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Ship Capsize Assessment and Nonlinear Dynamics

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Certain aspects of ship stability assessment in beam and in following seas are discussed. It is argued that the use of detailed numerical codes of ship motions cannot solve alone the assessment problem. On the other hand, whilst simplified models can be very useful for acquiring a fundamental understanding of the dynamics of capsize, still a good number of theoretical obstacles need to be overcome. In respect to beam sea capsize, firstly we discuss the structure of the mathematical model and the types of excitation. Then we consider the mechanism of roll damping very near to capsize angles and we point out a very interesting connection that exists with the specification of predictors of capsize based on Melnikov's method. Finally we sketch out a constrained design optimization procedure which can be used for finding those ship parameters' values where resistance to capsize is maximized. In respect to the following sea, we show that if capsize is examined in a transient sense, it should be possible to have a unified treatment of pure-loss and parametric instability. We also show what is the qualitative effect on the stability transition curves from bi-chromatic waves.

INTRODUCTION

Whilst one might think of many different methods for assessing the behaviour of a system, there is little doubt that the most reliable are those which are based on sufficient understanding of the system's key properties. For ship stability assessment however the application of this principle has been, so far at least, less than straightforward; because the behaviour of a ship in an extreme wave environment, where stability problems mostly arise, is often determined by very complex, hydrodynamic or ship dynamic, processes.

Ideally one would wish of course to have a full, meticulously developed and validated mathematical model of ship motions on which to carry out detailed analysis of dynamic behaviour and instability. Unfortunately this seems still to be well beyond our reach. But even if such a model were available, we would hardly know how to carry out in depth analysis for the nonlinear dynamical system in hand[2]. As a result, one can see two lines of research evolving, and it is essential that interaction between the two is encouraged: the first dealing with detailed mathematical modelling of the motion; whereas the second aiming to provide a better understanding of dynamic behaviour on the basis of simpler models that can capture however key features of the system's response. Areas of concern can be identified however in either of these directions: As the mathematical model of ship motion becomes larger, there is a cumulative effect from the uncertainties that often underlie the various assumptions and the unavoidable empiricism which lurks behind model development. On the other hand, when a simple model is used it is sometimes uncertain to what extent the observed behaviour corresponds to that of the real system.

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[2] As is well known, the motion of a body on the surface of the sea entails partial differential equations (PDEs) for its description. As evidenced from approximations of PDEs from systems of ordinary differential equations (ODEs), an infinite number of ODEs is required for absolute equivalence. This corresponds to the well known fact that memory effects (or frequency dependence of hydrodynamic coefficients) render the system's state-space infinite dimensional.

The first direction represents essentially the extension of the traditional seakeeping approach from small towards larger amplitude motions. However the second is quite novel in naval architecture. Its importance is owed to the fact that nonlinearity can make large amplitude responses follow completely different patterns that their smaller-amplitude counterparts. As is nowadays increasingly realized, a ship, like many other dynamical systems, can exhibit a very rich envelope of large-amplitude behaviour which is sometimes very difficult to unravel. In order to understand the underlying principles of safety-critical behaviour one needs to have an effective methodology which will guide his search and here is where the techniques of nonlinear dynamics' can provide truly valuable inputs.

These techniques enable, at first instance, better focus during physical model testing. This is essential because in extreme seas comparisons between theory and experiment are non-trivial due to the fact that the number of unknowns involved is very large. But perhaps the more far-reaching implication is that they offer a potential for developing effective methods of stability assessment that can combine scientific rigour with practicality for better design and safer operation. This potential allows us to start thinking also about intregrated stability assessment methods which will cover mechanisms of capsize associated with different environments and ship-wave encounters.

In the previous Workshop in Crete we have outlined some of our recent work along the above lines: We proposed a method of interfacing the findings of the nonlinear dynamics approach of ship capsize with design, in respect to the mechanism of capsize in resonant beam seas [1]. Also we continued our investigations of the instabilities of the following/quartering sea, discussing the interesting parallel that exists between yaw (related with broaching) and roll instabilities (related with pure-loss and parametric instability) [2].

The present paper consists of two parts: Firsly we discuss, very much in the spirit of this Workshop, some of the problems that exist in developing an effective stability assessment in beam-seas. Then we explain a practical assessment method for pure-loss and parametric instability in following seas.

BEAM-SEA CAPSIZE

A number of issues are currently under debate, such as the suitability of single roll or coupled models, the use of deterministic or stochastic-type of excitation; and the quantitative prediction of damping especially up to very large angles.

The suitability of the mathematical model

It is quite common, especially after Wright and Marshfield [3] to model roll motion in regular beam waves by expressing the roll angle relatively to the local wave slope. A single-degree roll equation is then used to describe roll dynamics with nonlinearities in damping and in restoring. For ships with small beam compared to the wave length, it is often reasonable to assume that, in sinusoidal beam waves they experience a fluctuating "effective gravitational field" $\mathbf{g}_e$ where the centrifugal acceleration of the water particle is combined with the acceleration of gravity $\mathbf{g}$ [4]. This says essentially that a small boat beam to long waves tends to follow the motion of the water particles and it allows direct use of the calm-sea restoring of the ship in the equation of relative roll. From an axes system tracking the motion of a water particle and having one axis always tangent to the wave surface the single roll model is then perfectly adequate.

But if it is intended to carry out model experiments, the physical model should rather not be constrained rigidly in sway because then the model cannot follow the motion of the water particles and direct comparison between theory and experiment becomes difficult. On the other hand, if the model is not constrained at all, it is likely to yaw and to have also a mean drift which also hinders comparisons with theory.

There is of course the possibility also that the ship "cannot" follow the motion of the water particles. Then the coupled roll sway and heave need to be considered along with the type of wave excitation as the above single roll model has encountered its limits. This is even more evident if the effect of non-regular waves is under consideration. However, one must bear in mind here that, unlike some seakeeping studies where we examine performance degradation during a voyage, in intact-ship capsize we are only concerned about an almost momentary event which is usually the result of encountering a small number of steep, often quite similar, waves with which the ship cannot cope.

The nature of the excitation deserves however some further attention: In our capsize studies we are usually restricting our analysis, one might think unjustifiably, in excitations produced by steep but non-breaking waves. This is an idealization which can result in unsafe predictions; because in the extreme environments where we investigate capsize, wave breaking is quite common. The nature of such excitations, a combination of smooth and impacting, and their magnitude can be very conducive for capsize.

But even if we assume that the structure of the conventional mathematical model is satisfactory, at least two further tasks need to be tackled: (a) To derive roll damping coefficients that can be applicable for near-capsize-angle motions; and (b) to identify capsize thresholds in terms of combinations of wave amplitude and frequency. Interestingly, the two tasks are, as shown below, in fact intrinsically connected.

Derivation of damping coefficients

Currently it is quite common to derive the damping coefficients from free roll decrement data under the assumption that the undamped roll would be basically harmonic. However, near capsize the nonlinearity of restoring is very strong rendering the response of a rather different type. This means that energy dissipation near capsize angles is not taken into account accurately when the coefficients are derived, although the values of these coefficients are critical in the theoretical investigation of capsize.

To explain these, let us consider a scaled equation of free roll with a quite general, quintic-type restoring curve:

$$\ddot{x} + D(\dot{x}) + x + \delta x^3 - (\delta + 1)x^5 = 0 \qquad (1)$$

where $x = \dfrac{\varphi}{\varphi_v}$ with φ the real roll angle and φ_v the vanishing angle. Differentiation is carried out in respect to scaled time $\tau = \omega_0 t$ where ω_0 is the 'undamped' natural frequency and t is the real time. $D(\dot{x})$ is the damping function that normally includes a linear plus an absolute quadratic or cubic component of roll velocity; and δ parametrizes the whole family of quintic restoring curves and therefore through δ we can establish a correspondence with the real (GZ) of our ship.

As has been shown in [5], if damping is neglected we can obtain the following exact "Hamiltonian" solution for large amplitude relative free roll (assuming that the "ship" was released with zero intial velocity):

$$x = x_0 \frac{\mathrm{cn}(u, k)}{\sqrt{1 - \lambda^2 \,\mathrm{sn}^2(u, k)}} \qquad (2)$$

where x_0 is the initial angle at $\tau = 0$; λ is a function of x_0 and δ; cn, sn are the so called jacobian elliptic functions (respectively elliptic cosine and elliptic sine) with argument $u = wt$, and modulus k; w is also a function of x_0 and δ. We note that when $k \to 0$ we have the linear case and the solution (2) becomes harmonic; whereas for $k \to 1$ we obtain the hyperbolic solution that defines the boundary of the Hamiltonian safe basin.

In order to find damping coefficients appropriate for extreme roll angles we need to know how energy is dissipated at these angles which requires to know the trajectory in $(x, \dot{x})$ from one peak (that is, one crossing of the zero velocity line) to the next, see Fig. 1.

For a linear roll equation such a solution is rather straightforward:

$$x = x_0 e^{-\zeta\tau} \sin\left(\sqrt{1-\zeta^2}\,\tau - \theta\right) \qquad (3)$$

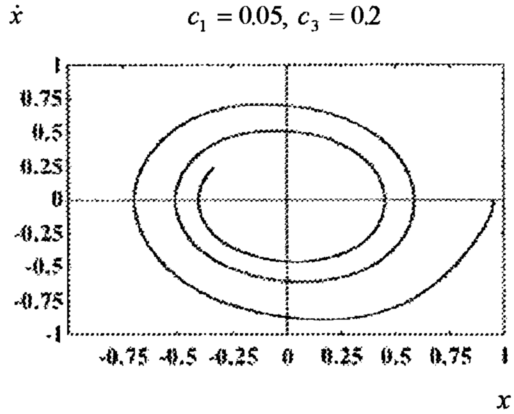

Fig. 1: Numerically derived roll decay for a quintic polynomial when $x_0 = 0.95$. By c_1 and c_3 are indicated respectively the linear and the cubic damping coefficients (nondimensional).

Expressions for "mildly" nonlinear (GZ) can also be derived through a perturbation approach. But for the strongly nonlinear case, if damping is present, exact analytical solution cannot be obtained; and a perturbation-like approach (with damping's nonlinearity as small quantity) involving elliptic functions is extremely complex whilst the accuracy achieved may be doubtful.

In [5] we have shown that it is possible to identify fully analytically the roll decrement per half-cycle for roll angles arbitrarily close to the vanishing angle if we assume the roll trajectory to constitute a perturbation of the Hamiltonian solution. As will be shown next this fits nicely with the Melnikov method of capsize assessment which is based on the same principle.

Predictors of capsize

Such predictors can be derived from an analysis either of steady-state or from transient roll responses [1]. To resolve an issue which was raised in last year's Workshop, by "steady-state capsize" we mean the absence of stable steady-state solution in the vessel's response. If such a state does not exist at a certain level of wave forcing and damping, the ship simply cannot stay upright. On the other hand, by "transient capsize" we mean that although a stable state might exist, at the initial transient stage the response is such that capsize occurs. As is obvious, the threshold wave slope of transient capsize should be lower than that of steady-state capsize. For this reason it is more sensible to predict capsize on the basis of transients [6].

A good criterion of incipient transient capsize can be derived from the so-called Melnikov's method through which we can find an analytical approximation of the critical wave slope, given the frequency ratio, where manifold tangencies arise and the domain of bounded roll motion starts becoming fractal, triggering rapid loss of the safe area of state space. Melnikov's method has been applied both in a deterministic and in a stochastic context.

It is remarkable that the critical condition derived from Melnikov's method can be interpreted also as an energy balance: Essentially, Melnikov's method "says" that to identify the critical wave slope given the damping, one should balance the work done by the forcing with the energy dissipated through damping *around the remotest orbit of bounded roll* (heteroclinic or homoclinic orbit depending on whether a symmetric or a biased in roll ship is studied). What makes such an interpretation particularly interesting is that it provides a connection with the widely debated in the early eighties method of energy balance for capsize assessment. That method however relied on harmonic or nearly harmonic responses.

Another observation on Melnikov is that it makes use of the perturbed Hamiltonian dynamics approach. This is the same fundamental assumption that has allowed, as discussed in the previous subsection, to find analytically the roll decrement during decay experiments for arbitrarily large initial roll.

From the above observations the conclusion may be drawn that the tasks of deriving damping coefficients and of predicting capsize are intrinsically connected and that, consistency in the followed approaches should be ensured.

Stability of symmetric and of biased ship

As has been pointed out by Thompson [6], the presence of even small bias, can reduce very considerably the critical wave slope where capsize occurs. It seems logical that a dynamic stability criterion should take into account this fact; but how much bias in needed in the assessment is very hard to define in a rational manner.

As is well known, a ship can become biased as the result of wind loading or cargo imbalance; but what is further notable is that a ship shows a "preference" to capsize towards the wave; and that in large waves an initially symmetric ship may develop also some "dynamic" list towards the wave. This would possibly require consideration of sway and higher order wave effects to explain but whether these matters should be taken into account in a capsize assessment is a rather open question at this stage.

About the design problem

It is of course highly desirable the information produced from the analysis of dynamics to be linked with the design process. Unfortunately, until recently this problem had not even been addressed, [1]. Generally, there are two main problems that need to be solved: Firstly, how to maximize the critical wave slope where manifold tangencies arise, over a range of wave frequencies; and secondly how to generate practical hull shapes given some desirable form of the restoring curve identified from the first task [essentially the inverse of the conventional task of deriving the (GZ) curve given a hull]. Here we shall discuss in further detail the first task.

Let's take the rather generic equation of roll with linearized damping and cubic-type (GZ) which has been thoroughly studied in the past:

$$\ddot{x} + 2\zeta\dot{x} + x - x^3 = F\sin(\Omega\tau) \tag{4}$$

$$F = \frac{I}{I+\Delta I}\frac{Ak\Omega^2}{\varphi_v}, \quad \zeta = \frac{B}{2\sqrt{Mg(GM)(I+\Delta I)}}$$

with B the dimensional equivalent damping, M the ship mass; and I, ΔI, respectively the roll moment of inertia and the added moment.

It can be noticed that in the expression of the equivalent damping ratio ζ, (GM) appears in the denominator which means that for our scaled equation increase of (GM) reduces ζ! However, at the same time the forcing is reduced even more since $F \propto \Omega^2 = \dfrac{\omega^2}{\dfrac{Mg(GM)}{I+\Delta I}}$.

From Melnikov we find the critical forcing F_M to be:

$$F_M = \frac{4\zeta \sinh\left(\frac{\pi\Omega}{\sqrt{2}}\right)}{3\pi\Omega} \quad (5)$$

In order to understand the meaning of this we should go back to dimensional quantities in which case we can obtain the following expression of critical wave slope $(Ak)_M$:

$$(Ak)_M = $$

$$= \frac{2B}{3\pi}\frac{Mg}{I(I+\Delta I)}\varphi_v(GM)\frac{\sinh\left(\frac{\pi\sqrt{I+\Delta I}}{\sqrt{2Mg(GM)}}\omega\right)}{\omega^3} \quad (6)$$

where ω is the wave frequency.

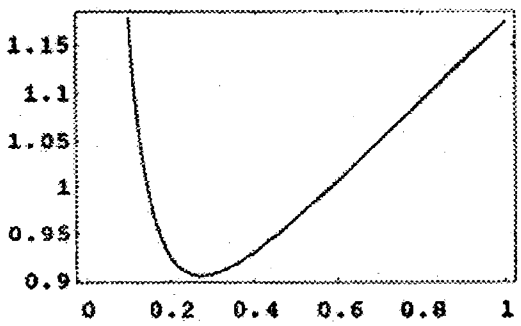

Fig.2: Basic trend of the dependence of (GM) on the critical wave slope

Increase of damping or of the vanishing angle are the typical ways to improve the resistance to capsize according to this mechanism [6]. However some more intriguing observations are possible also on the basis of Fig. 2: Expression (6) allows for having a situation where very low (GM) can, under certain circumstances, be beneficial! It is essential therefore that the findings are not applied blindly but an understanding about the physical mechanisms involved is developed, and areas of practical validity are established.

As for any resonance mechanism, it can be dealt with by increasing damping and/or by detuning our system from the excitation. As the natural frequency of the ship depends on (GM), such detuning can be achieved not only by increasing but also by reducing (GM). In fact, it is possible that if the phase of rolling response is nearly opposite to the phase of the wave, then the "absolute" roll motion (the wave slope plus the relative to it roll angle) can be very little, giving the impression that ship is "insensitive" to the excitation. Of course, under no circumstances could be advised to set low (GM) for the ship because then capsize can easily happen from other reasons.

In a practical context it is sensible, rather than trying to establish the capsize limits of the ship, to set threshold absolute roll angles beyond which the ship is in grave danger of capsize due for example to cargo shift. In such a case however, we must be very careful in the interpretation of the output of a roll motion equation like (2). Because a small relative angle could mean a quite substantial one in absolute terms given the wave slope; and on the other hand, as hinded earlier, the phase between the roll response and the wave can make absolute rolling to be very large or very small.

As has been outlined in [1] it should be possible to combine an expression like (6) with an optimisation process, given certain ship parameter constraints obtained from existing stability standards. For example, for the considered simplest possible case of cubic restoring, the Naval Engineering Standard 109 would produce as far as (GM) and φ_v are concerned, the following constraints:

The area criteria for (GZ) up to 30deg, 40deg and between 30 and 40deg give:

$$\text{(a) } (GM)\left(0.137 - \frac{0.019}{\varphi_v^{\,2}}\right) \geq 0.08 \quad (7)$$

$$\text{(b) } (GM)\left(0.244 - \frac{0.059}{\varphi_v^{\,2}}\right) \geq 0.133 \quad (8)$$

$$\text{(c) } (GM)\left(0.107 - \frac{0.04}{\varphi_v^{\,2}}\right) \geq 0.048 \quad (9)$$

Further constraints:

$$\max(GZ) \geq 0.3 \Rightarrow \text{(d) } 0.385(GM)\varphi_v \geq 0.3 \quad (10)$$

$$\text{(e) } (GM) \geq 0.3 \quad (11)$$

$$\varphi_{(GZ)_{max}} \geq 30\,\text{deg} \Rightarrow \quad \text{(f)}\ \varphi_v \geq 0.9064\,\text{rad} \quad (12)$$

(this is less than the recommended range of at least 70 deg).

The above lines are essentially sketching out an optimization process where Ak, expressed on the basis of (6), or preferably with a more detailed expression of the criterion taking better account of the hull, is sought to be maximized while making sure that realistic constraints like the above, are being satisfied.

It is very interesting that our concerns about the bias effects, expressed earlier, can be incorporated also into such a procedure. Let us consider the a - parametrized family of restoring curves with bias, where $a = 1$ means a symmetric system and $a = 0$ means a system allowing only one-sided escape [6]:

$$\ddot{x} + 2\zeta\dot{x} + x(1-x)(1+ax) = f\sin(\Omega\tau) \quad (13)$$

In [7] it has been shown that it is possible to find analytically an expression for the critical (Ak) for small and for large bias, respectively as following[3]:

Perturbation of symmetric system (small bias):

$$F_M = \frac{\sqrt{2}\sinh\left(\frac{\Omega\pi}{\sqrt{2}}\right)\left(2\sqrt{2}\zeta - 1 + a\right)}{3\Omega\pi} \quad (14)$$

Strongly "one-sided" escape (large bias):

$$F_M = 2\zeta\sqrt{a}$$

$$\frac{\left[3\alpha\beta^2\cosh^{-1}\left(\frac{\alpha}{\beta}\right) - \sqrt{\alpha^2-\beta^2}\left(\alpha^2+2\beta^2\right)\right]}{6\sqrt{2}\pi\mu\sqrt{\alpha^2-\beta^2}\sin\left[\mu\cosh^{-1}\frac{\alpha}{\beta}\right]} \quad (15)$$

$$\alpha = \frac{2(2a+1)}{3a}, \quad \beta = \sqrt{\frac{2(1-a)(2+a)}{3a}}, \quad \mu = \frac{\Omega}{\sqrt{1+a}} \quad (16)$$

Again, the quantities will have to be expressed in dimensional form in order to be able to find the true critical relationship of ship parameters.

[3] These analytical results are of importance also for [8] and [9] where Melnikov's critical wave slope had been identified only numerically.

CAPSIZE IN A FOLLOWING SEA

As is well known, in a following sea a ship may capsize due to severe fluctuations of its righting arm. Capsize can occur either from a sudden divergent roll ("pure-loss") or from a more dynamic process ("parametric"), where roll is built-up in an oscillatory and gradual manner. Traditionally, the two mechanisms are considered independently. However, as they are both the result of time-dependence of the roll righting arm (in fact dependence on the position of the ship on the wave), the propensity for capsize could be assessed more effectively if the two were treated in a unified manner.

Commonly, the parametric mechanism is examined on the basis of the principal and the fundamental resonance regions on the stability chart of a Mathieu-like equation. However, such a chart corresponds in fact to long term asymptotic behaviour which is rather unrealistic for a ship. This has created some controversy about the true relevance of the parametric scenario; because, although at realistic levels of ship roll damping the domain of the principal, and often of the fundamental resonance extend sometimes to feasible levels of restoring variation amplitude, this picture is correct if the considered number of wave cycles goes to infinity. Practically however, it is more important to know whether the instability becomes noticeable within a small number of wave cycles. But if the "allowed" number of wave cycles is small, the building-up of large roll requires very intensive variation of restoring which may, and one would indeed hope to, be unrealistic.

Another matter that needs to be taken also into account, more in respect to the pure-loss scenario, is the physical time required for capsize: At lower frequencies of encounter the ship may capsize more easily because it stays for longer time at unfavourable for stability regions of the wave. But because the ship is advancing very slowly relatively to the wave, the time for capsize can be excessively high. It is quite obvious in this case that for capsize assessment it becomes important where the ship was at $t = 0$. One possible way to deal with this dependence on the initial phase is to assume that the ship, at $t = 0$ is just entering the negative restoring region of the wave. For sinusoidal variation of (GM) this phase, say χ, is given from $\chi = -\arccos\left(\frac{1}{h}\right)$ where h is the amplitude of variation of (GM).

The major effect that the number of cycles has on the first resonances is shown clearly in Fig. 3 for a typical linear Mathieu-type roll equation which, on the basis of scaled quantities, takes the form:

$$\frac{d^2x}{d\tau^2}+\frac{2k\sqrt{a}}{\omega_0}\frac{dx}{d\tau}+a(1-h\cos 2\tau)x=0 \quad (17)$$

where $a=\frac{4\omega_0^{\;2}}{\omega_e^{\;2}}$, $x=\frac{\varphi}{\varphi_v}$ but this time $2\tau=\omega_e t$ (time nondimensionalized in respect to the encounter frequency ω_e). Also, ω_0 is the (dimensional) natural frequency and k is the equivalent damping factor ($2k=\frac{B}{I+\Delta I}$).

In Fig. 3 we examined whether the roll angle reaches the level of the vanishing angle within a prescribed number of wave cycles.

It is noted that if only four cycles are considered the h required is very high ($h=2.1$, not shown in the graph). As the order of the resonance increases the required amplitude becomes less dependent on the number of wave cycles; but the practical relevance of these resonances for a ship is rather minimal. It is also noted that, the lower the number of cycles the more influential becomes the initial position of the ship on the wave.

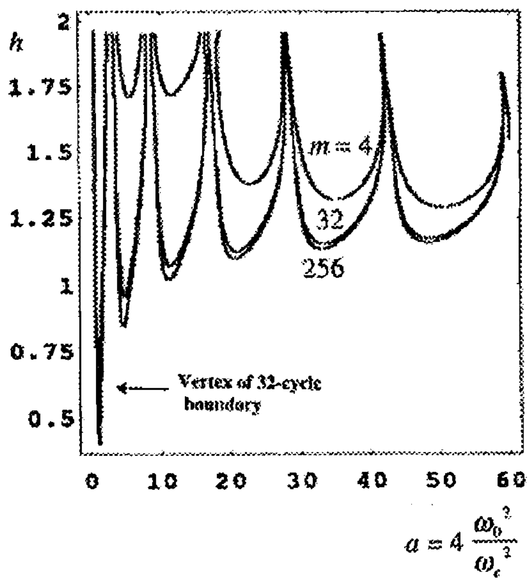

Fig. 3: Capsize regions in respect to the first six resonances, with parameter the considered number of encounter-wave cycles m. The initial heel was $x_0=0.01$ and as capsize was considered its 100-fold increase; $2k/\omega_0=0.025/0.144$.

Fig. 4 provides a unique combined view of regions of pure-loss and parametric instability on the basis of cubic-type restoring where the nonlinear term is time independent. This is allowed by the fact that behaviour is examined in a transient sense. Capsize occurrences are recorded if they happen in a small number of cycles and within limited physical time.

Of course, different hull forms will result in different restoring variation laws which, in turn, will give different arrangements of the capsize boundaries. At the moment, we are still lacking a systematic procedure for dealing with this fact. This is an area of research currently considered.

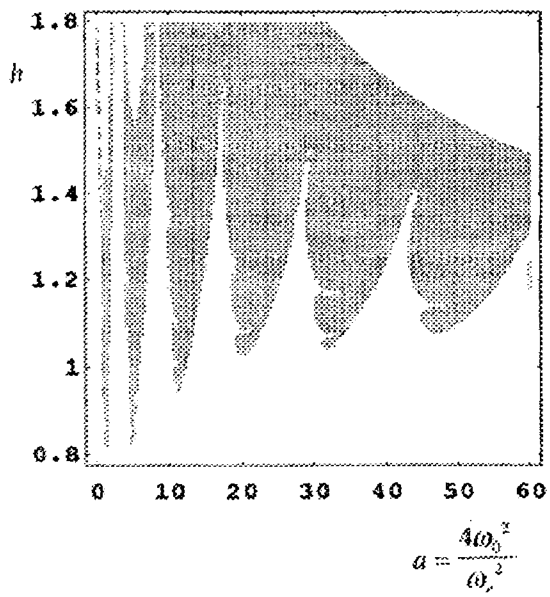

Fig. 4: Capsize regions for cubic-type restoring in less than 8 wave cycles and requiring less than 300 sec (natural frequency in calm sea $0.381\,\text{sec}^{-1}$). The dark regions correspond to capsize according to the parametric scenario. The white upper-right region is capsize in less than 50 sec and is according to the pure-loss mechanism. Quick capsizes ($t<50$ sec) occur also in the first two resonances and it is notable that the required amplitude h is comparable with that of pure loss. The graph is drawn with $2k=0.025$ and $x_0=0.1$.

Behaviour in bi-chromatic seas

A possible extension of the traditional examination of parametric instability on the basis of sinusoidal variation of (GM), is to study the behaviour of a ship under the effect of a wave group containing at least two independent frequencies . We shall assume that, in a qualitative sense, this could bring about a quasiperiodically varying restoring which, for two frequencies present, results in the following roll equation:

$$\frac{d^2x}{d\tau^2}+\frac{2k\sqrt{a}}{\omega_0}\frac{dx}{d\tau}+ \\ +a\left[1-r\,h\cos 2\tau-(1-r)h\cos 2\nu\tau\right]x=0 \tag{18}$$

In (18) the parameters r and ν represent respectively the relative strength of the basic frequency and the ratio of the second frequency to the basic.

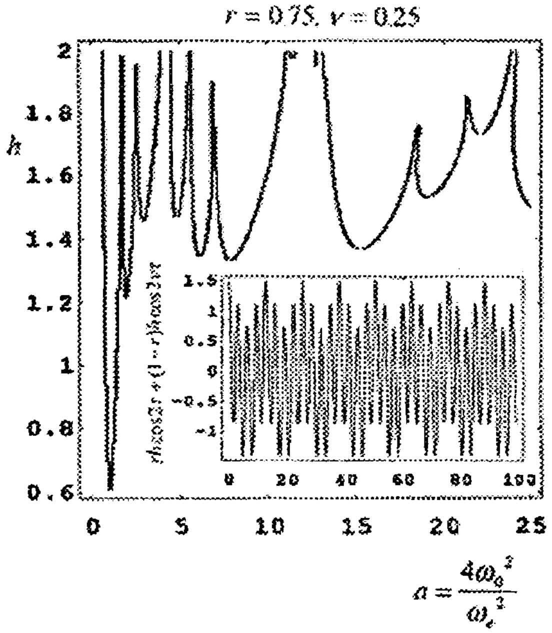

Fig. 5: Parametric instability in bi-chromatic waves for 32 wave cycles.

A general characteristic of the response is that a number of new "spikes" are growing on each primary resonant. However the effect of the extra frequency is not very influential on the principal resonance which extends at relatively low levels of required (GM) variation amplitude h.

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