Probabilistic assessment of intact stability

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Summary

This paper outlines a new concept for probabilistic intact ship stability assessment that bridges the deterministic and probabilistic approaches. In the paper is discussed in detail the method of calculation of the probability of instability. Furthermore, a new mathematical model in heave, sway and roll is introduced for numerical investigations of ship stability in beam seas.

1. Introduction

The intact stability of ships has returned to the forefront of research, as it seems that the time is ripe for establishing international regulations addressing explicitly and rationally the various modes of ship instability. Whilst the moderate step of ‘polishing’ the weather criterion is the one currently preferred by Administrations, the true challenge that lies ahead is the development of criteria that truly reflect the state-of-the-art in modelling and assessment of ship stability, that has progressed a long way beyond that of 1950. Toward this end, a new scientific basis is entailed that exploits the strengths of the finest modern approaches:

- Few could doubt that, solid understanding of the mechanics of the various modes of instability is a prerequisite for a sound approach. This could come through nonlinear dynamics analyses. Nonetheless this is still performed in a primarily deterministic context since the fully probabilistic appears to be premature (Spyrou & Thompson 2000). 'Threshold’ relationships in closed form between key parameters can be produced in certain cases (Thompson 1997; Spyrou et al 2002).
- To alleviate our difficulty for a ‘fully blown’ and practical probabilistic analysis of nonlinear ship motions, a meaningful interfacing of deterministic and probabilistic viewpoints should be established, exploiting advances in wave statistics and wave modelling (Myrhaug et al 2000; Stansell et al. 2002; Wist et al 2004; Spyrou 2005).
- It is desirable safety levels to reflect, quantitatively, levels of risk.
- Up-to-date mathematical models with sufficient detail and interactions should be used, to an extent that allows effective integration.

A concept for stability assessment that evolves along the above lines is proposed. It shall be explained below through a number of steps:

2. Description of proposed concept for stability assessment

2.1 *Type of assessment.*

An assessment could be “short” or “long-term”, depending on the time of exposure to the environment. A short-term assessment could be aimed at specifying the level of risk associated with a single trip. Thus, it could be an instrument in a system of departure control like the one applied in Greece for passenger ships (Spyrou et al. 2004), or internally within a ship management company. Also, it could be combined with an operational guidance and a system of weather routeing. Long-term assessment, on the other hand, supplies key data for design. For practical purposes it could refer to one year’s period; yet the probability of capsize should be calculated for a vessel’s lifetime. Short and long-term assessments need to be consistent with each other.

2.2 ***Type of service.***

For long-term assessment in particular, the service profile could be restricted, unrestricted, or combined. Restricted service means to examine only specific routes, for example a set of routes of a ferry in the Mediterranean. On the contrary, the unrestricted service sets no limits in the geographical area, and would be representative, e.g. of tramp shipping. A grid of spectra covering the geographical area as well as their seasonal variation at each node will be required.

2.3 ***Portfolio of stability criteria.***

Different ship types are prone to different modes of unstable behaviour. As a matter of fact the criteria should be ship-type specific.

Tendency for parametric rolling should be one of the assessment criteria, as also pure - loss of stability on a wave crest in following seas.

Resonant rolling in extreme beam wind/waves is more relevant for smaller vessels. The same applies for breaking waves from abeam that may lead to dangerous impact loads.

Broaching, including the so-called cumulative type should be examined for all ships.

2.4 ***Norms of unsafe behaviour.***

We envisage the setting of warning and failure levels per criterion and ship type, determined by threshold angular and linear displacements and accelerations, referring respectively to the safety of the cargo and the ship. Exceedance of the warning level could be permissible with controlled probability; whereas the failure level should never be exceeded for the acceptable level of risk. The setting of warning level per criterion should play a cautionary role. In Fig. 1 is shown an example of possible norms for a containership.

2.5 ***Bridging the deterministic and probabilistic ‘worlds’.***

The probability of occurrence of a certain instability could be assumed as equal to the probability of encounter of the critical (or worse) wave group that generates this instability. Observing the criteria mentioned earlier, one notes that they correspond either to manifestations of resonant behaviour

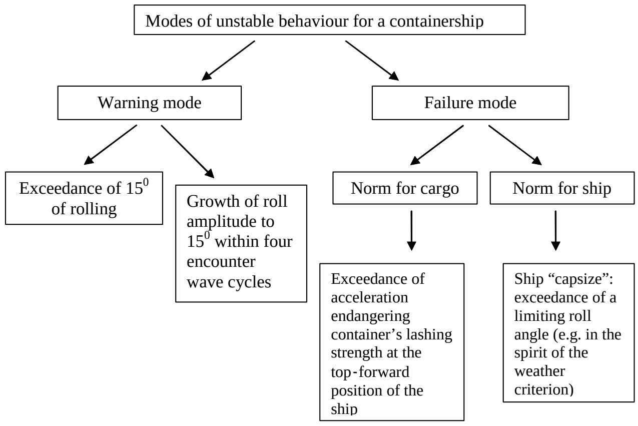

Figure 1: Tentative safety levels for a containership.

(which entails some regularity in the excitation), or to the encounter of a single critical wave. Therefore, on might dare to “break” the problem into two parts: one deterministic for deducing the specification of the critical wave group (meaning the height, period and run length) focused purely on ship motion dynamics; and one probabilistic centred on sea wave statistics for determining the probability of encounter of such a wave group.

Considering the deterministic part, it is essential to recognize that the character of the wave group that leads to exceedance of the norm of some criterion x could be found with more than one methods: A popular option is through numerical simulation. However several runs (and even runs on more than one codes) should be performed to count for effects of initial conditions etc. For an up-to-date review of existing numerical codes see the report of ITTC (2005). A second alternative is to apply analytical techniques to capture the key system dynamics. For example, it is feasible to seek analytical expressions for the growth of resonant roll in beam or in longitudinal seas (Spyrou, 2005) or to apply global stability analysis methods like Melnikov’s method in order to determine the ultimately critical combination of wave excitation, damping and restoring capability (Spyrou 2000).

2.6 ***Determining the probability of encounter of critical wave groups.***

For resonant - type criteria (beam-sea resonance, parametric rolling) we need to calculate the probability to encounter a wave group with a number of successive wave periods near the critical value and the corresponding wave heights consistently above the critical height.

Bivariate distributions of wave height and period have been proposed by a number of investigators (Longuet – Higgins 1975 & 1983; Cavanié et al. 1976; Tayfun 1993). The probability density function (pdf) proposed by Tayfun (1993) is:

$$f(h,\tau)=C_T\,h\left(1+\frac{1-\kappa^2}{32\,\kappa\,h^2}\right)e^{-\frac{1}{2}\left[\frac{4h^4}{1+\kappa}+\left(\frac{\tau-\mu_{\tau/h}}{\sigma_{\tau/h}}\right)^2\right]} \tag{1}$$

where:

$$\mu_{\tau/h}=1+\varepsilon^2(1+\varepsilon^2)^{-3/2} \tag{2}$$

$$\sigma_{\tau/h}=\frac{2\varepsilon}{\sqrt{8}h(1+\varepsilon^2)} \tag{3}$$

$$C_T=\frac{2\sqrt{2}}{2\sqrt{4\pi\kappa(1+\kappa)}\sigma_{\tau/h}} \tag{4}$$

$\mu_{\tau/h}$ and $\sigma_{\tau/h}$ are the conditional mean and standard deviation, C_T a normalizing factor, $h=\dfrac{H}{H_{rms}}$, $\tau=\dfrac{T}{T_m}$ the dimensionless wave height and period, $T_m=2\pi\dfrac{m_0}{m_1}$ the mean spectral period, m_j the j^{th} ordinary moment of wave spectrum. According to Longuet – Higgins (1975) the spectrum bandwidth ε is given by:

$$\varepsilon=\left(\frac{m_0m_2}{m_1^2}-1\right)^{-1/2} \tag{5}$$

The parameter κ depends on T_m and the frequency spectrum. According to (Stansell et al, 2002) is calculated:

$$\kappa(\tau)=\frac{1}{m_0}\left|\int_0^\infty S(f)e^{i2pf\tau}df\right|,\quad \tau=T_m \tag{6}$$

Tayfun (1993) approximated the conditional distribution of successive wave periods given the wave height on the basis of the Gaussian distribution for one wave period. Wist et al. (2004) noted that, for three wave periods at least, the multivariate Gaussian distribution is a satisfactory model of the conditional distribution. Their conditional pdf of p successive wave periods $T=\left[T_1,\ldots,T_p\right]^T$, given the corresponding wave heights exceeding the threshold h_{cr} is given as follows:

$$f_{\mathbf{T/H}}\left(\boldsymbol{\tau}/\mathbf{h}>h_{cr}\right)=\frac{e^{-\frac{1}{2}\left(\boldsymbol{\tau}-\boldsymbol{\mu}_{\tau/h_{cr}}\right)^T\Sigma^{-1}_{\tau/h_{cr}}\left(\boldsymbol{\tau}-\boldsymbol{\mu}_{\tau/h_{cr}}\right)}}{(2\pi)^{p/2}\left|\Sigma_{\tau/h_{cr}}\right|^{1/2}} \tag{7}$$

where the correlation matrix is given by:

$$\Sigma_{\tau/h_{cr}}=\begin{bmatrix}\sigma^2_{\tau/h_{cr,1}} & Cov\left[T_1,T_2/h_{cr}\right] & Cov\left[T_1,T_p/h_{cr}\right]\\ & \cdots & \\ Cov\left[T_1,T_p/h_{cr}\right] & \cdots & \sigma^2_{\tau/h_{cr,p}}\end{bmatrix} \tag{8}$$

and $Cov\left[T_i,T_j/h_{cr}\right]=\rho_{ij}\sigma_{\tau/h_{cr,i}}\sigma_{\tau/h_{cr,j}}$. The mean values $\mu_{\tau/h_{cr}}$ and the standard deviations $\sigma_{\tau/hcr,i}$ are calculated with equations (2) and (3). Assuming the Markov chain property, the correlation coefficients ρ_{ij} could be calculated as follows:

$$\rho_{1j}=\rho_{12}^{j-1} \tag{9}$$

The correlation coefficient ρ_{12} of two successive wave heights is calculated according to the next equation:

$$\rho_{12}=\frac{E(\kappa)-\left(1-k^2\right)\frac{K(\kappa)}{2}-\frac{\pi}{4}}{1-\frac{\pi}{4}}\approx \approx\frac{\pi}{16-4\pi}\left(\kappa^2+\frac{\kappa^4}{16}+\frac{\kappa^6}{64}\right) \tag{10}$$

where $E(\ )$, $K(\ )$ are complete elliptic integrals of the first and second kind, respectively.

It was noted earlier that, for resonance phenomena, it would be desirable to know the probability a number of successive wave periods to lie in some time interval $\left[\tau_1,\tau_2\right]$, given that the corresponding wave heights exceed the critical level h_{cr} that is already estimated from deterministic analysis. We have carried out calculation of such probabilities on the basis of the above procedure and the result is shown in Fig. 2. Specifically, it is shown the probability to encounter a wave group with $p=2,\ldots,6$ successive wave periods in a time interval around the spectral mean period (of course this value is not so critical for capsize, it is selected only for demonstration of the approach - we could have taken, instead, the time interval around the roll natural period, half of it, etc.) for a range of critical wave heights H_{cr} .

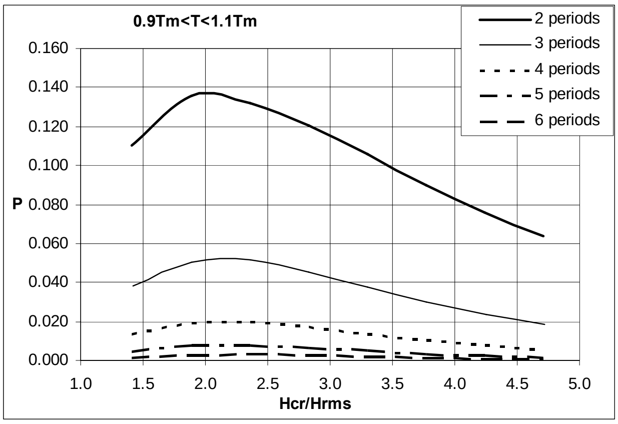

Figure 2: Calculated probability of encounter of specific wave groups

For other stability criteria (e.g. pure loss of stability) the joint distribution of wave height and period $p(H,T)$, like the ones of Longuet – Higgins or Kimura, are directly applicable. Furthermore, a pdf of wind speed $p(U_W)$ can be used (e.g. the Rayleigh distribution, see Bledermann 2004), through the calculation from the previous deterministic step of the critical wind speed. Assuming the events as independent (of course this is debatable), the probability to encounter the critical excitation from wind and wave can be calculated as (see also Fig. 3):

$$P[H,T,U] = \iiint p(H,T)p(U_W)dHdTdU \quad (11)$$

In summing up the probabilities, the percentage of time a ship spends in following, beam seas etc. should be taken into account as well as the distribution of operational time in terms of geographical areas.

2.7 *Risk assessment*

It is a matter of debate whether, at this stage, a stability assessment method should end with the calculation of the probability of instability; or whether the significant further step of calculating risks should be undertaken. A number of issues need to be sorted out concerning the quantification of the consequences of instability, in the various domains. This matter will be elaborated in a dedicated publication.

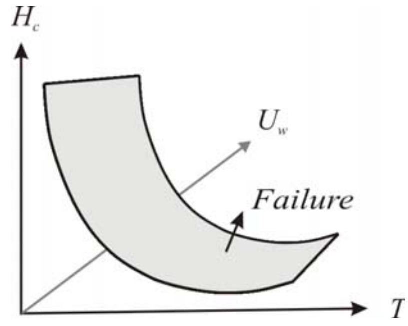

Figure 3: Locus of critical weather parameters

3. Mathematical model of coupled roll in beam seas.

A new mathematical model is currently under development that could be used for analysing coupled rolling motion in beam seas. This model is outlined in the following:

3.1 *Equations of motions*

From kinematics and in accordance to Fig. 4, the equations of motion in heave, sway and roll are written as follows:

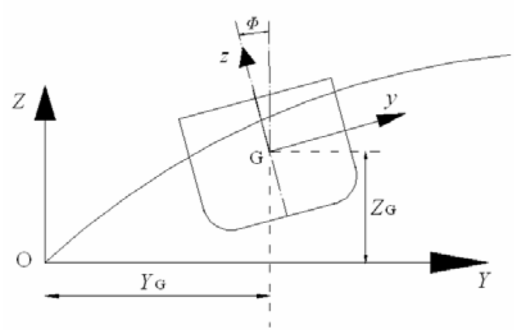

*Figure 4: The inertial (*OYZ*) and body fixed (*Gyz*) coordinate systems.*

$$m(\dot{v} - \dot{\phi}w) = \sum F_y \quad (12)$$

$$m(\dot{w} + \dot{\phi}v) = \sum F_z \quad (13)$$

$$I_G\ddot{\phi} = \sum M_G \quad (14)$$

where v, w are the sway and heave velocity of the ship's centre of gravity and $\dot{\phi}$ is the roll angular velocity, m and I_G are, mass and mass moment of inertia around x. The transformation between the inertial and body fixed coordinate systems is well-known:

$$\begin{pmatrix} \dot{Y}_G \\ \dot{Z}_G \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ w \\ \dot{\phi} \end{pmatrix} \qquad (15)$$

The two forces and the moment that appear at the right-hand-side of (12)-(14) can be decomposed as follows:

$$\sum F = F_{Hs} + F_W{}^{FK} + F_W{}^{D} + F_R + F_V \qquad (16)$$

F_{Hs} is hydrostatic, F_W^{FK} is Froude – Krylov, F_W^{D} is diffraction, F_R is radiation and F_V is the viscous force.

3.2 *Calculation of excitations*

In linear wave theory, the total wave velocity potential is the sum of the potentials of incident wave, diffraction and radiation. The hydrostatic and Froude – Krylov (hydrodynamic) forces are estimated by the integration of the incident wave pressure (static and dynamic respectively) over the wetted surface of the ship. For regular waves, the incident wave potential is calculated from:

$$\Phi_I = \frac{Ag}{\omega_w} e^{kZ^*} \sin(kY - \omega_w t) \qquad (17)$$

$$Z^* = Z - A\cos(kY - \omega_w t) \qquad (18)$$

From Bernoulli’s equation the pressure is:

$$P = -\rho \left(gZ^* + \frac{\partial \Phi_I}{\partial t} + \frac{1}{2} \nabla\Phi_I \nabla\Phi_I \right) \qquad (19)$$

The hydrostatic and Froude - Krylov forces are repetitively:

$$F_{HSi}(t) = -\rho g \iint_{S(t)} Z^* \vec{n}_i ds \text{ , for } i=2, 3, 4 \qquad (20)$$

$$F_W^{FK}(t) = -\iint_{S(t)} \rho \frac{\partial \Phi_I}{\partial t} \vec{n}_i ds \text{ , for } i=2, 3, 4 \qquad (21)$$

where i = 2, 3, 4 correspond to sway, heave and roll motion, ρ is seawater density and $S(t)$ is the instantaneous wetted surface. We should mention that the integration is performed over the instantaneous wetted surface and pressures are calculated from the exact wave elevation. As a matter of fact, the nonlinear part of the forces is taken into account, which is important for the accurate simulation of the large motions of the ship. The nonlinear Froude-Krylov force has a nonzero mean, in other words a drift force is present, which is likely to introduce a bias to the roll motion. The diffraction force should also bring about drift. It is well known that this drift force is proportional to the square of the wave amplitude and it increases in the short wave range where the reflection of waves and relative heave motion are more intense (e.g. Maruo, 1960; Newman, 1967). Kuroda & Ikeda (2002) have focused on this force in their investigation of ship roll with drift. This component is currently implemented in our model.

The radiation forces are frequency dependent. In order to study transient behaviour it is necessary to transform these from the frequency domain to the time domain. Using the impulse response function, obtained as the Fourier transform of the frequency dependent radiation transfer function, the radiation forces will be (Cummins, 1962):

$$F_{Rj}(t) = -a_{jk}(\infty)\ddot{s}_k - \int_0^{+\infty} K_{jk}(\tau)\dot{s}_k(t-\tau)d\tau \qquad (22)$$

for j, k=2, 3, 4

$$K_{jk}(\tau) = \frac{2}{\pi}\int_0^{\infty} b_{jk}(\omega_e)\cos(\omega_e \tau)d\omega \qquad (23)$$

The convolution integral is the well-known memory effect. a_{jk}, b_{jk} are the added mass and damping coefficients. $\dot{s}_k$, $\ddot{s}_k$ are velocity and acceleration of the ship in the k direction of motion and ω_e is the encounter frequency. In our model we use a state-space approximation of the radiation force in order to maintain the mathematical model in the form of a system of o.d.e.s which enables easier consideration of nonlinear dynamics.

Our model calculates also the sway drag force, roll damping, and cross coupling forces between sway, heave and roll. For example, the drag force due to bilge keels is calculated as follows (see also Fig. 4):

$$F_{BY} = \frac{1}{2}\rho(\dot{Y}_G - r_A\dot{\phi}\cos(\theta) - u_2)\left|\dot{Y}_G - r_A\dot{\phi}\cos(\theta) - u_2\right| C_D A_{BK}$$

$$F_{BZ} = \frac{1}{2}\rho(\dot{Z}_G - r_A\dot{\phi}\sin(\theta) - u_3)\left|\dot{Z}_G - r_A\dot{\phi}\sin(\theta) - u_3\right| C_D A_{BK}$$

$$M_B = -r_A\,[F_{SZ}\cos(\theta_1) + F_{SY}\sin(\theta_1)] \qquad (24)$$

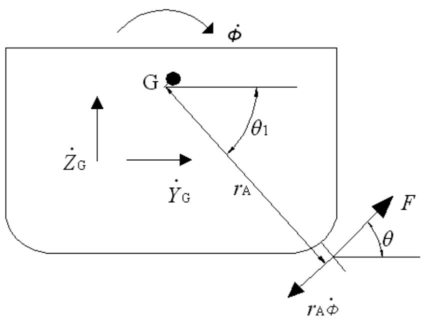

Figure 5: Bilge keel damping forces.

C_D is the drag coefficient and A_{BK} the total bilge keel area. Other symbols are explained in Fig. 5. The method takes into account the local relative velocities along the hull, using the sway, heave and roll velocities ($\dot{Y}_G, \dot{Z}_G, \dot{\phi}$), the wave particle velocities u_2 and u_3 (eqn. 25 and 26 below) as well as the detailed geometry of the hull.

$$u_2 = \frac{\partial \Phi_I}{\partial y} \qquad (25)$$

$$u_3 = \frac{\partial \Phi_I}{\partial z} \qquad (26)$$

The numerical model is programmed completely in a Mathematica environment. As said, the system contains only ordinary differential equations (the convolution integrals are approximated by sets of o.d.e.s) which enable stability analysis in a straightforward way. Input data are, concerning the ship: the hull geometry, her mass and the distribution of mass; and for the incident wave, its height and frequency. The code creates panels over the hull whereon the static and dynamic pressures are calculated at successive time steps, as well as the angle between the horizontal plane and the normal vector of the panel.

3.3. *Application of the mathematical model*

As application we have studied the Japanese fishing vessel that has been the object of a benchmarking exercise by the Stability in Waves Committee of ITTC (ITTC 2005). Her panelization is shown in Fig.6. Some preliminary output from the mathematical model is shown below. Firstly, simulation of a roll decay test from extreme angle of release, approximately 85% of the angle of vanishing stability (Fig. 7). Also, numerical simulation of roll response near resonance for moderate wave steepness is shown in Fig. 8. Of interest is also the sway response as there is drift motion of the vessel (Fig. 9). Finally, we examined the effect of wave steepness on the mean roll angle (Fig. 10). As noticed, there seems to be an almost linear increase of the mean roll angle (towards the weather side), which, for $H/\lambda = 1/12$, could reach nearly 16% of the roll amplitude. The mean roll angle, as we believe, comes from the lateral wave drift force combined with the lateral resistance force and the pair produces an extra roll moment that tends to rotate the ship, in the present case, to the weather side. This bias in rolling motion should be seriously taken into account as it may reduce disproportionally the dynamic stability of the ship (Thompson, 1997). The drift motion tends also to lower the encounter frequency and, as a result, larger amplitude motion appears later in terms of wave frequency. This effect is shown in Fig. 9 where we present points of the roll response curve taking into account the mean drift velocity in the calculation of encounter frequency.

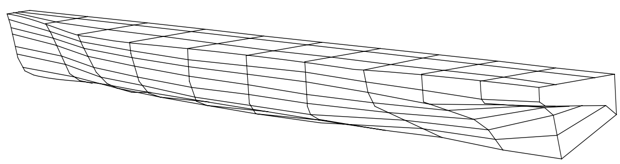

Figure 6: Half-hull panelization.

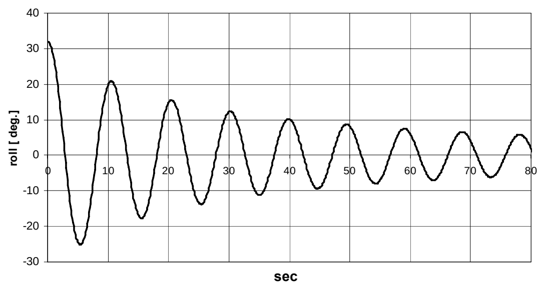

Figure 7: Roll decay test

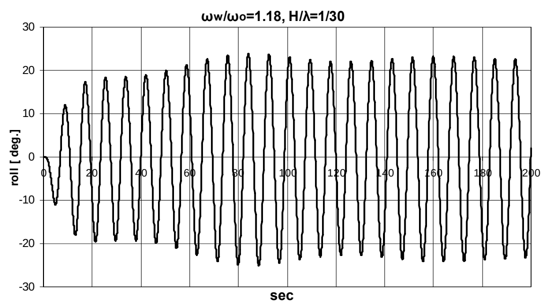

Figure 8: Roll response in beam waves

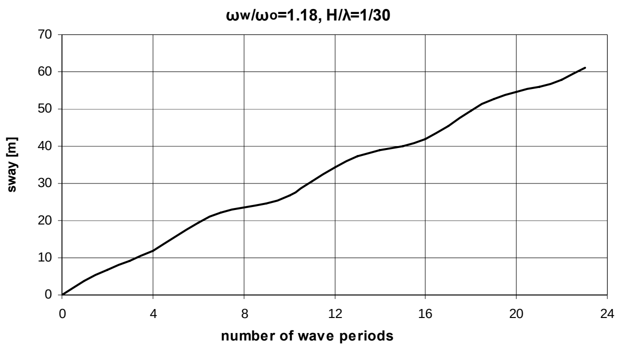

Figure 9: Sway response.

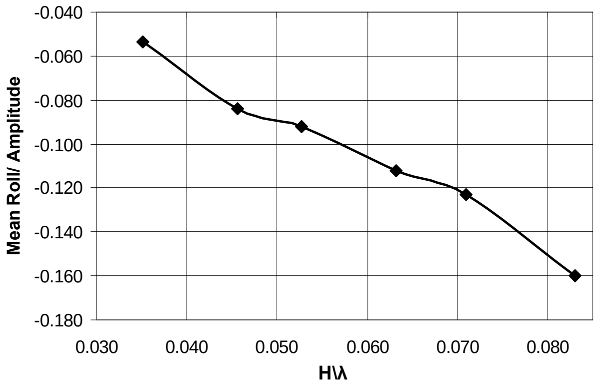

Figure 10: Effect of wave steepness on the mean roll angle ($\omega_w / \omega_o = 1.312$).

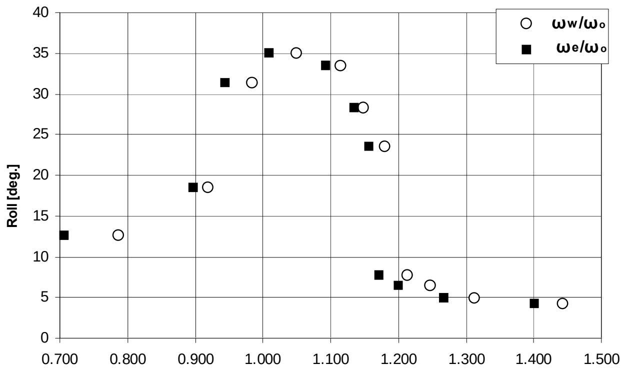

Figure 11: Roll response diagram for $H/\lambda = 1/30$.

4. A simplified worked example

Type of ship:

Fishing vessel (the one used previously).

Type of assessment:

Short term.

Assessment criteria:

Resonant rolling (without wind effect).

Norm of unsafe ship behaviour:

Warning mode: 20^0 roll angle at steady state.

Specification of critical wave group:

From our numerical code, we calculate that we need six successive waves with encounter periods in the range $T_e = (7.813, 8.667)$ sec and wave height above $H_{cr} = 3.518m$ in order to satisfy the above warning mode. In fact, we determine an area of the critical surface that consists of triplets of (H, T_e, n), (like the locus of Fig. 3) where n the number of waves.

Probability to encounter the specified wave group or 'worse'.

We use for example the I.T.T.C. spectrum with $H_S = 2.5m$. The calculated probability is:

$$P[T_1, T_2, ...T_6 / H > H_{cr}] = 0.0000143262$$

where $T_1, T_2, ...T_6 = T_e$.

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