Direct Assessment Methods for Nonlinear Ship Response in Severe Seas

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ABSTRACT

The principle of separation is considered as a framework for the direct assessment of failure events related to ship motion in severe seas. The idea is to separately consider the nonlinear phenomena resulting in large response and the conditions which lead to the occurrence of such phenomena. The fundamental aspects of three methods which each use the principle of separation are reviewed: the peaks-over-threshold / envelope peaks-over-threshold method, the split-time method, and the wave group method. The application of the principle of separation is also discussed for the validation of numerical simulation tools used for large-amplitude ship motions.

Keywords: principle of separation; problem of rarity; split-time method; wave groups; envelope peaks above threshold (EPOT); direct assessment methods; Monte Carlo simulations

1. INTRODUCTION

Dangerous ship behavior in waves is most often caused by either extremely high or extremely steep waves, or a sequence of waves with particular frequencies and amplitudes. These wave events are rare and assessing their probability of occurrence remains a difficult problem. The response of a ship to these wave events is expected to have large amplitude motion and to be significantly influenced by nonlinearities in wave forcing, damping, and hydrostatic restoring. When a dynamical system has significant nonlinearities, its behavior becomes very sensitive to initial conditions (Poincaré, 1890; Lorenz, 1963). Depending on the initial conditions, the ship response to a large wave may range from merely "contouring" the wave, to catastrophic motions including capsizing. The main difficulty with the assessment of dynamically-related undesirable events, or dynamic "failures," is both their rarity and significant nonlinearity, which need to be addressed simultaneously.

1.1 Nonlinearities and the Problem of Rarity

Failures related to a ship's motions and loads in severe seas are characterized by both their rarity of occurrence and significant nonlinearity. Because of this, the accurate evaluation of the ship response in these conditions becomes difficult and impractical with the use of traditional "brute-force" direct assessment methods– Monte Carlo simulations and/or a large number of experimental realizations in the basin.

Assessing the dynamical response to these wave sequences constitutes the general problem of rarity – when the time between events is long, compared to a relative time-scale (Belenky, *et al.*, 2008). The problem of rarity may be solved by separating the ship response into sub-problems, according to their time scale. For ship motions, the simplest example of an implementation using this approach is the piecewise-linear method for calculating capsizing probability (Belenky, 1993; Paroka & Umeda, 2006; Paroka, *et al.*, 2006; Belenky, *et al.*, 2009). The same principle has also been applied to determine nonlinear response using numerical simulations (Belenky, *et al.*, 2008).

For example, large roll motion response (*i.e.* roll near, or beyond, the maximum of the GZ curve) appears when a dynamical system is characterized by significantly nonlinear stiffness. By its nature, the point of maximum is when the oscillator behavior changes from an attractor to a repeller. Additionally, large roll angles are typically the result of specific phenomena – nonlinear excitation, which may be exhibited in the form of fold bifurcation. Such phenomena are not limited to roll motion. Large yaw angles may also be the result of fold bifurcation (Spyrou, 1997), such as in the case of direct broaching.

This nonlinearity makes it difficult to use traditional techniques to determine values associated with rare events, such as extreme value distributions. While the theory of extreme distributions is still applicable, the fitting of these distributions may be difficult, due to the insufficiency of the available data where these nonlinearities are significant. This situation can be resolved with the explicit modeling of nonlinear phenomena, but this would require consideration of the influence of random initial conditions and could be influenced by the occurrence of previous nonlinear events, depending on the time-scale. These considerations lead to the concept of a separation between the nonlinear phenomena resulting in large response and the conditions which lead to the occurrence of such phenomena.

1.2 The Principle of Separation

This separation leads to a modeling of the ship response problem as a combination of two sub-problems: non-rare and rare. The non-rare problem is focused on determining the probability of occurrence of the conditions which may lead to the nonlinear phenomena resulting in severe response, as well as determining the distribution of the appropriate initial conditions. The rare problem is focused on determining whether large responses occur for particular initial conditions.

In principle, if the failure is the result of a chain of events, there may be several rare problems. For example, in broaching due to surf-riding, the occurrence of surf-riding is required for the broaching event to manifest itself for the given environmental conditions. The non-rare problem would define the conditions where surf-riding is possible, while the rare problem represents the probability that surf-riding will occur, given the existence of the necessary conditions. The inception of broaching, given the occurrence of surf-riding (yaw repelling), is a function of the manifestation of instability in yaw after the occurrence of the surge equilibrium.

The main assumption behind the separation principle is that a mechanical system can be “restarted” at any moment of time, *if* the state variables at the instant of “restarting” are fully determined. For the case of a body moving in vacuum, this is an exact statement. For a ship on the free surface, however, it is an assumption, because the hydrodynamic memory effect cannot be fully realized. In this sense, all of the necessary memory effect is contained within the initial conditions at the initialization of the rare problem.

Three methods which use the principle of separation are reviewed: the peaks-over-threshold / envelope peaks-over-threshold method, the split-time method, and the wave group method. Each of these methods utilizes data from simulations and/or model experiments. The POT/EPOT method uses existing data from simulations and/or model experiments. The split-time method may use existing data for the non-rare problem to establish the conditions for simulation of the rare problem. The wave group method gives ship-specific consideration to the identification and generation of the data set of interest.

1.3 Relation with Time

A failure event is assumed to follow the assumption of Poisson flow, so that the probability of at least one failure during time T is expressed as:

$$P(T) = 1 - \exp(-\lambda T) \quad (1)$$

Here, λ is the rate of events. The assumption of Poisson flow is only applicable if the failure events may be considered as independent events. By considering the rarity of a failure, this assumption seems to be reasonable and can be explicitly checked. The problem of determining the probability of a failure may be considered to be solved completely, if the rate of an event is found.

2. PEAKS-OVER-THRESHOLD METHOD

Statistical extrapolation, as is obvious from the term itself, is focused on the use of observed statistics for the prediction of the statistical characteristics of an event which is too rare to observe directly. In principle, extreme value theory (Gumbel, 1958) allows one to derive a distribution of the largest value observed during a given time. However, these derivations require exact knowledge of the distribution of the value and are quite lengthy even for a normal distribution. At the same time, formulae for the distribution of an extreme itself are quite simple. Depending on the distribution of the value, it could be one of three extreme value distributions: Gumbel, Freschet, and Weibull. As a result, the practical solution is to fit one of these distribution using either experimental or simulation data. This approach has been used by McTaggart (2000, 2000a) and MacTaggart & deKat (2000) to evaluate the probability of stability failure of an intact vessel.

The main difficulty with this approach is that collected motion data are statistically dominated by small motions, which may make a purely statistical prediction quite questionable. This difficulty can be avoided by applying the Principle of Separation. In terms of a statistical fit, this means that only the data above the threshold are used for extrapolation. The non-rare problem consists of a simple counting of the exceedances of a process over a given threshold. The threshold is chosen to separate regions where a linear solution is applicable from the regions where nonlinearity may be significant for the failure event of interest. The rare problem is solved by fitting an extreme value distribution to the data over the threshold. The method is generally known as the Peaks-Over-Threshold (POT) method. The application of the POT method for stability failures is considered by Campbell & Belenky (2010). The concept of the method is illustrated in Figure 1.

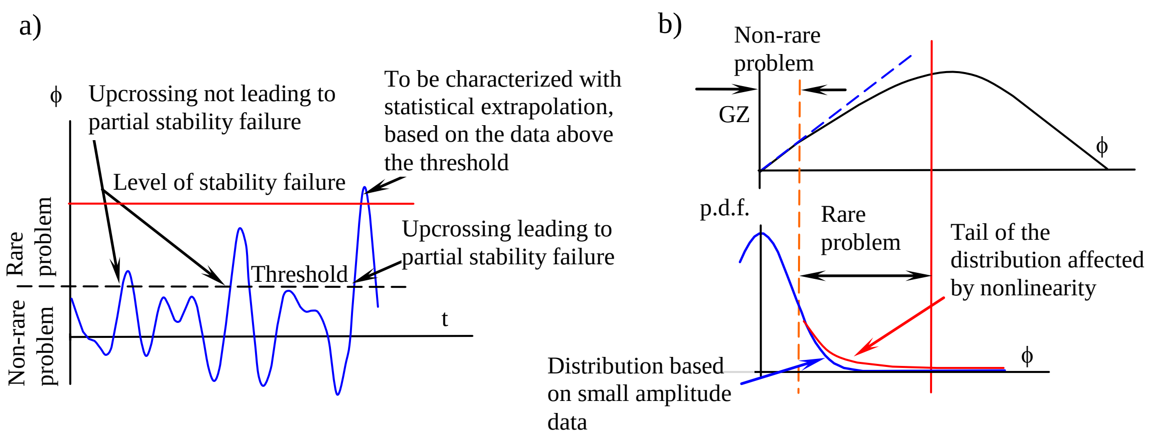

Fig. 1. The concept of the Peaks-Over-Threshold Method, a) the general scheme; b) influence of the threshold

The POT method separates the solution based on a threshold. The rate of events is determined in the form:

$$\lambda = \xi \cdot P_C \tag{2}$$

Here, ξ is exceedance rate of a threshold and P_C is a conditional probability of a given failure if the threshold has been crossed. It can also be considered as the fraction of upcrossings which lead to a failure. The evaluation of the upcrossing rate is the objective of the non-rare problem, while the conditional probability of failure is the objective of the rare problem.

The non-rare problem is well known from the theory of stochastic processes (e.g. Kramer & Leadbetter, 1967). If the distribution of a stationary process and its derivative are known, then the upcrossing rate can be expressed as:

$$\xi = f(\phi_{m0}) \int_0^{\infty} \dot{\phi} f(\dot{\phi}) d\dot{\phi} \tag{3}$$

The problem of modeling the distribution, based on the results of numerical simulations accounting for statistical uncertainty, is considered in Belenky & Weems (2008). The non-rare-problem can be solved statistically by counting the number of observed upcrossings (upcrossing rate is the mean number of events per unit of time). Confidence intervals for the estimate can be evaluated using the binomial distribution of an auxiliary random variable (Campbell & Belenky, 2010).

There are two possible formulations for the rare problem: using an extreme value distribution, or using a statistical fit of the peaks above the threshold. The formulation for the rate of events using extreme value distribution (Campbell & Belenky, 2010a) is given as:

$$\lambda = -\frac{1}{T_W} \ln\left(\exp(-\xi T_W) + (1 - \exp(-\xi T_W)) F_{EV}(\phi_{m2})\right) \tag{4}$$

Here, the level ϕ_{m2} is associated with stability failure and T_W is the observation time used to fit the extreme value distribution F_{EV}.

It is also possible to fit a distribution using a sample of the peaks that exceed the threshold. In this case, the formulation becomes very similar to the split-time method:

$$\lambda = \xi \cdot P_C \; ; \quad P_C = \int_{\phi_{m2}}^{\infty} f_{POT}(\phi) d\phi = 1 - F_{POT}(\phi_{m2}) \tag{5}$$

Here, f_{POT} is a distribution fitted using the available data of peaks over the threshold and F_{POT} is the corresponding cumulative distribution function.

The application of the POT method is limited by relatively mild nonlinearity. Roughly, this means that the level ϕ_{m2} associated with stability failure should not exceed the maximum of GZ curve. The data used for the rare problem may not contain enough statistical information on the behavior of the system beyond that point. The range around the maximum of the GZ curve is characterized by severe nonlinearity, caused by the simultaneous influence of the attractor at upright equilibrium and the repeller at the angle of vanishing stability. This severe nonlinearity is manifested in a very significant sensitivity to initial conditions, resulting in tremendous physical uncertainty for data collected in this range.

As the intended use of the POT method is the evaluation of the probability of a partial stability failure, the method has been generalized to handle cases when the Poisson flow assumption may not be directly applicable. This includes cases with following and stern quartering seas, parametric roll resonance, and other cases when the response spectrum becomes narrow and the response itself becomes clustered. It also includes cases when the failure is defined as the crossing of a level on either side: port or starboard. As the Poisson flow requirements must be met to relate the probability of failure with the time of exposure, an envelope is used instead of the actual process.

As the process of motions is not necessarily narrow banded, the upcrossing of a theoretical envelope may overestimate the rate of failures, therefore, a piecewise linear approximation can be used instead (see Figure 2).

All the calculations, including the counting of upcrossing and the fitting of distributions, are performed on the peak-based envelope rather then the process itself. This version of the POT method is known as the Envelope Peaks-Over–Threshold, or EPOT, method (Campbell & Belenky, 2010a).

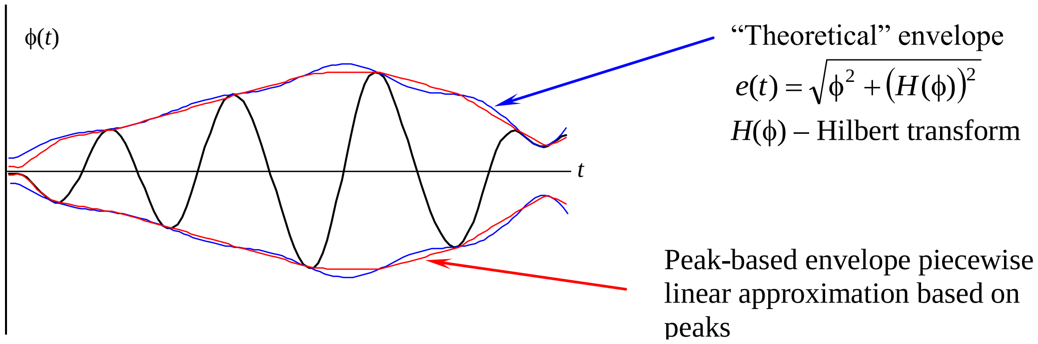

Fig. 2. Approximation of the envelope for a non-narrow banded process

The POT/EPOT method can utilize data from numerical simulation and/or physical model tests, but may not be applicable to conditions with severe nonlinearity, such as roll angles above the maximum of the GZ curve, as it does not contain an explicit model of extremely nonlinear motion.

3. SPLIT-TIME METHOD

The split-time method also separates the solution based on a threshold, however the method is meant to be applicable for severe nonlinearity, up to capsize. The rate of events is determined by formula (2), while the application of the split-time method for the evaluation of capsizing probability is illustrated in Figure 3.

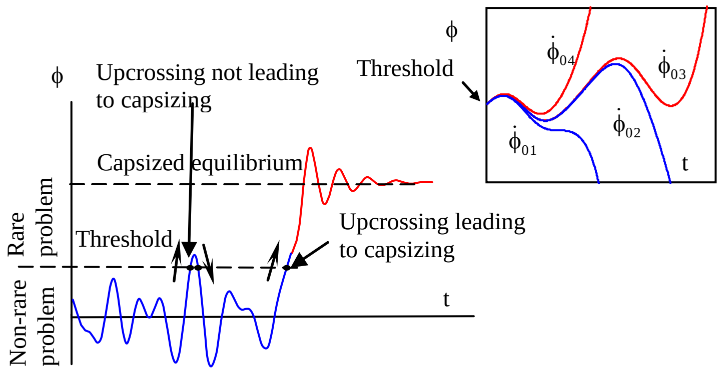

Fig. 3. Application of the split-time method for evaluating capsizing probability

The formulation of the non-rare problem is identical to that of the split-time method. The solution of the rare problem is found by a set of short simulations, which are focused on finding the initial conditions at upcrossing which lead to a response event of interest (e.g. a large roll angle or slamming event). For example, when the capsizing problem is considered with just one degree of freedom, the only initial condition needed is the roll rate at the upcrossing of the specified threshold. A value of the roll rate at upcrossing that exceeds the rate that leads to capsizing is the critical roll rate. Its value can be determined by a bisection-line method, as illustrated in the insert to Figure 3. Once the critical roll rate is determined, the conditional probability of capsizing after upcrossing is expressed as:

$$P_C = \int_{\dot{\phi}_{cr}}^{\infty} f_u(\dot{\phi})d\dot{\phi} \tag{6}$$

Here, $f_u(\dot{\phi})$ is the distribution of roll rate at upcrossing. It is not equal to the probability density function (pdf) of roll rates, as an instant of upcrossing is not just any occurrence. The distribution of roll rate at upcrossing can be expressed as follows (Belenky, *et al.*, 2008):

$$f_u(\dot{\phi}) = \frac{\dot{\phi}f(\dot{\phi})}{\int_0^{\infty}\dot{\phi}f(\dot{\phi})d\dot{\phi}} \tag{7}$$

This method can be applied for cases of extreme nonlinearity, as it contains an explicit model of very large motions. The method has been generalized for problems related to changing stability in waves, such as pure loss of stability, by tracking the change of the GZ curve in time (Belenky, *et al.*, 2009; 2010). An algorithm for these calculations was described by Belenky & Weems (2008a) and has been implemented in Large Amplitude Motion Program (LAMP) ship motion simulation code (Lin and Yue 1990, 1993). An example of the GZ curve change for the ONR Topside Series, tumblehome configuration (ONRTH) (Bishop, *et al.*, 2005) is shown in Fig. 4.

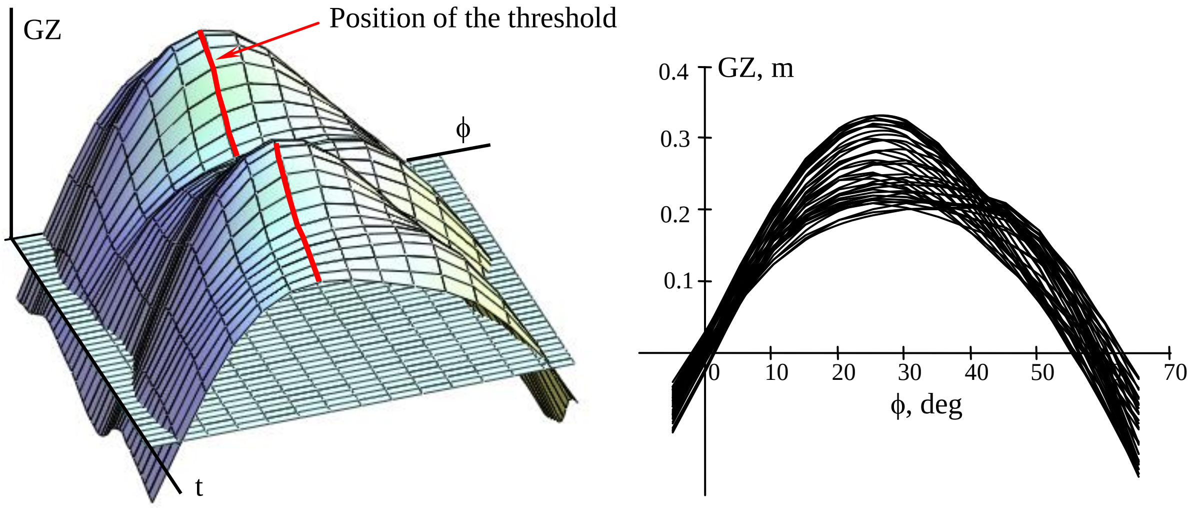

Fig. 4. Change of the GZ curve in time, ONRTH in stern quartering seas, sea state 7, speed 15 knots

The random changes of the GZ curve in irregular waves results in the necessity of modeling the threshold roll angle as a stochastic process. In principle, this does not change the general scheme of application of the split-time method (Fig. 5). The critical roll rate also becomes a stochastic process. To express probability of capsizing in this case, three stochastic processes must be introduced:

$$x(t) = \phi(t) - \phi_m(t) + \phi_{m0}\ ; \quad y(t) = \dot{\phi}_{cr}(t) - \dot{\phi}(t)\ ; \quad \dot{x}(t) = \dot{\phi}(t) - \dot{\phi}_m(t) \tag{8}$$

Here, ϕ_m(t) is the changing threshold, while ϕ_{m0} is a position of the threshold in calm water. The process $x(t)$ shows the distance to the moving thresholds, the process $\dot{x}(t)$ is its derivative, and the process $y(t)$ is the difference between the instantaneous and critical roll rate. Then, the rate of capsizing can be expressed as:

$$\lambda = \xi \cdot P_C\ ; \quad \xi = f(\phi_{m0})\int_0^{\infty}\dot{x}f(\dot{x})d\dot{x}\ ; \quad P_C = \int_{-\infty}^{0} f_u(y)dy \tag{9}$$

Here, $f_u(y)$ is a distribution of process $y(t)$, at an instant when the process $x(t)$ upcrosses the threshold. It has been shown (Belenky, *et al.*, 2009) that this distribution can be expressed as:

$$f_u(y) = \frac{\int_0^\infty \dot{x} f(\phi_{m0}, \dot{x}, y) d\dot{x} dy}{f(\phi_{m0},) \int_0^\infty \dot{x} f(\dot{x}) d\dot{x}} \tag{10}$$

In this case, the capsizing event is considered as an upcrossing through the time-dependent threshold, where the instantaneous roll rate exceeds the critical roll rate (Fig. 6).

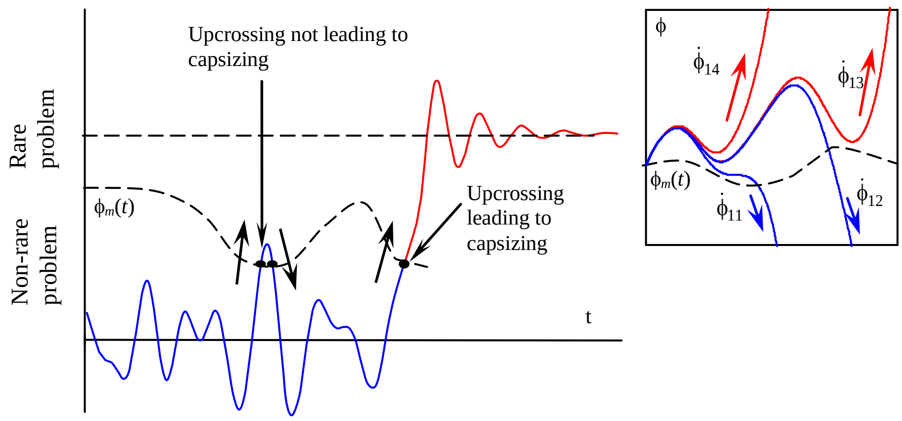

Fig. 5. Application of split-time method for the case of changing stability in waves

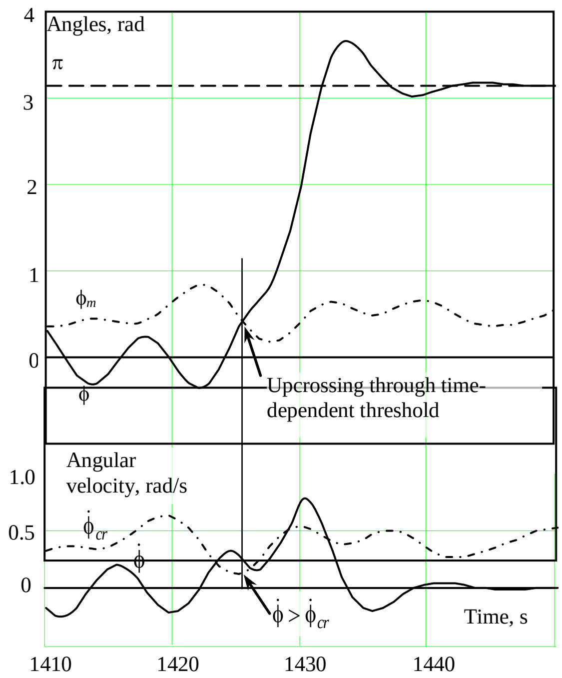

Fig. 6. Definition of capsizing with critical roll rate

The testing of the concept of the split-time method with changing stability has been performed with a piecewise linear system, where the decreasing part of stiffness was random. A special formulation of a piecewise linear term for stiffness allows for the derivation of a closed form solution. The convergence of statistics to the theoretical solution has been demonstrated (Belenky, *et al*., 2009).

It may be possible to extend the split time method for surf-riding by considering spatial phase portrait described by Spyrou (1996) as a frozen frame in time. A similar approach was used by Vishnubhota *et al.*, (2000) for the definition of invariant manifolds for irregular waves.

In principle, the split-time method can be used with numerical simulations and/or model test data. The solution of the non-rare problem does not encounter any significant difficulties, although the experimental implementation of the rare problem may be challenging, as it requires full control of initial conditions. Some additional discussion on this topic occurs later in this paper.

4. METHOD OF WAVE GROUPS

In contrast with the two previous approaches, the wave group method separates the problem in the time-domain, rather than in the state-space of the variables. The concept for this method is to extract a sequence of waves which can result in large amplitude excitation and evaluate the dynamic response to these particular sequences of waves, or "wave groups," with random initial conditions.

The occurrence and characteristics of wave groups has been studied extensively in oceanography; a brief review of these works is available from (Bassler, *et al.*, 2010). From the oceanographic point of view, there are two principle approaches to define wave groups: envelope theory (Longuet-Higgins, 1957) and using a Markov chain representation (Kimura, 1980). The formulations typically consider wave events to occur above a given threshold. However, from the ship response perspective, the characteristics which are important are different from the ones typically used to consider wave groups in the oceanographic context. Here both the amplitude and duration of the wave events must be considered. A definition of this wave sequence, or wave group, from the ship response perspective is proposed in Bassler, *et al.* (2010a) and is briefly discussed below (Figure 7).

It was shown that groups of large waves, as well as single large waves, can be reproduced deterministically in an experimental basin (e.g. Davis & Zarnick, 1964; Clauss, 2000; Bassler, *et al.*, 2009). Different aspects related to the application of assessing the response to wave groups and single large waves were discussed by Blocki (1980), Tikka & Paulling (1990), Boukhanovky & Degrtyarev (1996), and Alford, *et al.*, (2007). The first complete implementation of this type of approach with quantitative results was proposed during the SAFEDOR project (Spyrou & Themelis, 2005; Themelis & Spyrou, 2007; 2008). Similar approaches were followed more recently by Umeda, *et al.* (2007) and Bassler, *et al.* (2010, 2010a).

A failure can be caused by a single wave, or by a wave group, each resulting in different dynamical response characteristics for the ship. Therefore, the rate of failures must be expressed as a combination of both types of excitation events:

$$\lambda = \lambda_G \cdot P_{FEG} + \lambda_S \cdot P_{FES} \qquad (11)$$

Here, λ_G is the rate of encounter of a wave group, and λ_S is the rate of encounter of a single wave. P_{FEG} is the probability of failure if a wave group is encountered and P_{FES} is the probability of failure if a single wave is encountered.

The use of formula (1) for relating the probability of failure with the time of exposure implies the independence of encounters with either a wave group, or a single wave event. This leads to the definition in formula (11) of a wave group or a single wave, as shown in Figure 7. In this case, from the ship dynamics perspective, all three waves in the 1st group must be considered as one excitation sequence event, or wave group event. While all six waves in the 2nd group are considered another event.

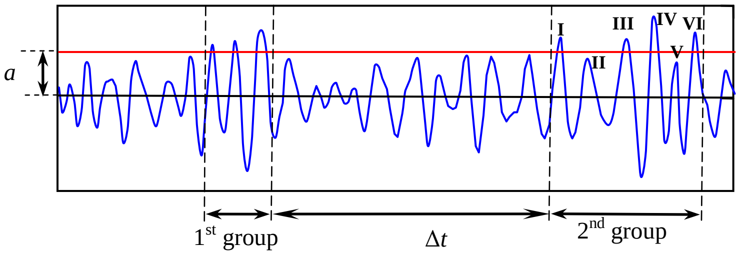

Fig. 7. Definition of wave groups from the ship dynamics perspective: wave events must occur far enough apart in time, so that the autocorrelation function of ship response effectively dies out.

In order to consider the response to a wave group encounter as a single random event, the response to the current wave group should be independent from the response to the previous group. As a result, there should be enough time between these groups for the autocorrelation function of the response to effectively die out. Therefore, large waves that are close to each other in sequence should be considered as part of the same sequence, or group, even if they are intermittently separated by a few small waves.

Two values are needed for this definition from the ship dynamics perspective, the threshold, *a*, and the time duration, Δt. The value of the threshold is defined as the amplitude of the excitation that leads to a significantly nonlinear response. One way to define this amplitude for roll motion is to use the roll response curve (Figure 8a), where ϕ_a/ϕ_v is the ratio of the amplitude of response and the angle of vanishing stability. For this motion, significant nonlinearity can be characterized as the theoretical possibility of fold bifurcation; this requires the existence of at least one point on the response curve where the tangent is vertical. The smallest amplitude of excitation, α_1, resulting in the appearance of such a point can be used to determine the amplitude of wave steepness, and to define the threshold. It may also be observed that this threshold also corresponds to the onset of nonlinearity in the ship-specific GZ curve.

The interval between the wave events, groups of large waves, or a single large wave, can be evaluated using the autocorrelation function, $R_\phi(\tau)$, of the linear or linearized response (Figure 8b) from the non-rare problem. The use of the linear or linearized response is fully justified, as the large amplitude response is only expected as a result of a single or small group of large waves. As a result, a linear or linearized model can be used to determine the response between the excitation events of interest.

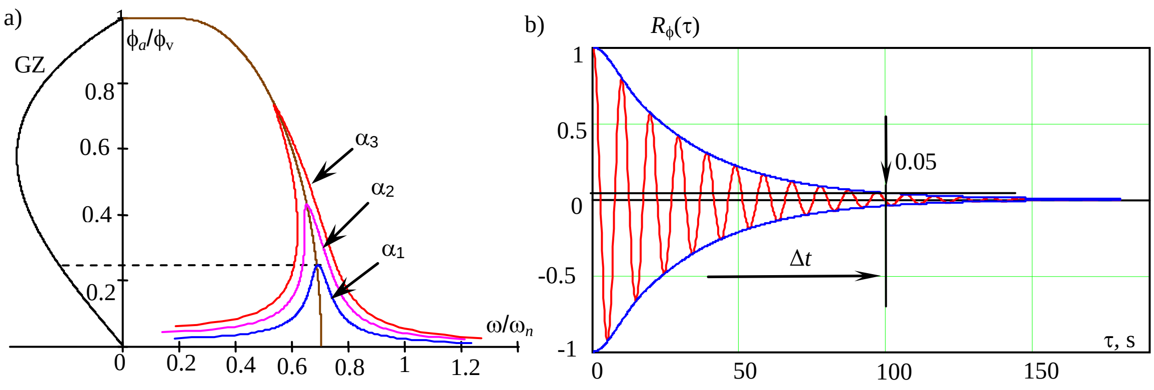

Fig. 8. The definition of wave groups: determining the threshold (a) and time duration (b)

The non-rare problem is simply the evaluation of the response of the linear or linearized system in the frequency domain. It produces the autocorrelation function that is used for the definition of the wave groups and characterizes the initial conditions for an encounter with the wave group, or a single large wave. The rare problem consists of the evaluation of the response of a nonlinear dynamical system to a deterministic group of waves, or to a single large deterministic wave. The initial conditions of the dynamical system at the moment of encounter with the wave event are random and have a normal distribution. The variance and mean, if any, are known from the non-rare problem.

Bassler, *et al.* (2010) described statistical testing of the concept using simulated wave elevation data. It was shown that a random event of encountering a wave group and a single large wave follows Poisson flow, as the time between these events has an exponential distribution (Figure 9a). A method to estimate rates of encounter for a group and a single wave was also proposed. This can be performed using the distribution of the number of waves in a group, or the probability mass function (pmf), where the first bin corresponds to the single large wave events (Figure 9b). A series of distributions of wave parameters were also studied; including amplitude, period, and steepness of the first, second, and third waves in a group. These data may be useful to help formulate a model of a wave group based on ship-specific characteristics and consideration for the different dynamical response mechanisms associated with single wave and multiple wave encounters.

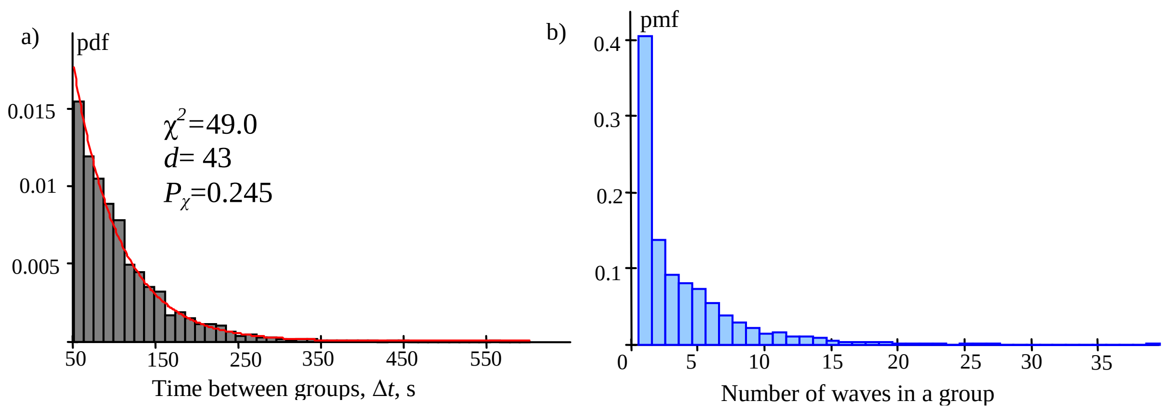

Fig. 9. Distribution of the time duration between groups (a), and the number of wave in a group (b); statistics estimated based on 200 simulated records of wave elevation, 30 min each; the threshold was a = 5 m, time between groups Δt = 50 s

The wave group method can be applied to model tests and/or numerical simulations. Using either technique, the probability of failure due to encounter, P_{FEG} and P_{FES}, as given in formula (10), can be determined. However, because of the formulation of the principle of separation in this method, precise control of initial conditions is necessary. For numerical simulations, one realization for each set of initial conditions can be used to determine the probability of failure due to the deterministic wave sequence. For model tests, because of inherent experimental uncertainties, a set of runs for each initial condition can be used to determine the probability of failure. The number of necessary experimental realizations is determined by the precision of initial condition control that is possible in a basin with deterministic wave generation capability.

5. THE PRINICPLE OF SEPARATION FOR VALIDATION

The validation of numerical tools intended to characterize rare events is more than just a challenging task. Some considerations on how the principle of separation can be used to assist with this task are discussed below. However, the practical implementation of these ideas remains the subject of future work.

Reed (2009) reviews different aspects related to the validation of simulation tools in context with two related processes: verification and accreditation. As emphasized by Reed, bifurcation analysis is important as it allows a demonstration that the theoretical basis of a simulation tool correctly reproduces the qualitative behavior of the nonlinear dynamical system. Quantitative validation may include comparisons with experimental measurements of the forces acting on ship and the resulting motions, including trajectories for maneuvering in steep waves.

The validation of simulation tools for large motions in irregular waves presents significant additional challenges related to the stochastic nature of the processes and the rarity of events, and also the problems related with nonlinear behaviors. An application of the principle of separation can simplify the required validation by allowing separate validation of non-rare and rare problems.

5.1 Validation of the Wave Model

Initial consideration is given to the validation of the wave model. The usual procedure is to compare spectra for the environmental conditions of interest. However, this may be insufficient for the simulation of rare events.

The wave model used in a simulation tool must provide a reasonable representation of the statistical characteristics of real waves, taking into account unavoidable uncertainties caused by the finite volume of experimental and simulated sample data. The first issue is related to the reliable comparison of two variance estimates, while both of them are random numbers.

A comparison of the distribution of wave elevation with the theoretical normal distribution may also prove useful. Because a wavemaker is also a nonlinear system, it may disturb the normality of the distribution. If an experiment is carried out in natural waves, such as in a large-scale or full-scale environment), the normality of the distribution can be disturbed by influences due to current, the shoreline, bottom effects, etc. If this is the case, the expectations for the accuracy of validation may need to be adjusted.

Because the interest is in simulation of the nonlinear ship response, consideration of the wave effects on the instantaneously submerged portion of the hull is necessary. Particularly for large, steep waves, the fluid pressures and orbital velocities below the free-surface may vary significantly. The wave model used in the simulation must have sufficient accuracy to represent the fluid behavior for the wave conditions of interest. Although difficult, model experiments may be performed to determine the velocity-field characteristics for these types of events (Minnick, *et al*. 2010) and then used to validate the selected wave model.

Another aspect to be addressed is the stationarity of experimental wave data. While it is not considered to be a problem for an experiment in a controlled environment, the stationarity of natural waves may be an issue. A metric used to assess the degree of stationarity in these conditions could be very useful for validation. One possible metric could be the use of the “run test” to evaluate the duration of stationarity, as discussed by Bendat and Piersol (2010).

If wave elevations are determined with the traditional inverse Fourier transform of the wave spectrum, the resulting time history is valid as a model of a stochastic process for a limited time. This time depends on the number of frequencies considered in the model. In the case of an insufficient number of frequencies, the restored time history of wave elevations may suffer from self-repeating effects (Belenky & Sevastianov, 2007). The presence of the self-repeating effect can be revealed by calculation of the autocorrelation function, using the cosine Fourier transform from the given spectrum, with an accepted frequency set.

5.2 Validation of Non-Rare Solutions

Validation of the solution for the non-rare problem has a mostly statistical character and may be different for each method.

The split-time method was originally developed to evaluate the probability of capsizing. However, it can be used to calculate the probability of partial stability failure (e.g. a large roll or yaw angle) as well. The threshold used in this method is fairly high, relative to the degree of nonlinearity of the system, and is random. The threshold is located on randomly changing GZ curve and therefore, depends on the method used to calculate the GZ curve in waves.

A direct validation of the calculated GZ curve in waves may not be simple. However, several key points can be checked experimentally. In one key stability condition of interest, a ship model travels with the wave celerity, close to the wave crest. The position of the ship model relative to the wave crest can be estimated from a video record. The model has an asymmetric load and therefore, is heeled. The angle of heel depends on the instantaneous righting arm in waves and can be compared with calculated value. Such an experiment could also reveal how much influence the local waterplane distortion has on the stability in waves and how accurate quasi-static calculations of the instantaneous GZ curve (Belenky & Weems, 2008a) really are.

Nevertheless, it may be possible to compare experimental and numerical solutions of the non-rare problem using a so-called “equivalent” threshold. This threshold is defined as follows: the same number of upcrossings of roll motion through an equivalent threshold exists as the roll process has through the random threshold. Then the rate of upcrossing through the equivalent threshold can be compared with experimental data. A similar approach may be taken towards the distribution of roll rates at upcrossing.

Another aspect of the validation of the non-rare solution is the direct comparison of the statistical characteristics of motions between an experiment and numerical simulation. As the threshold is relatively high, the motion response may be influenced by nonlinearity, including practical non-ergodicity (Belenky & Sevastianov, 2007). The effect of practical non-ergodicity may be observed as the increased difference between the statistical characteristics of different records belonging to the same ensemble. It is desirable to quantify the effect of non-ergodicity, as it is unrealistic to expect that the difference between the experiment and simulation can be smaller than the one caused by practical non-ergodicity.

In contrast to the split-time method, the non-rare solution of the peaks-over-threshold method is expected to be within the linear range. However, the tail of the distribution remains above the threshold. Therefore, the distribution of motions is, in fact, truncated. This must be accounted for when making a comparison of the variance estimate of the motion. The expected accuracy of the statistical estimate below the threshold is higher than the estimates of the whole process. The same can be observed about the distributions – a comparison of the distribution of values below the threshold is expected to yield a more definitive answer since the influence on nonlinearity and the associated uncertainties are minimal. The distribution of both motions and velocities are expected to be close to normal.

In the case of the EPOT method, the distribution of the peak-based envelope values is expected to be close to Rayleigh. In the case of a narrow banded process, the derivatives of a peak-based envelope are expected to be close to normal.

In both cases, the statistical comparison of the estimates of upcrossing rates is meant to be a very important validation parameter.

In principle, the validation of the non-rare problem for the wave group method is similar to the peaks-over-threshold. The difference is that the threshold is defined in terms of excitation, rather than the motion displacement. For this method, the distribution of motion and its derivative at upcrossing of the excitation process are the focus for validation.

5. 3 Validation of Rare Solutions

To validate the solution of the rare problem in the split-time method, one should demonstrate that a ship capsizes if a critical roll rate is exceeded. As it is very difficult to control initial roll rate, it may be attempted backwards by checking the roll rate at the instant of threshold crossing for a time-series where capsizing was actually observed. This experiment can be done in steep regular waves, where observing capsizing is not so difficult, and the instantaneous waterline is relatively easy to estimate– reducing the uncertainty of the calculations of the GZ curve in waves and the critical roll rate.

Validation of the rare solution for the POT/EPOT method appears to be rather straight forward. Two distributions of peaks (or envelope peaks) above the threshold can be compared using the Pierson chi-square goodness-of-fit test. Additionally, statistical frequencies which exceed a certain level above the threshold can be compared. A significant difference between them can be evaluated to determine if such a difference is caused by random factors.

Validation of the rare solution for the wave group method has two components. First, it must be demonstrated that the proposed model of wave groups is a true representation, supported by statistical data from realistic seaway conditions of interest. Second, the response of a ship model being excited by the wave group agrees well with the behavior obtained with the numerical simulation. This can be achieved by direct comparison with experimental results in a basin capable of reproducing deterministic wave groups (Bassler, *et al.*, 2009). However, as mentioned previously, the precise control of initial conditions is an essential component to this experimental validation.

6. SUMMARY

Failures related to large ship responses (motions and/or loads) in waves are rare, and large-amplitude ship motions are significantly influenced by the nonlinearity of the dynamical system. The necessity of modeling these significant nonlinearities results in only one option for simulation – the Monte-Carlo method in the time-domain, while the rarity of occurrence of the failure events makes direct “brute-force” approaches computationally cost prohibitive.

The principle of separation seems to provide an alternative to overcome this difficulty. The concept is to consider, separately, the nonlinear phenomena resulting in a large response and the conditions which result in the occurrence of such phenomena. This can be achieved by introducing an intermediate threshold, the crossing of which is frequent enough to be observable. The probabilistic characteristics of the conditions leading to a failure are considered at the instant of the crossing of the threshold. As a result, the problem is separated into two sub-problems: non-rare (crossing of the threshold) and rare (evaluation of conditions at the threshold which result in a failure).

Three methods that use this approach, are being developed for dynamic stability problems: the split-time method (where the stability failure is associated with the upcrossing of a time-variant roll-angle-threshold, with roll rate exceeding the critical value), the peaks-over-threshold method (using a fitted distribution of the peaks exceeding a fixed roll angle threshold), and the wave group method (where the ship response is evaluated) for a series of deterministic sequence of waves with random initial conditions.

An additional advantage of applying the principle of separation is the ability to perform validation separately for the non-rare and rare sub-problems. This separation allows both the physical and statistical uncertainty to be reduced, while also providing a robust validation technique for nonlinear phenomena.

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