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# Principle and Application of Continuation Methods for Ship Design and Operability Analysis

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## Abstract

The numerical technique of continuation is often used in the exploration of nonlinear dynamical systems. In the current paper it is promoted as a more general purpose engineering investigation tool that could produce direct benefits for a variety of ship design and operability studies. Continuation has been tried in university environments, for example in aerospace and chemical engineering, and in some focused naval architecture research. However the industry does not seem to be aware of the capabilities offered. Two specific themes are selected for demonstrating its potential: parametric rolling of a post-panamax containership; and controllability of a ropax ferry. Emphasis is placed on explaining the steps that a non-expert should take when implementing this technique for a ship-related investigation.

## Keywords

Ship; stability; roll; capsize; assessment; design; continuation; dynamics; bifurcation

## Introduction

In ship design, "performance-based assessments" are perceived, by-and-large, as numerical simulation studies, ideally complemented by some limited scale physical model testing. Whether for reproducing specific scenarios or for assessing ship performance in a probabilistic sense, use of simulations driven by mathematical models is nowadays recognized as a firm step towards the future and beyond the prescriptive approaches of the past, despite some concern about the degree of accuracy of numerical predictions.

Use of a mathematical model is superficially identified as synonymous to simulation. Such an impression, purportedly acquired through the "linear theory" curricula that have dominated ship dynamics engineering courses, is basically deceptive. It is true of course that the dynamic behavior of linear harmonically excited systems could be completely characterized by means of a limited number of simulations. For whatever the initial conditions, not more than a single type of long-term response could be reached, which after all should evolve in proportion to the applied excitation.

As well known, such a notion may not be extended to nonlinear systems: multiplicity of responses and dynamical "interactions" between them; as well as the occasional complexity of the response pattern itself, entail a much more organized approach to performance assessment. There is no assurance that a limited number of simulations could capture all sorts of critical system behavior. Furthermore, a massive campaign of simulations from many different initial conditions is self-defeating, as soon as the active dimensions of system's state space have been increased in the hope of attaining a more reasonable representation of the physics. The efficiency of popular probabilistic techniques, like the Monte-Carlo method, could be enhanced if prior to their application some basic understanding of the underlying state-space organization of the system had been acquired.

Circumstances are known where strong nonlinearity influences ship motions, suggesting that such a discussion is pertinent: As well known, roll restoration at large inclinations is determined by geometrical nonlinearity. For some ship forms, hydrodynamic nonlinearity affects turning motion even in a calm sea. Nonlinear pendulum-like surging can be incurred by a steep following seaway (Spyrou, 2006). The connection of nonlinearity with instability is prevalent and in most cases instabilities of seaworthy ships could be considered as realizations of erratic behavior under extreme conditions. Thus, significant magnitudes of excitation, large departures from desired equilibria and modal interactions should constitute their likely setting.

It is thus argued that, "brute-force" simulation alone is a hopeless path in eliciting useful quantitative connections between the design or control parameters of a ship with her propensity for extreme behavior and instability. Yet, little has been achieved towards incorporating new advanced tools of analysis surpassing simulation.

A case is made below for the adoption of the numerical technique of continuation (alternatively known also as path-following) as a tool of maritime engineering analysis (Keller, 1977; Doedel et al., 1997). It is not surprising that similar arguments have been heard also for the analysis of "industrial-scale" aircraft models (MacMillan, 1998). In that field, the use of continuation for research purposes spans a period of thirty years (Mehra & Carroll, 1980). Continuation schemes have been used also in some rail and vehicle dynamics research (Schupp, 2006; Catino et al., 2003). In the maritime field, earlier implementations targeted initially the loci of equilibrium states under hydrodynamic/aerodynamic loading (Spyrou, 1990 & 1996; Falzarano, 1990). However, the recent use of continuation for tracing the steady periodic responses of a ship and for direct detection of stability boundaries expanded the horizons of application and showed some promise of implementation by non specialists (Spyrou et al., 2007).

The forthcoming section provides a brief introduction to continuation. Then applications are undertaken in two directions: Firstly, for locating the boundary of parametric instability of a containership. Secondly, for identifying the course keeping and turning potential of a ropax ship excited by unidirectional wind.

## About continuation

Fundamentally, continuation can be applied in order to determine how the steady-states of a dynamical system evolve as one or more system control parameters are varied. Knowledge of "long-term" behavior, specifically of the type and arrangement of stable and unstable states of the system, is essential for defining an effective strategy for performance assessment. Its current caliber does not cater for studying transients, although it could seriously support this, through the tracing of boundaries of domains of attraction and also by identifying the critical areas of state-space where a focused simulation effort would prove very effective. Thus this focus on steady-states should not be interpreted narrowly and lead to underestimation of the scope of the approach. The technique currently works for stationary and periodic states. Unstable solutions could be traced en par with stable ones. A key strength of continuation is that it can be used also for following bifurcation points as selected parameters are varied. These bifurcation loci represent, in actual fact, stability boundaries. A basic background on nonlinear dynamical systems can be obtained for example from the books of Strogatz (1994) and Thomson & Stewart (2002).

A mathematical model brought into the canonical form of a system of ordinary differential equations (possibly combined with algebraic equations) represents the usual input. Mathematical details about the principle of continuation for such systems can be found in Spyrou et al. (2007). Efforts are noted also towards implementing continuation schemes for dynamical systems described by partial differential equations (Shroff & Keller, 1993; Davidson, 1997). In continuation the length of the solution curve is often used as a parameter of the numerical scheme in order to pass over bifurcation points, where for example, the solution curve might fold back or continue in more than one direction. This technique is referred to in the literature as *'pseudo-arc-length'* continuation. An accessible introduction to the mathematics involved in popular continuation analysis algorithms can be found, for example, in Seydel (1994).

A few packages currently exist for conducting continuation analysis. *DsTool* (Back et al., 1992) from Cornell University for example, is a user friendly dynamics investigation tool with some limited continuation capability. It worked initially in *UNIX* but currently is operational on several platforms. It also incorporates a simple GUI (Graphic User Interface) capability. The package *LOCBIF* can perform continuation of equilibrium and periodic solutions of low-dimensional (below order 10) dynamical systems (Khibnik et al., 1993). *CONTENT* is designed to perform simulation, continuation, and normal form analysis of dynamical systems (Kuznetsov & Levitin, 1995-1997). It is able to predict bifurcations of equilibria. It operates in *UNIX*, *Windows*, *Linux* and other environments. The most computationally advanced algorithm until recently was *AUTO 86/87* (Doedel et al., 1997). It can be used for performing continuation of equilibrium, periodic and homoclinic solutions. Also, it can locate most types of bifurcation points. It operates in *UNIX* and it was initially written in *Fortran*. *AUTO 2000* is a more recent version written in *C* programming language. Unfortunately, the interface is not particularly suitable for a non-specialist and this imposes difficulties when analyzing systems showing complex behavior.

The algorithm that will be used in the current study is known as *MatCont* (Dhooge et al., 2003) and it exploits *MATLAB*'S powerful capabilities including its *GUI*. Its predecessor was the *CONTENT* algorithm; however the model of the dynamical system can now be introduced through a more convenient format. The *GUI* of *MATLAB* allows very effective visualization of the evolution of the branches of solutions. The location of bifurcation points is also indicated. Possibly emerging new branches of stationary or periodic responses may be captured from there on. The tool is in fact under continuous development and new program functions are gradually introduced.

*MatCont* is capable to trace equilibria and identify associated bifurcations such as saddle-node, pitchfork and Hopf points. Once such singular points of the system are detected, there is possibility to perform the so-called "codimension-2" continuation in order to discover bifurcation points that could arise when two control parameters of the system are altered simultaneously. This does not mean of course that the parameters are varied simultaneously in a physical sense, but rather that regions of system's parameter space are associated with specific types of behavior, separated by the obtained continuation curves. Characteristic bifurcations that could be identified from such a function and yet be relevant to ship motions are: *cusps* where two loci of saddle-nodes coalesce; and Bogdanov-Takens when a Hopf curve intersects with a curve of saddle-nodes.

Either from a Hopf bifurcation point or from a limit cycle obtained by simulation, *MatCont* offers the possibility to conduct continuation of limit cycles. For this function, the period of oscillation is treated internally as a new state variable. The periodicity of these steady-state solutions entails the construction of a boundary value problem. Continuation of periodic responses is likely to reveal for some systems folds of the amplitude response curve, flips (period-doublings) and Neimark-Sacker bifurcations (transition to toroidal type of response). Further codimension-2 bifurcations could be detected from there. Continuation of homoclinic orbits has recently being introduced in *MatCont* by using *HOMCONT* (Kuznetsov & Levitin, 1995-1997) toolbox which was formerly developed in *AUTO* (Doedel et al., 1997). With the *HOMCONT* toolbox, *MatCont* is able to identify also homoclinic-to-saddle-node ("omega-explosion") bifurcation points.

## Parametric rolling

In this section will be outlined the process of extracting directly the boundary of parametric rolling by means of numerical continuation. This serves the purpose of demonstrating, by means of a simplified example, the use of continuation for studying a well-known ship stability problem that has received a lot of attention recently (see for example Belenky et al. 2003). For a practical level introduction on how continuation works, one can find some rudimentary examples based on very simple differential equations in the user manual of MatCont (Dhooge et al. 2003).

The nature of constituent boundary segments of parametric rolling, which from our perspective are loci of bifurcations, is illustrated in Fig. 1 (the diagram is generic for a ship with initially hardening roll restoring). In this diagram, by $a$ is meant the frequency ratio $4\omega_0^2/\omega_e^2$ where $\omega_0$ is the natural roll frequency and $\omega_e$ is the encounter frequency. Parameter $h$ represents the intensity of fluctuation of restoring which, given a hull, is function of the wave.

To demonstrate the potential of the technique we shall use at this stage a simple Mathieu-type mathematical model of the roll motion of a fictitious post-panamax containership, however with a fairly accurate representation of the variation of her roll restoring in waves. A rendered view of the hull is shown in Fig. 2. The nonlinear variation of $GZ$ in waves is approximated through Fourier series whose coefficients are functions of wave height $H$ (Scanferla, 2006).

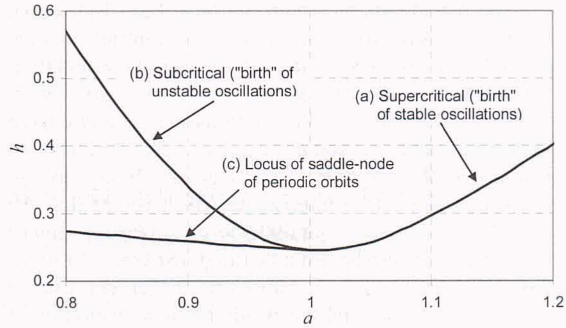


**Fig. 1: Generic representation of boundary lines of principal parametric instability.**

$$\begin{aligned} GZ(\phi,\omega_e t;H) = & \sum_{i=1}^{N_p} A_{0,i}\phi^i + \left(\sum_{i=1}^{N_p} A_{1,i}\phi^i\right)\left(\sum_{j=1}^{M_p} B_{1,j}H^j\right) + \\ & + \sum_{n=1}^{n_{MAX}}\left[\left(\sum_{i=1}^{N_p} A_{2n,i}\phi^i\right)\left(\sum_{j=1}^{M_p} B_{2n,j}H^j\right)\cos(n\omega_e t) + \right. \\ & \left. + \left(\sum_{i=1}^{N_p} A_{3n,i}\phi^i\right)\left(\sum_{j=1}^{M_p} B_{3n,j}H^j\right)\sin(n\omega_e t)\right] \end{aligned} \tag{1}$$

Symbols appear with their usual meaning: $\phi$ , $t$ are respectively roll angle and time. $A_{kl}$, $B_{kl}$ are coefficients determined from the Fourier analysis of the "true" $GZ$ surface in waves, determined from some standard ship design software (in our case this was *Maxsurf*). To reduce the number of parameters, wave length was fixed equal to 1.5 times the ship length. The fitted $GZ$ surface is shown in Fig. 3.

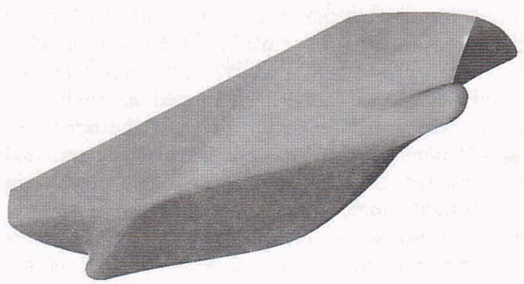

**Fig. 2: Hull of investigated post-panamax vessel.**

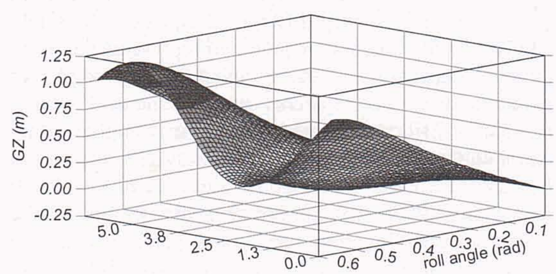


**Fig. 3: Fourier series approximation of $GZ$ up to $35^0$.**

The above model is characterized by explicit time-dependence that could be removed by introducing the so-called "FitzHugh-Nagumo" equations. The incurred cost of two extra degrees of freedom is necessary for bringing the model in a consistent format for introduction into *MatCont*; yet this is no hindrance for the investigation which thereafter proceeds as follows: As first step, simulation is performed from some initial condition and for a relatively large wave height, targeting a response within the domain of principal parametric instability (therefore the value of $a$ is set around 1.0). Our specific selection in this case was: $\omega_e = 2\omega_0$, $H = 5m$, an initial roll angle $\phi_0 = 0.1$ rad and initial roll velocity $\dot{\phi}_0 = 0$ rad/s.

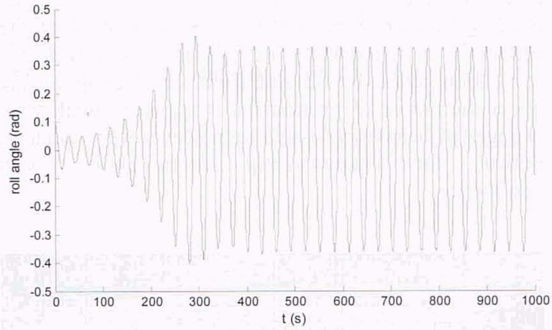


**Fig. 4: Simulation of parametric rolling performed in order to capture a limit-cycle.**

The obtained response appears in Fig. 4. An arbitrary point is then selected, belonging on the steady limit-cycle whereupon the system has settled after transient effects died-out. From there, numerical continuation of the roll response amplitude can be performed; firstly to the left and then to the right of the captured cycle, using as independent variable the wave height.

The character of the realized evolution of roll amplitude is described schematically by the diagram of Fig. 5. In this case we have captured the creation of parametric oscillations that takes place on boundary (c) of Fig. 1. The exact layout of the steady periodic responses obtained is shown, in 3-D format, in Fig. 6. Several observations are worthy to be made in this instance:

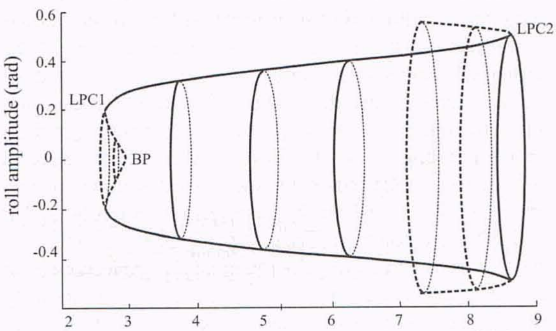


**Fig. 5: Schematic description of evolution of parametric oscillations showing also key bifurcations.**

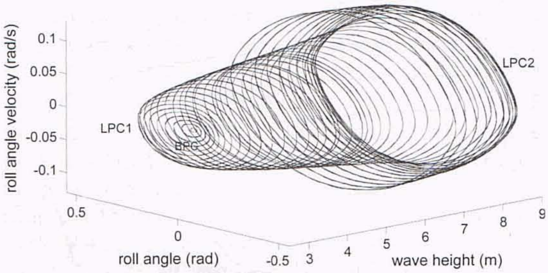


**Fig. 6: Result of continuation of parametric oscillations as captured with *MatCont*.**

Firstly, the point of generation of parametric oscillations has been captured (it is denoted by BP). Secondly it is revealed that, once generated somewhere on curve (c) of Fig. 1, the response amplitude evolves initially towards lower wave heights. Concomitant stability analysis confirms that these responses are unstable and they revert to stable at a folding of the curve ("saddle-node of periodic orbits" or "limit-point of cycles"), denoted by LPC1, which plays an important role: LPC1 defines the lowest wave height that gives rise to parametric oscillations. At higher wave heights the stability reverts back to unstable at LPC2, where the branch of solutions folds back.

Once identified, bifurcation points may be evolved for all frequency ratios. The obtained curves, defined on the plane of frequency ratio versus wave height, should constitute parts of the boundary of parametric rolling. The loci of BP, LPC1 and LPC2, captured through continuation, are shown collectively in Fig. 7. It is confirmed that, BP is evolving along the curves (a) and (b) of Fig. 1 while LPC1 is evolving along curve (c). In other words the locus of BP defines the linear boundary of parametric rolling while that of LPC1 defines the nonlinear one which is often unjustifiably disregarded. The evolution of LPC2, and possibly of other bifurcation points concerning very high waves, determine secondary boundaries that lie inside the region of parametric rolling and, as a matter of fact, may not be of immediate interest in the current work.

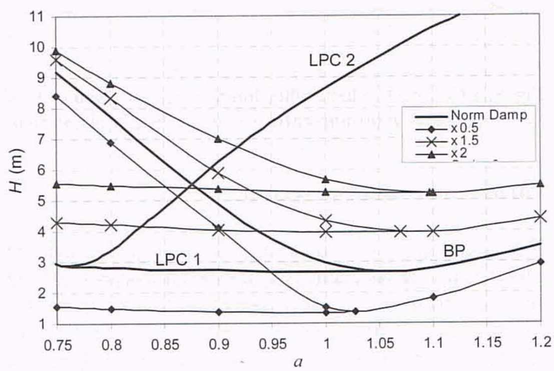


**Fig. 7: Stability boundaries of parametric rolling for different levels of damping. The locus of LPC2 is also shown for the original damping.**

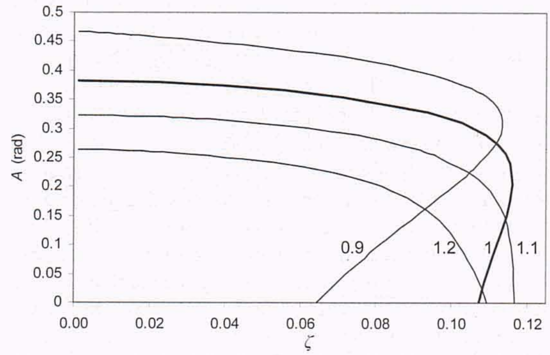


**Fig. 8: Varying damping ratio for $H$ =5m.**

To enable some comparison, similar curves for different linear damping values have been superimposed in Fig. 7. In the diagram was inserted also the locus of LPC2 with reference to the original damping value. It can be observed that an increase in the damping not only increases the minimum wave height required for initiation of parametric rolling but also shifts the critical encounter frequency.

It is remarked finally that several other parametric studies could be performed at this stage, in an essentially automated manner. Such an example is presented in Fig. 8 where wave height was fixed and linear damping was varied. The evolution of the steady roll amplitude around principal resonance is shown. Another set of interesting curves is obtained from the evolution of BP, LPC1 and LPC2 when $a$ is set to 1, using again the damping as varying parameter (Fig. 9).

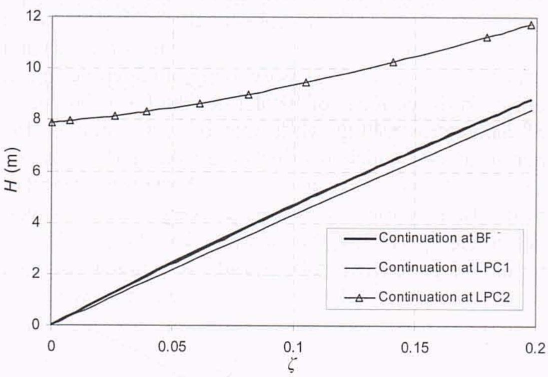


**Fig. 9: Effect on the instability boundaries for fixed $a$=1 as both damping ratio and wave height are varied.**

## Turning and course-keeping

The strength of continuation may also be illustrated through an assessment of the course-keeping and turning capability of a ship. We came to appreciate this potential as we investigated recently the controllability of a ropax ferry in strong unidirectional wind (Fig. 10). The mathematical model together with some key results from this investigation referring to course-keeping, and a list of references can be found in Spyrou et al. (2007).

The mathematical model was a typical modular one with 4-degrees-of-freedom (horizontal plus roll), addressing hull reaction, propeller, rudder and wind excitations. In particular for the wind module, the proposal of Blendermann (1996) was implemented.

A number of interesting stability problems underlie the specification of the limiting environment where a desired heading can be maintained and also turning maneuvers can be executed in an effective manner. Intuitively, one conjectures that given a steady wind environment and setting of rudder to a specific angle, the ship should in some cases stabilize herself on a straight-line course whereas in other cases she should be engaged in turning motion even in the long term. Earlier work has indicated also the possibility of additional types of response, like oscillations around a mean heading in head wind, unless effective control of the rudder is exercised (Spyrou, 1995). It is argued that simulation alone would fall short of offering a clue about the nature of phenomena that govern qualitative changes of behavior; something however that would be critical for characterizing controllability with confidence.

**Fig. 10: The investigated ropax.**

### *a) Capability to maintain the heading*

Given the above problem setup, a possible way to proceed for a continuation-based investigation of course-keeping is as follows: Let the steady wind velocity be set at a relatively high value (e.g. 23 m/s). Selecting as control parameter the rudder angle, the locus of equilibrium headings could be automatically produced (see Fig. 11). This curve is comprised basically of equilibria and their identification through *MatCont* is straightforward. In general, to initiate continuation some initial guess of a starting point is required, located in the neighborhood of the targeted curve. It is recalled that in the investigation of parametric rolling this was identified from simulation and selection of a point from the long-term part of the produced time-series. Here however no real need of such a guess existed, because the state representing forward motion of the ship with no turning or drifting (corresponding to $\delta$=0), lies on this curve. Several bifurcation points (basically saddle-nodes (limit points) and Hopfs, denoted on the diagram respectively by LP and H) were found to arise. As a matter of fact, only some parts of this curve correspond to stable condition (there is a generic pattern for the changes of stability that is seemingly obeyed also by other ship types, see Spyrou (1995).

Let us now turn our attention on LP (or equivalently its symmetric LP′) which defines the threshold heading relatively to the wind and also the utmost rudder deflection where straight-line motion of this ship could be sustained, for a certain specification of the wind environment. Normally, this point corresponds to a nearly beam wind condition and it is worth noting that in the past, it had been proposed to base the selection of rudder on a requirement of keeping the course in strong beam wind (Mikelis, 1991). Continuation can easily produce the locus of this point under simultaneous variation of rudder angle and wind velocity. The outcome, for a range of speeds, is illustrated in Fig. 12. The format seems to be suitable for use as operational guidance of the Master. It is noticed that the curves tend to become vertical at stronger wind, i.e. rudder saturation occurs and, given the speed, further deflection of rudder proves ineffective beyond a certain limit. Influential ship design parameters might also be selected for continuation in order to optimize the maneuvering characteristics and/or meet specific criteria.

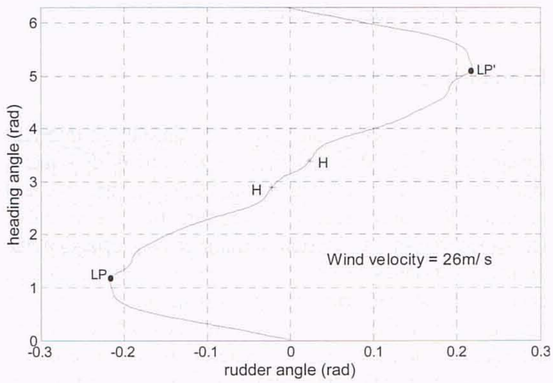


**Fig. 11: Variation of the rudder angle produces the locus of equilibrium headings.**

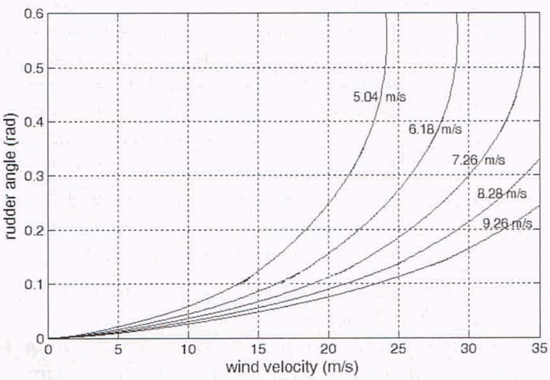


**Fig. 12: Evolution of LP1 on the plane of rudder angle and wind velocity, with parameter the speed of the ship. Wind from stern quartering.**

The prospect of tracing the dependence of key dynamical phenomena upon design parameters by an automated procedure is worthy of attention. One simple example is provided by the diagram of Fig. 13 where the rudder area was picked as control parameter of the continuation curve. The maximum wind velocity where course-keeping can be sustained in strong beam wind seems to be almost proportional to the rudder area. Hull parameters could also have been selected for a sensitivity analysis of this nature.

As noted already, in the head wind range planar oscillatory behavior of the ship is possible to emerge due to a Hopf bifurcation phenomenon (see Fig. 11). These oscillations could be suppressed through suitable active control of the rudder. A sufficient combination of gain values of an assumed "proportional-differential" controller can be determined without difficulty by using these gains as the pair of control parameters of a "codimension-2" continuation run. Such loci have been produced automatically, according to the discussed procedure, and they are shown in Fig. 14. Complete extinction of the unwanted oscillations is achieved by selecting a combination of gains that falls to the right of the presented curves.

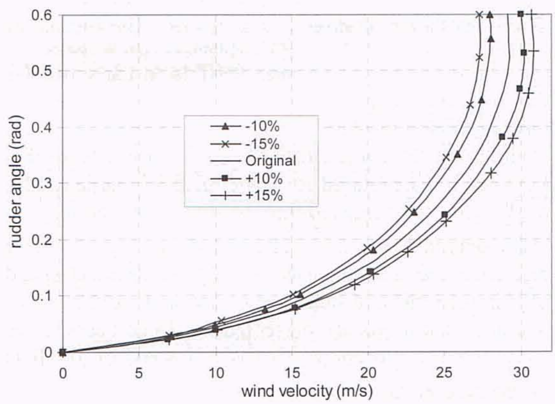


**Fig. 13: As above, with parameter the area of the rudder.**

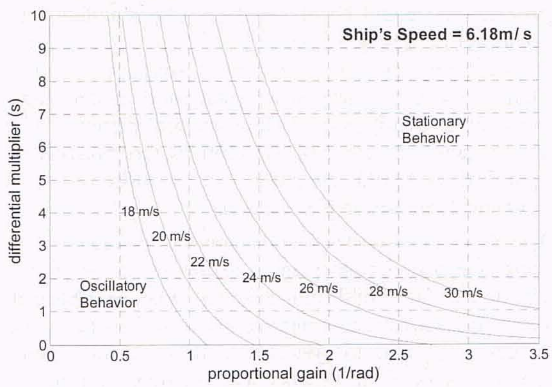


**Fig. 14: "Sufficient" combinations of gain values for annihilating parasitic oscillations in head wind. Different wind velocities are shown (from Spyrou et al 2007).**

### *b) Turning in wind*

Another issue of interest is the capability to execute effectively turning maneuvers in wind as well as the dynamic character of such turns. As well-known, in wind, a turn invoked by some constant deflection of the rudder does not converge to circular motion. Unfortunately, continuation of periodic states cannot be performed directly upon the system of ordinary differential equations presented in Section 2 because, although a pattern of repetition is ultimately obtained concerning for example the yaw rate, the variable $\psi$ that represents the heading and appears explicitly in the state-vector of our system increases monotonically i.e. it is not a periodic function of time.

In order to overcome this computational barrier, some suitable transformation of the variable that represents the heading was contrived. Specifically, the following pair of dummy variables was introduced: $a = \cos\psi,\ b = \sin\psi$, simultaneously ensuring of course that the condition $a^2 + b^2 = 1$ is satisfied. Then the kinematic relationship $\dot{\psi} = r$ that appears in the mathematical model needs to be substituted by the pair $\dot{a} = \frac{d}{dt}(\cos\psi) = -\dot{\psi}\sin\psi = -rb,\ \dot{b} = ra$. By means of this transformation it became feasible to carry out for the first time continuation of limit sets representing turning motion in wind, as the rudder angle was varied. To initialize the continuation, a periodic limit set of this kind had to be captured for some relatively large rudder angle. This was achieved through direct simulation.

The obtained result is illustrated in Fig. 15 (for reference, the diagram was drawn for a nominal speed of ship 6.18 m/s and steady unidirectional wind 26 m/s). We note that orbits are presented as closed because we have opted to plot $\cos\psi$, that is $a$, instead of $\psi$. It is observed that as the rudder angle is decreased below a certain critical value, the closed orbit is broken and thus the ship should be unable to complete the turn. In fact, the run of the continuation algorithm is stopped at that point abruptly with a message of bifurcation. Thus the broken orbit could be obtained only by simulation (it could not be otherwise because this represents, in fact, transient response: in a dynamical sense, the ship is "left" to seek a nearby steady state and, in this case, possibly a steady straight-line motion pattern). The considerable distortion of the orbit as the threshold is approached should be noted as reminiscent of a dynamical interaction phenomenon.

The above thus substantiate the conjecture made earlier: there is a critical rudder angle below which the ship is directed to an equilibrium heading, whereas above it, the ship is capable to complete her turn. It is remarked however that due to system's nonlinearity, this value may not coincide necessarily with the maximum rudder angle where the straight-line course can be maintained. Therefore, continuation helped us to determine efficiently the minimal rudder angles required for accomplishing turning maneuvers in wind.

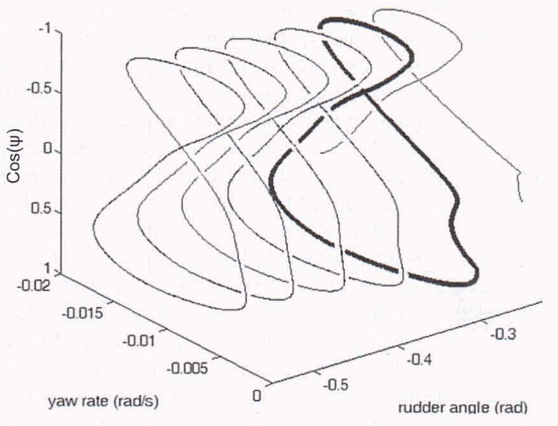


**Fig. 15: A 3-D view of the periodic and stationary states as rudder angle is varied**

## Concluding remarks

Continuation tools outperform simple numerical simulation when the behavior of nonlinear dynamical systems is under investigation. Despite the limited attention they have received thus far, they are valuable for supplying in an automated manner the dependencies of ship responses to critical environmental and design parameters and for identifying conditions where system behavior exhibits qualitative changes of character. Simulation would entail a strenuous procedure for capturing the relationship between responses and system parameters when the system is nonlinear. Usually multiple simulations are run, and before each run a new value of the independent parameter has to be set. But it is well known that nonlinear systems may have more than one branch of solutions. In this inefficient and time-consuming way it is not certain that all branches of solutions and bifurcation points have been identified; and if this could be achieved for simple systems, it is practically impossible to define the topology of a complicated system by such means. Moreover, unstable solutions usually play a very important role in dividing system's state space into sub-spaces leading to different long-term behavior. Thus it is essential to be able to locate also branches of unstable states efficiently.

Continuation allows someone to vary two or more parameters simultaneously and thus follow the evolution of system's bifurcation points. When continuation is used in combination with simulation, the maximum potential is gained as the simulation effort can be focused on sub-spaces, where complex dynamical activity has been predicted through continuation to be taking place. Despite the current inability of continuation to deal with transients, it could enhance in some instances the potential also of probabilistic simulations by helping to identify beforehand the dynamical structure of the system, so that probabilities could be more easily assigned.

Engineers that work in industry often face a gap in terminology and, even if continuation packages had reached a degree of user-friendliness that allowed wider adoption (it seems that we are quite near to this stage), this communication barrier would still need to be overcome. It is felt that awareness through education about these novel instruments and underlying theory by young engineers who enter the profession would be the best way for exploiting these advanced capabilities in an industrial context.

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