SOME RECENT ADVANCES IN THE ANALYSIS OF SHIP ROLL MOTION

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ABSTRACT

In an effort to place our previous investigations of ship roll dynamics within physically based limits, we extend a numerical steady state analysis to higher frequency forcing. Working with a simple nonlinear roll model, a number of different phenomena are discussed at above resonant frequencies, including sub-critical flip bifurcations and a *second resonance region*.

We then discuss a highly generalised approach to roll decay data analysis that does not require us to predefine damping or restoring functions. The problem is approached from a local fitting standpoint. As a result the method has potential for further extension to more complex models of damping as well as restoring force curves.

INTRODUCTION

Previous studies of beam sea roll models [1, 2, 3] have focussed on the resonant region, where linear theory would predict capsize to be most likely. Here, we explore the steady state dynamics at higher frequencies of forcing and discuss some new features of the control space. In particular we discuss capsizing wave slopes at high forcing frequencies. Interestingly, the capsizing slopes are of similar magnitude to those at resonance.

The derivation of accurate representations of damping functions as part of a ship roll model is highly desirable in the study of roll dynamics. Roll damping functions, however, are extremely difficult to obtain by theory or experiment. The tendency has been to remain with simple linear or low order nonlinear velocity dependent models [4, 5]. To test the validity of such approaches we must be able to obtain damping functions from experimental data efficiently and accurately. However the difficulty in separating parameters in any such analysis has hindered improvement on existing ideas. Here we approach the problem from a local fitting standpoint using linear approximations to reconstruct a globally nonlinear curve.

Although the approach discussed is applied over all the data, separating angle and velocity dependent terms remains a serious problem. We conclude by briefly discussing some ideas for improving our ability to deal with these difficulties.

HIGH FREQUENCY FORCING

During the design of roll experiments it is necessary to ascertain the forcing parameter ranges over which our nonlinear oscillator model is valid. In particular we need to consider two limits;

the maximum wave slope and frequency. The former is a consequence of the nature of waves and simple to evaluate. The latter is a more subtle problem related to the fact that the beam of a ship must be small compared to the wavelength for the model to be applicable.

Firstly we write our roll equation as,

$$I\theta'' + B(\theta') + mgGZ(\theta) = IAk\omega^2 \sin(\omega_f \tau) \tag{1}$$

where the prime denotes differentiation with respect to real (unscaled) time, τ, I is the rotational moment of inertia about the centre of gravity (incorporating any added hydrodynamic mass), θ is the roll angle relative to the wave normal, $B(\theta')$ is the non-linear damping function, $GZ(\theta)$ is the roll restoring force, Ak is the wave slope amplitude (A is the wave height and k the wave number) and ω_f is the wave frequency. We also write ω_n as the natural frequency of linearised ship motions.

We then utilise a simple non-dimensionalised model for roll motion, the Helmholtz-Thompson equation [2, 6]

$$\ddot{x} + \beta\dot{x} + x - x^2 = F\sin\omega t \tag{2}$$

where, in terms of (1), our two parameters are $F = Ak\omega^2/\theta_V$ and $\omega = \omega_f/\omega_n$ with $x = \theta/\theta_V$. We also introduce the parameter $J = Ak/(2\zeta\theta_V) = F/\omega^2$ which is a scaled measure of wave slope based on a linear capsize analysis, [7]. Here, θ_V is the angle of vanishing stability and ζ the effective linear damping coefficient. We also set $\beta = 2\zeta = 0.1$.

The first limit is a consequence of the nature of water waves. For a steepness above $H/\lambda \approx 1/7$ the wave will break and the use of a simple sinusoidal forcing is no longer valid. Thus, with wave slope $Ak = \pi H/\lambda$, we can write,

$$J^{max} \approx \frac{\pi}{14\zeta\theta_V} \tag{3}$$

The model assumes that the ship tries to follow the motions of the water particles in the wave and does not interfere with the pressures in the wave. This is only valid when the beam of the ship is small compared to the wavelength. We can thus write a minimum wavelength, λ^{min}, permissable in terms of the beam, b

$$\lambda^{min} = \epsilon b \tag{4}$$

where we take, as a first estimate, $\epsilon \approx 6$. This in turn gives us a maximum forcing frequency

$$\omega_f^{max} = \sqrt{\frac{2\pi g}{\lambda^{min}}} = \sqrt{\frac{2\pi g}{\epsilon b}} \tag{5}$$

leading to

$$\omega^{max} = \frac{\omega_f^{max}}{\omega_n} = T_n\sqrt{\frac{g}{2\pi\epsilon b}} \tag{6}$$

where ω_n and T_n are the natural roll frequency and period of the ship. Note that this second limit is due to the approximations of our roll model whereas the first is a feature of wave behaviour.

Substituting in two real ship values (a purse seiner [8] and a container [9]) for beam dimension and natural frequency we find,

Ship	θ_V [degrees]	T_n [s]	b[m]	ω^{max}	J^{max}
Purse Seiner	40	7.47	7.6	1.4	3.2
Container	-	19.4	25.4	1.9	-

Therefore as a first step we extend previous steady state analyses to frequencies up to $\omega \approx 2$ with the additional limit $J^{max} \approx 3$. Using numerical techniques we are able to plot the development of steady state oscillations whilst varying wave amplitude (or slope). This process is repeated for a range of forcing frequencies. For below resonance frequencies it has been shown [10] that for (2), as F is increased, escape (corresponding to capsize) occurs with a jump from a fold bifurcation. Above resonance the system escapes from a chaotic orbit after a period doubling cascade. For the latter case the initial flip bifurcation is often taken to be a sufficiently accurate indicator of capsize in the control space.

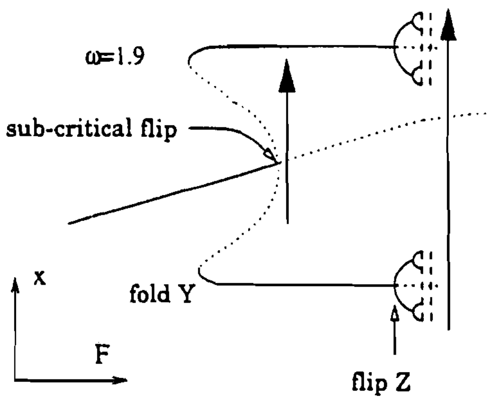

Figure 1: Schematic example of a high frequency capsize mechanism

Figure 1 shows a schematic example of a discontinuous jump found at higher frequency forcing. The solution path shows restabilisation after a sub-critical flip bifurcation onto a period 2 oscillation. Here we would see a sudden increase in roll amplitude. In this case the flip bifurcation is not a good estimate of capsize. With further increase in F, the system undergoes a period doubling cascade to chaos, before escaping. Note that the fold Y and the subsequent flip Z are bifurcations of the period 2 oscillation.

We illustrate the high frequency bifurcations in a control space diagram, figure 2. The steady state capsize line show the wave slope at which capsize occurs when J is increased in small steps from zero. The ragged nature of this line is primarily due to the computational approximations required in the numerical procedure.

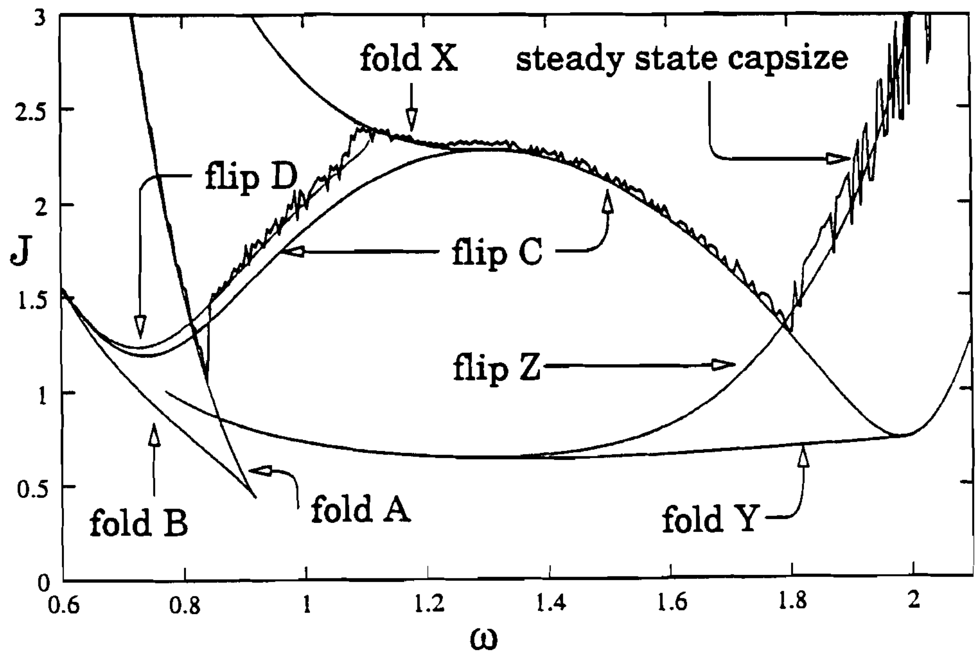

Figure 2: Bifurcation diagram for the capsize equation (2), extended to higher ω. The flip C is super-critical at large and small frequencies: it is sub-critical betweeen the codimension 2 events at which is meets fold X ($\omega \approx 1.3$) and fold Y ($\omega \approx 2$). Damping coefficient, $\beta = 0.1$.

Importantly we find that the flip C is a good estimate of capsize only below $\omega \approx 1.8$. However, the discontinuous jump (at the sub-critical flip) for $\omega > 1.8$ must be considered a highly dangerous phenomenon.

Of further interest is the existence of an effective *second resonance region* at $\omega \approx 1.8$ which shows qualitative similarity to the 'wedge' at resonant frequencies. At this second resonance capsizability of the model (as measured by J rather than F) is comparable to that at resonance. Note that the use of the scaled wave slope, J, rather than the amplitude, F, gives the correct emphasis to capsize in this higher frequency region. A simple design formula (based on a linear analysis), [2], predicts capsize at $J = 1$, which is a reasonable lower bound in the above case. For higher damping, this $J = 1$ formula is found to be more accurate.

ROLL TIME SERIES ANALYSIS

We have recently been considering whether we can extract the damping and restoring curves from simple roll decay data. In general, given a roll decay time series we can take two basic approaches to fitting our nonlinear model to the data; global or local. A global approach predefines a polynomial to describe the damping (or restoring) functions. The predictions of such a model can thus be fitted to the data over some number of roll cycles. A local method does not require the pre-definition of these functions and instead fits local linear approximations over small sections of the data. These local approximations are then combined to reconstruct a global, nonlinear fit. Here we present the basic method and discuss its failings as well as their possible solutions.

The first step is to model the time series so that we can obtain estimates for its derivatives. At time τ_i the time series will have some value θ_i. Using the surrounding points we can also approximate $\dot{\theta}_i$ and $\ddot{\theta}_i$. We may employ a number of different methods to do this. Here we employ a Savitsky-Golay filter [11] that we have succesfully used to obtain double derivatives from experimental roll decay data. We again use our roll motion model (1) and assume that we can write the two functions $B(\dot{\theta})$ and $GZ(\theta)$ as locally linear. We can now write our equation of motion *locally* as,

$$I\ddot{\theta} + B_0 + B_1\dot{\theta} + mg(\lambda + \mu\theta) = 0 \tag{7}$$

and

$$GZ(\theta)_{local} = \lambda + \mu\theta \tag{8}$$

$$B(\dot{\theta})_{local} = B_0 + B_1\dot{\theta} \tag{9}$$

If we write $B_0 + mg\lambda = C$, we are left with three unknowns (B_1, μ, C) and thus require three equations to find these unknowns.

Therefore we simply need to sample the time series at three nearby points. Nearby here means that they must be close enough in phase space such that our local dynamical model is valid.

This gives the local slopes for $B(\dot{\theta})$ and $GZ(\theta)$ and the constant C. Since we cannot easily separate C we instead specify $GZ(0) = 0$ and $B(0) = 0$, and integrate over our local slopes to reconstruct the restoring and damping curves.

We then scan through our time series selecting three consecutive points every step and solving the equations to obtain locally fitted parameters over a wide range of phase space. We then reconstruct the curves by integrating over the local slopes.

EXAMPLES AND IMPROVEMENTS

As an example we have taken some numerically generated data from a model with known restoring and damping functions (the symmetric escape equation, [2], which is similar to (2) but with a restoring force of $x - x^3$). Here we have reconstructed damping and restoring simultaneously. Figure 3 show the reconstructed GZ curve.

Note that for this method velocity and angle dependent parameter separation remains a problem (the equations we are solving to find B_1, μ and C become ill-conditioned and much of the data series proves unusable for this method. Therefore we have applied the method carefully over parts of the data set for which it succeeds.

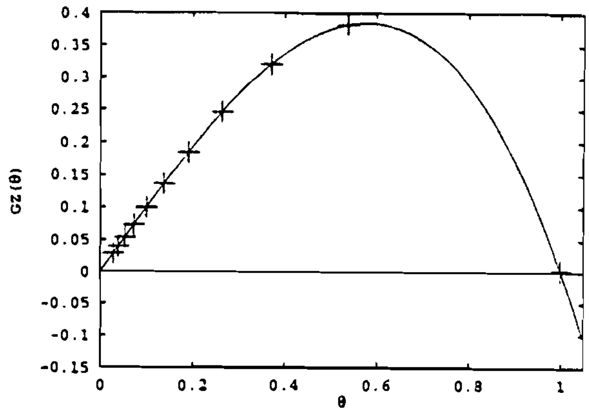

Figure 3: Reconstruction of restoring force curve for the symmetric escape equation, the reconstructed points are shown with the original curve

In figure 4 we plot a reconstructed nonlinear damping curve. Here the restoring function was pre-specified and the damping taken to be dependent only on velocity.

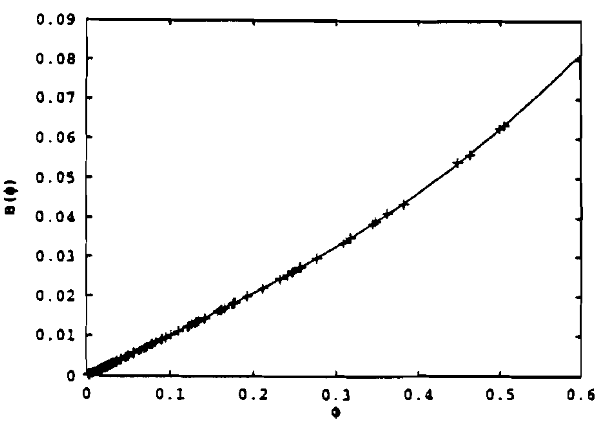

Figure 4: Reconstruction of a linear plus cubic damping curve with specified GZ

Therefore parameter separation was not a problem and all of the data was used. The routine has also been applied to some experimental roll decay data and was found to perform well in the presence of limited precision and noise. This experimental data was from a low angle decay test and so the restored functions were very close to linear. It was found that calculations of natural frequency using the reconstructed GZ gave results accurate to within 1% of the measured values.

We can improve our ability to deal with the parameter separation problem by employing singular systems analysis [12], to provide us with more information on how and where the method fails. Treating the fitting as a matrix inversion problem we can rewrite our set of equations as,

$$\begin{pmatrix} \dot\theta_1 & \theta_1 & 1 \\ \dot\theta_2 & \theta_2 & 1 \\ \dot\theta_3 & \theta_3 & 1 \end{pmatrix} \begin{pmatrix} B_1 \\ mg\mu \\ C \end{pmatrix} = -I \begin{pmatrix} \ddot\theta_1 \\ \ddot\theta_2 \\ \ddot\theta_3 \end{pmatrix} \qquad (10)$$

or

$$Ax = b \qquad (11)$$

By expressing the problem is such a way, we are able to utilise Singular Value Decomposition (SVD) which can be used to both solve for x and also provide information on separability of the parameters. When the data does not distinguish well between two or more parameters then A becomes ill-conditioned and this can be detected with SVD [11].

The solution is obtained by decomposing A and then back-substituting given b (it is similar in application to solution by standard matrix decomposition methods). If A is ill-conditioned then SVD will provide the best approximation to a solution in the least squares sense. Thus we are able to go further than is possible with the simpler approach.

A further reason for employing SVD is that we can add additional rows to A and solve for x with a reduced likelihood of ill-conditioning. We can do this by simply selecting more nearby data points to provide local roll equations. A still more powerful addition is to include further rows representing energy balance equations for the sampled data points.

CONCLUSIONS

A steady state bifurcation analysis of a simple roll model has been extended to higher forcing frequencies. We have discussed a number of new phenomena, with particular reference to capsize mechanisms. The higher frequency region has been shown to bear qualitative similarities to that around resonance and we have identified a *second resonance region*. Capsizing wave slope at frequencies around $\omega \approx 1.8$ is found to be comparable to that at resonance, although the feasibilty of such conditions occuring must be considered. Furthermore we have shown that the usage of the flip bifurcation as a capsize estimate must be made carefully in this high frequency regime.

Secondly, we have applied a local fitting method to numerically generated roll decay data and succesfully recovered a nonlinear damping function. The method has been extended to the simultaneous reconstruction of restoring and damping curves, but in this case parameter separation problems remain. The basic difficulty is the separation of velocity and angle dependent terms over the whole data series. We have discussed the application of Singular Systems Analysis to improve our ability to deal with this problem and sketched out how it may be applied.

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