

# SHIP DESIGN FOR DYNAMIC STABILITY

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## ABSTRACT

Is it possible to use in ship design the latest findings from the modern analyses of capsize based on the theory of nonlinear dynamics? This is the question which we are attempting to address in the present paper. Our goal is the establishment of a rigorous scientific basis for quantitative assessment of dynamic stability which will cover all the known types of ship capsize. The paper begins with an overview of the field of ship stability and then follows a summary of the known types of capsize of intact ships, classified on the basis of the relative wave direction. The main part of the paper is divided into two Sections. In the first are discussed several recently developed simple design formulae, which are suitable for a check of stability at a very early stage of design. In the following Section are demonstrated some numerical stability analyses tools which can be used for detailed design guidance. In the paper we do not discuss only recent achievements but we also identify areas where further development is urgently required.

## 1. INTRODUCTION

The area of dynamic stability of ship motions in waves is multifaceted and rather loosely defined in naval architecture; but the essential aspect is that in the open sea some ships tend to perform dangerously large motions which from time to time result in the tragic event of a capsize<sup>1</sup>. Ship safety against capsize is a combination of good design with prudent seamanship and in our view it goes beyond mere rule compliance. We think that a rational approach about ship safety entails the best available scientific knowledge to be "infused" with the current practices of design, operation and rule setting. These notwithstanding, we are urged to note the profound lack of a proper methodological framework of ship stability assessment which would exploit the recent progress in understanding the dynamic origins of capsize and play the role of an interface between practice and research. The development of such a framework is nontrivial because the process of ship capsize is often determined by nonlinear phenomena and is not a simple task to develop scientifically sound and yet simple-to-understand and practical, quantitative measures of dynamic stability covering all possible types of capsize.

The regulatory regime of stability determines the range of choices of the designer; but it does not tell him much about how to compare designs or how to identify a ship geometry for optimally resisting capsize for a variety of dynamic environments. The current criteria for intact commercial vessels have been recently collected in IMO's Resolution A749(18) [1]. These criteria should be criticised not so much for being unsafe but rather for reflecting very little of today's level of understanding about the process of ship capsize in different environments. For general application we rely on empirical and static type criteria that were adopted in late sixties (resolution A.167) and

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<sup>1</sup> It is outside the scope of the present paper to comment on the stability of, very obviously, badly designed and/or operated ships, which might capsize even in calm water situations.

follow the ideas of *Rahola*, developed on the basis of statistical data collected before the 2<sup>nd</sup> World War. To close the gap created by the absence of any consideration of roll motion dynamics the “weather criterion” was introduced in mid-eighties and following long debate [IMO Resolution A562(14)]. For many, the scientific basis of this criterion is very, if not over-, simplistic. The criteria for naval vessels of several western Navies are based on *Sarchin & Goldberg* [2]. These criteria are characterised by a higher degree of stringency; but fundamentally, they follow the same principles as these of commercial vessels and seem today partly outdated.

The problem of dynamic instability is very relevant also for a damaged ship [3]. However in this case the difficulties of the analysis do not lie so much on the mechanism of capsizing but rather on the dynamics of water accumulation and its movement inside a damaged compartment. Quite often such a capsizing is simply the result of a nearly static loss of stability occurring soon after “enough” quantity of water has flooded high compartments of a ship [4]. Considerations of dynamics are not explicitly present in the current criteria although provisions have been made for ensuring a margin of safety above the level of damage stability in still water. This is more apparent in the “regional” criteria of the *Stockholm Agreement* that apply only for the countries of north-western Europe. The stability criteria of damaged ships have undergone a significant evolution in recent years. For passenger ships we have, after 1974, probabilistic-type criteria as an alternative to the deterministic criteria of SOLAS. Probabilistic damage stability criteria are the only IMO criteria applicable to cargo ships. These dichotomies are likely to disappear in the future thanks to an initiative currently underway at IMO for developing a “harmonised” set of rules.

The dubious account for dynamic types of ship capsizing in the current stability criteria has been of concern also for the U.S. Navy. In a recent publication at the SNAME Annual Meeting a dynamic capsizing assessment methodology for intact ships was proposed which is entrusted upon simulations combined with the application of risk assessment techniques such as fault-tree analysis [5]. The approach that we advocate on the other hand is embedded on the understanding of the dynamic effects that generate the various types of capsizing. The idea that we want to introduce is that, it is well possible to develop a unified and rational basis of ship dynamic stability assessment which will synthesise the recent advances achieved from the application of the rigorous stability analysis of dynamical systems’ theory.

In this paper we shall take a first step towards popularising some of the most recent research findings about the various mechanisms of ship capsizing, hoping to make obvious the great potential and the new opportunities that have been created for design. Our approach will be comprised of two levels: The first refers to a very early stage of design where it is desirable to have simple analytical predictors of dynamic stability (or, for a certain standard of stability, of the required values of influential parameters such as damping), while our knowledge about the ship is still limited. The detailed account of a ship’s form takes place at a second level where the stability analysis is performed with suitable numerical methods. It is remarked that the presented measures of stability could be relevant also for the operational side of the problem which however should be the subject of another publication.

## **2. CLASSIFICATION OF THE TYPES OF CAPSIZING OF INTACT SHIPS**

Some well-known types of ship capsizing which may happen in the open sea are summarised in Fig. 1. They are classified on the basis of the relative wave direction. In recent years better understanding of the dynamics of ship capsizing for various wave environments has been achieved,

through application of a combination of geometric, numerical and analytical methods. An important development was the analysis of dynamic capsize as a problem similar to an escape from a *potential well* which is an intrinsically transient phenomenon [6]. A number of new concepts and techniques of stability assessment, such as “basin erosion” and “the transient capsize diagram” became part of the stability vocabulary. We should single-out the proposal of a simple design formula for safety against resonant beam-seas [7]. It was derived on the basis of the amplitude magnification factor of the linear roll motion but its validity was confirmed for the nonlinear system by using results of the so-called theory of *Melnikov* [8]. Another finding with important repercussions was that the maximum wave slope sustained by a ship in beam waves is significantly reduced from the existence of even a small amount of bias.

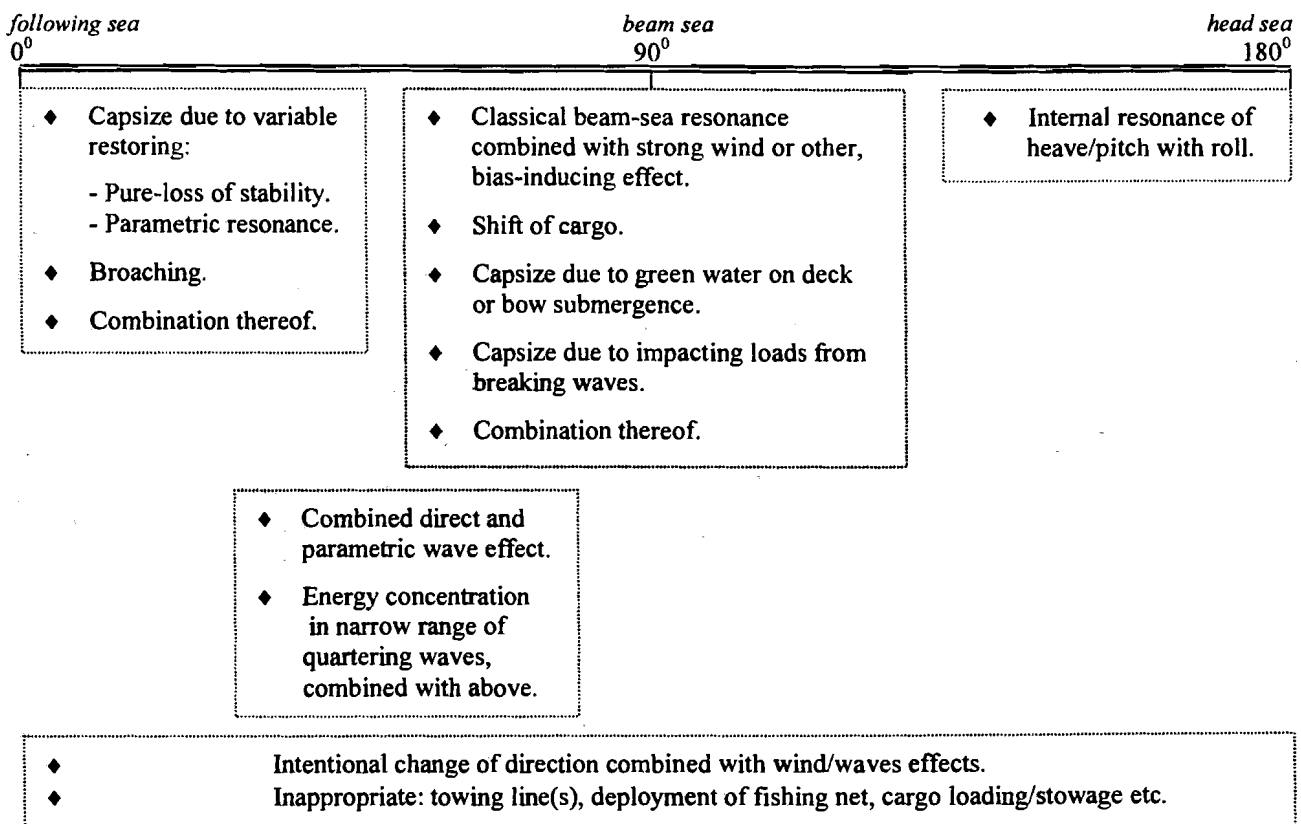


Fig. 1: Types of ship capsize

An approach using insights from nonlinear dynamics is very suitable also for analysing capsize in astern seas. According to a popular classification, this can be due to pure-loss of stability, parametric resonance, or broaching. Such a classification represents of course an idealisation since additional effects, such as water-on-deck, coupled motions etc. could also play an important role. Pure-loss and parametric instability have the same origin and, fundamentally, they can be studied on the basis of a single roll equation. They represent respectively semi-static and fully dynamic types of instability exhibited due to time-dependence of the restoring lever [9]. The dynamics of broaching and the onset of capsize are however more complex. Generally, a multi-degree approach is required in order to study the actively controlled, large-amplitude motion of the ship as it is advancing in an environment of steep quartering waves [10].

Capsize in head seas is perhaps the least likely to occur, although one mechanism of dynamic instability is known to exist which depends on the tuning of the encounter frequency with the natural frequencies of roll, heave and pitch [11].

### 3. LEVEL 1: SIMPLE ANALYTICAL FORMULAE

#### 3.1 Beam-sea resonance

A ship responds to long and beam regular waves like a rotational oscillator, coupled to the wave normal by a nonlinear rotational spring [7]. It is easily shown that when only linear dynamics is considered, the critical wave slope where the amplitude of resonant rolling reaches the capsize limit is given by the relationship:

$$Ak = \mu 2\zeta \varphi_v \quad (1)$$

where  $Ak$  is the wave slope,  $\zeta$  is the damping ratio and  $\varphi_v$  is the angle of vanishing stability. Also,  $\mu = (I + \Delta I)/I$  where  $I$ ,  $\Delta I$  are respectively the roll moment of inertia and the hydrodynamic “added” component. The damping ratio is expressed as  $2\zeta = B/\sqrt{mg(I + \Delta I)\overline{GM}}$ . Thus, in addition to the dimensional damping  $B$  it incorporates the metacentric height  $\overline{GM}$ , the moment of inertia  $I + \Delta I$  and the ship weight  $mg$ . It is preferable to use as  $\overline{GM}$  an “equivalent” one, giving the same potential energy with the actual  $\overline{GZ}$  at the angle of vanishing stability.

Consideration of a generic cubic-type  $\overline{GZ}$  and application of the *Melnikov* theory for transient capsize results in the following improved expression of critical wave slope, given for a range of scaled frequencies  $\Omega = \omega/\omega_0$  around resonance ( $\omega$  is the wave frequency and  $\omega_0$  the natural roll frequency):

$$Ak = \frac{4}{3} \frac{I + \Delta I}{I} \frac{\zeta \varphi_v \sinh(\pi \Omega \sqrt{2})}{\pi \Omega^3} \quad (2)$$

The importance of having simple measures like the above, is that they can be used for maximising dynamic stability while using as constraints the existing stability criteria (Fig. 1). This idea was first introduced in [8]. We should remark here however that expression (2) is based on cubic restoring which does not allow much freedom in terms of  $\overline{GZ}$  shape; because this shape is determined completely as soon as only two parameters, as for example the metacentric height and the vanishing angle, are known. Also, if we opted to fix the dynamic lever at the vanishing angle, as it might look logical in the first instance, even that limited freedom would be lost. An urgently needed step to be taken therefore is the development of an improved analytical formula like (2), parameterised with respect to the shape of  $\overline{GZ}$ . This would be very important for design. One approach that we are currently considering is to introduce a “ $\overline{GZ}$ -form” function  $f$  multiplying the right-hand-side of (2). Our research has shown that it may not be viable to derive such a function through analytical means; but it can be obtained from detailed numerical studies with families of  $\overline{GZ}$  curves.

*Thompson* prefers to use a stability measure that is based on asymmetric (quadratic-type) restoring and accounts for one-sided capsize (to the lee-side). The main reason for this is because extensive simulation studies have shown that even a small wind-like bias could reduce very appreciably the

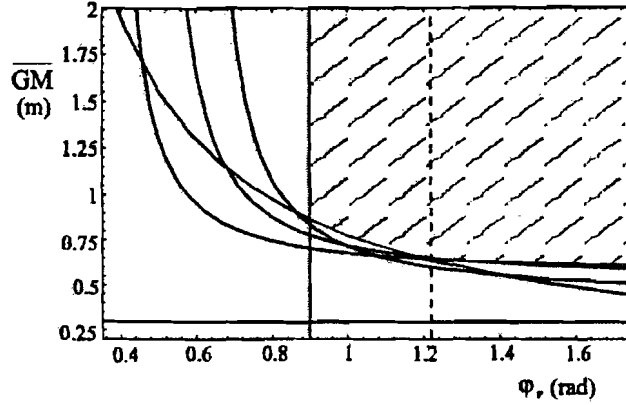


Fig. 1: For a cubic GZ function, the GZ criteria of the Naval Engineering Standard 109 produce the constraints shown in this graph. The hatched region is the permitted. The dashed vertical line is a recommended minimum of the vanishing angle.

sustainable wave slope. Therefore a predictor of critical  $Ak$  based on symmetric  $\overline{GZ}$  could result in an unsafe design. Quadratic restoring is the limiting case and leads to the following value of maximum sustainable wave slope:

$$Ak = \frac{2}{5} \frac{I + \Delta I}{I} \frac{\zeta \varphi_v \sinh(\pi \Omega)}{\pi \Omega^4} \quad (3)$$

In a recent MSc Thesis at UCL [12] was investigated in detail the effect of different magnitudes of bias on the critical  $Ak$ , using the following simple generic expression of asymmetric restoring proposed earlier by Thompson:  $R(z) = z(1-z)(1+az)$  where  $a$  is the bias parameter (for  $a = 1$  we obtain the cubic symmetric and for  $a = 0$  we obtain the quadratic). The maximum sustainable wave slope parameterised in terms of  $a$ , but with  $a$  relatively near to 1, was found to satisfy the expression,

$$Ak = \frac{\sqrt{2}}{3} \frac{I + \Delta I}{I} \frac{(2\sqrt{2}\zeta + a - 1) \varphi_v \sinh(\pi \Omega \sqrt{2})}{\pi \Omega^3} \quad (4)$$

A problem that exists in the analysis presented so far is that roll damping is treated as linear although it is well known that for large amplitude oscillations it becomes nonlinear. In order to overcome this and at the same time retain the simplicity of the expressions a novel method for deriving an equivalent damping was recently proposed, targetting rolling amplitudes at the capsize range [13]. For symmetric cubic-type restoring and a nondimensionalised damping having the customary quadratic form  $D = b_1 \dot{z} + b_2 \dot{z}|\dot{z}|$  (where  $z = \varphi/\varphi_v$ , and the differentiation is with respect to the scaled time  $\tau = \omega_0 t$ ), the new expression of equivalent linear damping is,

$$\zeta = \frac{1}{2} b_1 + \frac{\sqrt{2}}{5} b_2 \quad (5)$$

The above “equivalent” linear  $\zeta$  should be used when applying any of the formulae (1) to (4) for prediction of the critical  $Ak$ .

### 3.2 Parametric instability

This is a type of dynamic instability exhibited by systems subjected to “internal” forcing that is caused by time variability in some system parameter. For ships this lurks usually in the stiffness term which, in long and steep longitudinal waves, tends to fluctuate depending on where the ship lies on the wave. It has been the rule to interpret this position dependence as time dependence. The most fundamental mathematical model that could represent the resulting dynamic behaviour is a linear Mathieu equation with some low damping. It is well known that for the upright state of a ship, asymptotic-type instability should occur when the natural frequency lies near to half multiples of the modulus of the encounter frequency ( $4\omega_0^2/\omega_e^2 = n^2$  where  $n$  is an integer), if the amplitude of  $\overline{GM}$  variation, represented by a parameter  $h$ , be above a certain level. This critical level is quite sensitive to the magnitude of damping. In [9] were collected several expressions which could be used for predicting the critical  $h$ . For the first instability region ( $n = 1$ ), a good approximation is the following simple expression:

$$h = 4\zeta \quad (6)$$

For the higher instability regions ( $n \geq 2$ ) a good predictor of the critical  $h$  is,

$$h = \frac{8}{n^2} \sqrt{\frac{\zeta \sqrt{n!^2}}{2}} \quad (7)$$

Less severe criteria could be derived by examining the condition for a capsizing occurrence instead of the instability of the upright state of equilibrium. These two are not equivalent because the instability of the upright state might lead to bounded roll oscillations and not necessarily to a capsizing. The critical  $h$  for transient capsizing of a parametrically forced ship with a cubic restoring function and time-dependence only in the linear term is [14]:

$$h = \frac{4\sqrt{2}}{3\pi} \frac{\omega_0^2}{\omega_e^2} \zeta \sinh\left(2\pi \frac{\omega_e}{\omega_0}\right) \quad (8)$$

### 3.3 Pure-loss of stability

By considering the statical stability of a ship on a wave crest we derive that pure-loss should not happen if  $h$  remains always less than 1.0, even for the severest wave profile. However, such a condition takes no account of the roll dynamics and it says nothing about the range of encounter frequencies around zero where a pure-loss capsizing should be expected. It is possible to develop a better criterion by interfacing: the time required for reaching a capsizing heel angle from some small initial roll bias; with the time that the ship spends under the condition of negative  $\overline{GZ}$  which depends on the frequency of encounter. This idea is discussed in detail in [15]. For a simple sinusoidal variation of  $\overline{GZ}$  the time spent under negative restoring is:

$$t_{neg} = \frac{2 \arccos \frac{1}{h}}{\omega_e} \quad (9)$$

with  $h \geq 1$ . Following our argument, for pure-loss the heel angle should reach capsizing levels within a time period which should not exceed the value obtained from (9). If we consider an

“averaged” linearised system for the region of negative stiffness the critical time required for reaching the vanishing angle is given approximately by:

$$t_{cap} = \frac{-\ln \left[ \frac{\gamma + \sqrt{\gamma^2 - c}}{2\sqrt{\gamma^2 - c}} \varepsilon_1 + \frac{1}{2\sqrt{\gamma^2 - c}} \varepsilon_2 \right]}{-\gamma + c\sqrt{\gamma^2 - c}} \quad (10)$$

where  $c$  is the average stiffness in the region where it is negative,  $\gamma$  is a damping parameter expressed as  $\gamma = \zeta\sqrt{-c}$ ; and  $\varepsilon_1, \varepsilon_2$  represent respectively the initial bias in terms of heel angle and roll velocity. By equating the right-hand-sides of (9) and (10) and solving in terms of  $\omega_e$  we can obtain an approximate prediction of the critical encounter frequency below which capsize is possible.

### 3.4 Broaching

A capsize due to broaching is induced by the loss of directional stability. This entails, before any discussion about roll behaviour is made, to consider the development of a criterion of yaw stability. The simplest method to accomplish this is, to introduce the wave effect on a simple model of yaw motion like Nomoto’s equation. This results in a parametric-type system having the following structure [16]:

$$\ddot{\psi}' + \underbrace{\frac{(1 + k_2'K')}{T'}}_{2\zeta\omega_0^2} \dot{\psi}' + \underbrace{\frac{k_1K'}{T'}}_{\omega_0^2} \left[ 1 - \underbrace{\frac{A'}{k_1K'}}_h \cos(\omega_e t) \right] \psi = \underbrace{\frac{k_1K'}{T'}}_{\varepsilon} \psi_r \quad (11)$$

where  $\psi$  is the heading angle,  $k_1, k_2$  are the proportional resp. differential gain of the rudder’s control system,  $K, T$  are the system gain resp. time constant of the ship;  $A$  is the wave forcing and  $\psi_r$  is the desired heading. The prime indicates a nondimensionalised quantity. For  $\varepsilon = 0$  (11) becomes a classical Mathieu equation with damping. In fact, as has been discussed in detail in [17], there is an analogy between the yaw and roll instabilities in a following sea. As for the case of a “pure-loss” in roll, the condition of static stability for (11) is that the stiffness component never turns negative, i.e.  $h < 1$ . From this condition we can obtain the critical value of the proportional gain of rudder control for avoidance of a growing yaw divergence in a following sea (note that the differential one is not involved in this static criterion):

$$k_1 = \frac{A'}{K'} \quad (12)$$

System (11) may also exhibit however parametric instability in which case the critical conditions are given again by equations (6) and (7), expressed of course this time in terms of the yaw parameters rather than of roll. Considering for example the first region of instability [eq. (6)], the marginal combination of gain values, when  $\psi_r = 0$ , is given by the following equation:

$$h = 4\zeta \rightarrow \frac{A'}{k_1 K'} = 2 \frac{1 + k_2 K'}{\sqrt{k_1} \frac{K'}{T'}} \rightarrow \sqrt{k_1} (1 + k_2 K') = \frac{A'}{2 \sqrt{\frac{K'}{T'}}} \quad (13)$$

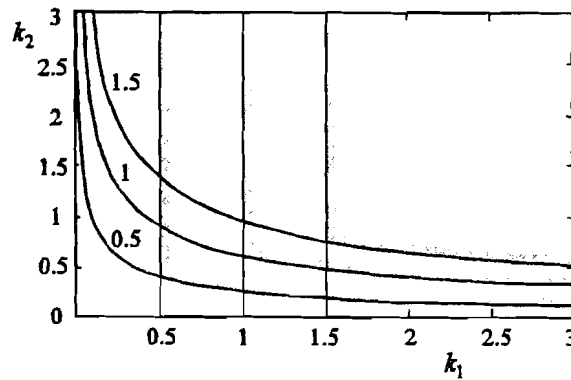


Fig. 2: Regions of stability satisfying both the static criterion (12), creating boundaries of straight vertical lines, and the dynamic criterion (13) which involves  $k_2$  as well as  $k_1$ . The parameter on the “dynamic” curves is the ratio  $A'/K'$ .

There are several semi-empirical expressions linking the main dimensions with the manoeuvring indices [18]. Similar expressions can be developed for  $A'$ , parameterised in terms of wave length and steepness. This would make possible to have a quick check of yaw stability in certain critical waves as soon as the main particulars are known. In Fig. 2 can be seen the  $k_1, k_2$  combinations which guarantee course stability, on the basis of (13), using the linear ITTC relation between the  $K'$  and  $T'$  indices. The arrows point towards the region of stability. The stepped parameter is the ratio  $A'/K'$ . All three curves are drawn for  $K'/T' = 0.5$ . For a comparison, on the same graph are presented the stability regions corresponding to the static criterion (12).

#### 4. LEVEL 2: MORE DETAILED GUIDANCE BASED ON NUMERICAL MODELS

Due to their simplicity, the formulae of Section 3 could not possibly “contain” much information about ship geometry. Moreover, it may not be forgotten that these formulae are derived after a number of simplifying assumptions without which analytical solutions are quite unlikely. The main weaknesses of these formulae stem therefore from their lack of detail and their approximate account of a ship’s dynamic behaviour. The way to overcome these is by solving the equations of motion numerically and by providing, during the modelling process, as many links with the geometry of the hull and of the appendages as the current state of knowledge allows. A second level design guidance should involve therefore a suite of computer programs, used in the following way: From a ship’s lines plan are obtained the hydrostatic and the (approximate) hydrodynamic data of the ship. These appear as coefficients in the equations of motion. We may identify the added mass and potential damping characteristics on the basis of some appropriate seakeeping code such as NEWDRIFT [19]. However for viscous damping is perhaps more practical to use semi-empirical



methods. This data is introduced into another routine where the numerical stability analysis is carried out. The output is a set of characteristic curves which determine the capsize boundaries for various wave/wind environments.

Let's see some results from applications of this methodology:

#### 4.1 Beam-sea capsize

A simple stability assessment procedure devised at UCL was to "sweep" a wide range of frequencies around resonance and trace numerically the curve of the critical wave slope separating transient capsize from survival. According to this procedure, initially the ship is assumed to be at rest and then it is exposed suddenly to a small number of regular waves cycles which come from the beam. In Fig. 3 the two curves indicate the numerical capsize boundaries obtained with a fully nonlinear (NL) and also with the previously defined "equivalent" linear (MED) damping. The tested body was a prismatic model derived from a section of the ship model on which Marshfield conducted his well known capsize experiments. These curves are superimposed on a large set of points representing experimental results and the coincidence appears rather good. Details of this work can be found in [13].

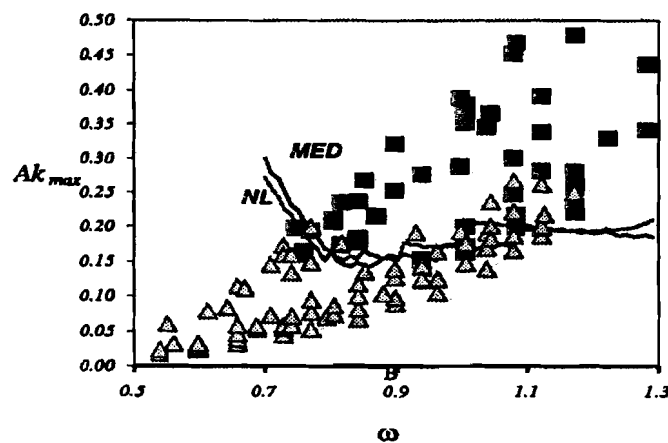


Fig. 3: Numerical transient capsize boundaries in terms of the critical wave slope, for a range of frequencies around resonance [13].

#### 4.2 A unified treatment for capsize due to pure-loss and parametric instability

With a numerical approach we can obtain a quantitative assessment of a ship's tendency for capsize without discriminating between the pure-loss and the parametric mechanisms, by exploiting the fact that they are both described by an equation of motion with position-dependent restoring. The key for this combined treatment is the consideration of the parametric capsize as a transient phenomenon which should be examined for a limited number of wave cycles as well as for a limited time (the latter is relevant also for pure-loss). Another advantage of this approach is that it allows to incorporate additional dynamic effects, such as nonlinear surging, which can play a detrimental for stability role. A stability diagram based on an assessment of this kind can be seen in Fig. 4 for a simplified case based on a sinusoidally varying  $\overline{GZ}$ . It noted however that a similar analysis based on the exact laws of  $\overline{GZ}$  variation could equally be carried out without any problem. With graphs like that of Fig. 4 we can determine for example the required damping so that even in

the severest wave profile there will be no capsize. Or we may concentrate more on the  $\overline{GZ}$  shape and see how it affects, for a given level of damping, the capsize boundary. It is perhaps logical to “design” in terms of the lowest value of wave slope which gives capsize.

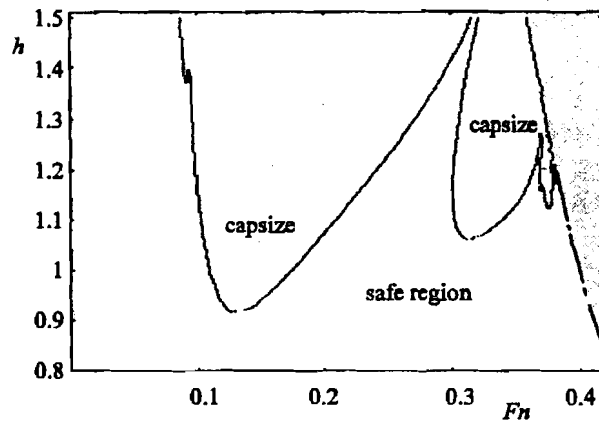


Fig. 4: A unified treatment of roll stability in following seas. The dark region represents surf-riding. Such diagrams allow to identify the required damping of a candidate hull form in order to ensure that capsize is avoided [17].

### 4.3 Broaching and capsize

The derivation of numerical measures for stability against broaching is perhaps the most complex case of all because coupled motions need to be considered. As a minimum, the mathematical model should involve the yaw/sway pair for predicting the lateral instability; the surge motion in order to be able to account for asymmetric surging and surf-riding as well as their effect on the lateral motions; and of course roll for capsize. The main problem is not so much on the modelling aspect, as it is about how to extract the required information out of this model in an effective and consistent way. In stability analyses of multi-degree models, such as the one required for broaching, it becomes much harder to maintain the rigour of the analysis due to the very large number of parameters involved. So it is not uncommon, these studies to end up as simulation exercises without having determined any global measures characterising the stability of the system which could be used for design or other purposes.

In previous publications we have shown how to develop simple and yet reliable procedures for quantifying the tendency of a ship for broaching and capsize [20]. Perhaps the most characteristic diagram out of this work is shown in Fig. 5. Diagrams of this kind may be derived at the design stage, with some uncertainty lying basically only with regard to the exact values of the manoeuvring coefficients for which a “potential flow” approach is not workable. In the first instance however, semi-empirical formulae could be used while higher accuracy could be achieved later through model testing.

## 5. CONCLUDING REMARKS

Recent advances in the study of ship dynamics have allowed us to develop a two-level framework for a rigorous quantitative assessment of ship stability. This framework can be useful to a designer who wants to determine, along with other design considerations, a hull geometry and appendages

that maximise safety against capsizes. A procedure for automatic identification of optimal restoring and damping characteristics, such as the one outlined in [8] for beam-sea capsizes, would be a useful addition to this framework. Finally, it must be noted that although due to length limitations of this paper we have not discussed in detail all the possible types of capsizes, and especially we have not presented any analysis for the damage stability problem, such matters are currently under consideration.

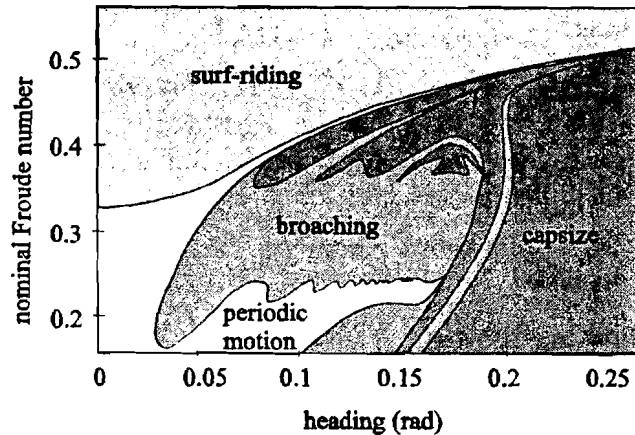


Fig. 5: The above diagram of broaching and capsizes, first published in [20], is quite revealing of the complex character of the boundaries separating the various types of motion in astern seas. The ratio of the area of broaching (and similarly for capsizes) versus the area of the rectangle having as one side a suitable heading range (e.g.  $-45$  to  $+45$  deg) and as other the operational range of Froude numbers, is an effective quantitative measure of the tendency of a ship for broaching.

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