roll damping,

- τ is nondimensional time, $\tau = \omega_{0(roll)}t$

- t is real time,

The reaction related component of the roll moment should be regarded as the critical one for capsize due to broaching for the following reason: If the ship is captured in surf-riding, it attains a forward speed that is often well above its design speed. Surf-riding at non-zero angle of encounter can be followed, as explained in the next section, by escape involving turning and drifting. During this transient, considerable energy will be transferred from the surge direction into yaw and further into roll.

5. STUDIES OF GLOBAL BEHAVIOUR

Broaching at the transition towards surf-riding

Consider a steered ship in stable overtaking-wave periodic motion, with a certain non-zero encounter angle and operating near to the threshold of surf-riding. A slight increase in either propeller thrust or wave steepness can cause the crossing of the inset of the saddle of crest and will generate the usual jump of surf-riding. However, the stability characteristics of stationary surf-riding are different from those of the previous periodic response. In other words, the autopilot gain values that guaranteed stability for the periodic motion are not necessarily sufficient for the stationary one [11]. But even if the corresponding surf-riding point is stable this still cannot guarantee that the ship will be attracted towards this point because there is, in a dynamical sense, 'competition' with turning motion. The type of motion will depend on the initial condition and in order to be able to predict the outcome one needs to understand first the global organization of the system's state space through the analysis of *transient dynamics*.

Such analysis for multi-dimensional systems is a largely unresolved matter and in order to derive some practically useful information we developed a method based on the assumption that the ship was initially in steady periodic motion. Then, as the ship passes from a trough the propeller is suddenly set at a higher rate. We should remark at this point that, it would be more relevant physically to vary a parameter linked with the wave rather than with the ship. To implement this however in a consistent way is a difficult problem and this is now under further consideration. Our new method can be much more easily demonstrated by selecting the propeller rate as the varied parameter. With repetition for a very large number of desired headings and final propeller rates (the latter represented by the corresponding nominal Froude numbers) and recording of the long-term response characteristics, a mapping is established between the points of the parameters' plane (ψ_r, Fn) and response types, Figs. 5 and 6. This mapping method produces a very clear picture of how the domains of stable surf-riding, periodic motion, broaching and capsize are organized, capturing the key information of a multi-dimensional system on a simple plane.

The ship that is taken as the basis of our study is a 34.5m purse-seiner [2]. In Fig. 5 we can compare the surf-riding domains for two different time scales, $t = 200$s and $t = 400$s. To obtain an order of magnitude we note that the encounter period in directly following seas before the increase of Fn was 11 s. In Fig. 6 on the other hand, we present the organization of all four domains with duration of simulation $t = 400$s.

For GM =1.51m broaching is mostly followed by capsize with the exception of a narrow band where capsize is avoided. The method of control influences however considerably these boundaries. Here the autopilot gains used were, in dimensional form, 3 for the proportional gain and 3 for the differential while the time constant was 1/3 [2]. Quantification of the tendency of a ship for broaching is possible, by measuring relative areas and their change under the effect of selected control parameters, in a similar fashion to the "integrity curves" concept of Thompson [32].

Voluntary escape from surf-riding

Here the scenario is in a sense opposite from the one presented in the last subsection. The ship is assumed caught in stable surf-riding and then some control is applied with the intention to return to the periodic mode. The possible outcomes of such control action are :

(i) to restabilize at some other surf-riding state, that corresponds to the new control settings;

(ii) to leave surf-riding and return to the periodic mode (the desirable outcome);

(iii) to be engaged in forced turning motion that corresponds to broaching;

(iv) to capsize.

Control parameter changes can be realized either in respect to the nominal Froude number, or , in respect to the desired heading of the ship. When the control change concerns *Fn* the boundaries have a clearly nonlinear character, Fig. 7. These boundaries are discussed in further detail in [3] and [33].

6. CONCLUDING REMARKS

The behaviour of a ship in abnormally large astern seas has a very rich dynamical content. While interesting dynamic effects appear independently in surge, sway/yaw and roll, the existence of hydrodynamic couplings creates a very challenging multidimensional problem where energy can be transferred from one mode into another. Local and global bifurcations, instabilities of Mathieu type and phenomena of escape from a safe basin have been theoretically shown to underlie broaching behaviour. Further studies will now be needed in a number of areas, in order to enhance the theoretical foundations of the approach and in order to understand the dynamics at greater depth. For example, the frequency dependence when the encounter frequency is far from zero must be assessed.

Also, some further insights into the dynamics should be derived at more fundamental level on the basis of elementary mathematical models that capture however the key features of system response. Generally, studies based on simple and on more detailed mathematical models have their own individual merit and they should go hand-in-hand if substantial progress is to be achieved. A major advance that stems from the current work is that, in the future, experimental efforts can be much better focused since the conditions that create the instability are now fairly predictable. Indeed there is a compelling need for carrying out extensive free running model tests in order to verify the presented theoretical findings. Such experiments are however rather non-customary and they would constitute a major advance in their own right. Another challenge lying ahead is, how the new information can be brought into a form that can be effectively utilized by naval architects in the design stage of a ship as well as by ship masters during ship operation.

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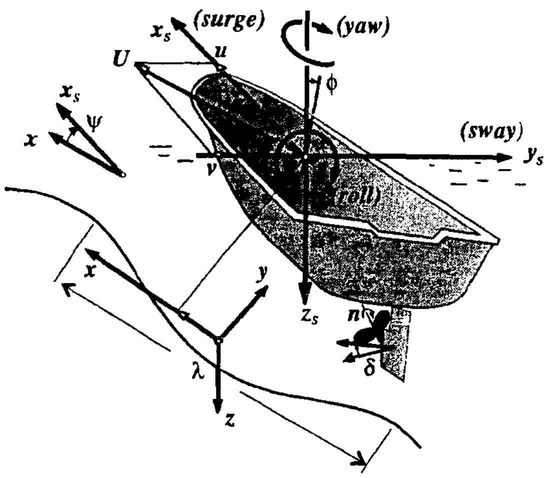
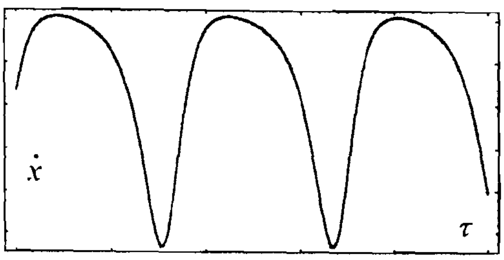
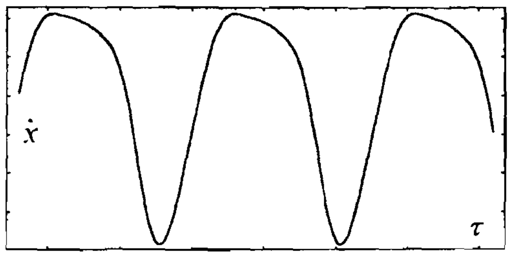


Fig. 1: Basic definitions.





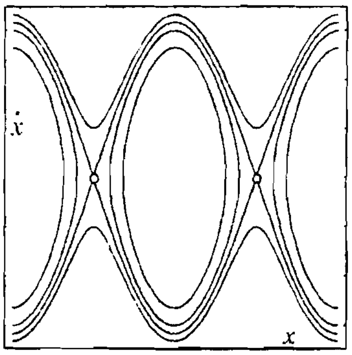
(2a) Surge velocity versus time: Typical asymmetric response near to the threshold of surf-riding.

(2b) Qualitatively similar behaviour with (2a) can be obtained by using the equation:

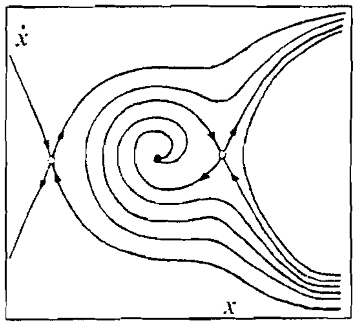
$$u=1.4+0.2\sin(\tau)+0.2\cos(\tau)-0.09\sin(2\tau)$$
$$+0.01\cos(2\tau)+0.015\sin(3\tau)-0.009\sin(4\tau)$$
$$+0.0015\sin(5\tau)$$

where $\tau=\omega_e t$.

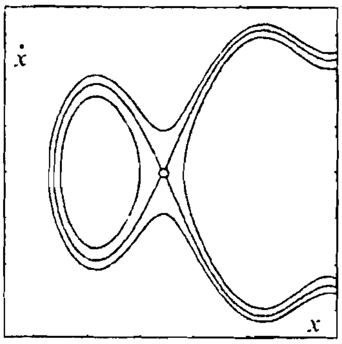
Fig. 2: Large-amplitude surging



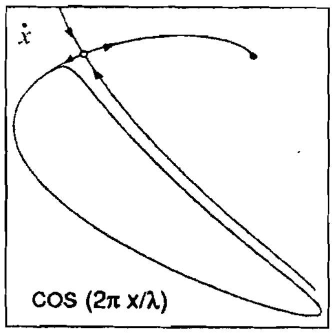
(3a) Hamiltonian system



(3c) Damped system



(3b) 'Break-of-symmetry' due to the introduction of external torque.



(3d) The saddle inset separates trajectories ending either on the periodic or on the stationary attractor.

Fig. 3: Integral curves

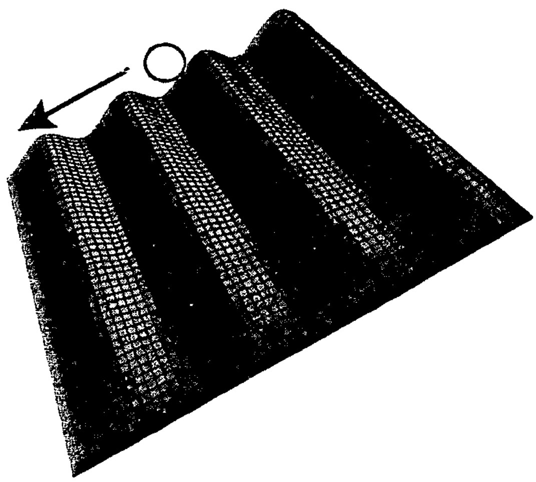


Fig. 4: Ball rolling in tilted potential wells.

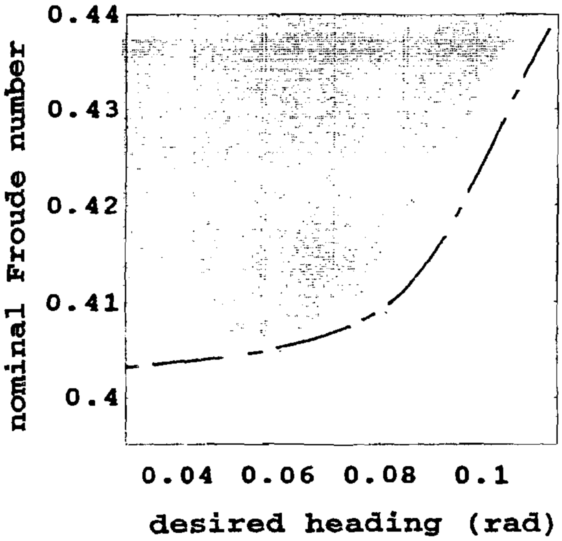


Fig. 5: The domain of surf-riding after 200s (grey area); and after 400s (higher than the dashed line).

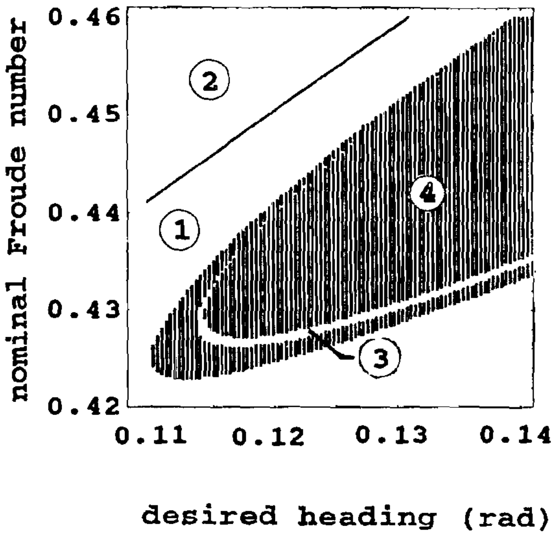


Fig. 6: The organization of the different types of response, on the plane (ψ_r, Fn).
1 periodic response, 2 surf-riding,
3 broaching without capsize, 4 broaching and capsize.

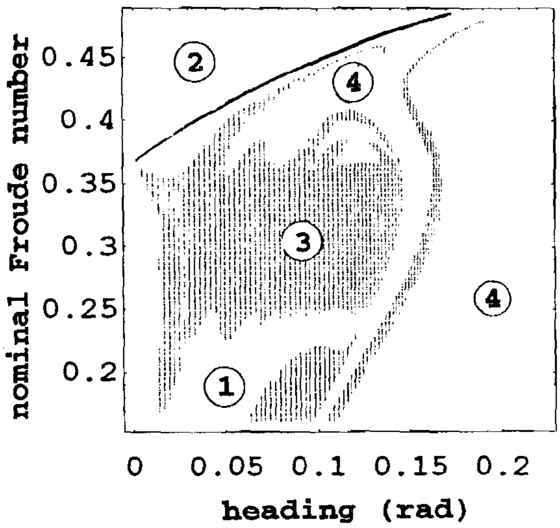


Fig. 7: Escape from surf-riding. The representation of response types is similar with Fig. 6. Also, GM=1.51m; autopilot gains (dimensional): proportional 2, differential 2, time constant 1/3.