16th International Conference on Hydrodynamics in Ship Design
3rd International Symposium on Ship Manoeuvring
Gdansk – Ostróda, Poland
7 – 10 September 2005

A coupled roll – sway – heave model for analysing ship capsize in beam seas on the basis of a nonlinear dynamics approach

N. Themelis & K.J. Spyrou

School of Naval Architecture and Marine Engineering
National Technical University of Athens
9 Iroon Polytechneiou, Zographou, Athens 15773, Greece.

Abstract

We study the effect of heave and sway on roll dynamics for a ship in beam waves. In the first part of the paper we discuss quantitatively the limitations of single-roll models that neglect these couplings, in terms of wave steepness and the long wave assumption. Thereafter we present a roll-sway-heave mathematical model that is currently developed and is suitable for systematic investigations of large amplitude rolling and capsize. Preliminary predictions of response will be shown for a fishing vessel exposed to harmonic beam waves, including her non-zero mean roll and drift as function of wave steepness.

Introduction

Capsizing of a ship in waves is one of the most basic fears for mariners, passengers as well as for anyone with a vested interest on maritime transportation. The keen interest of the international community is reflected by the current efforts at IMO to improve the main ship stability criterion, the so-called “weather criterion” [Francescutto, 2002] as well as by abundance of research papers on the topic [Perez-Rojas, 2003]. Approaches based on mathematical modelling are nowadays established as the main route towards improving understanding and developing effective design criteria. Contemporary investigations range between rigorous analyses of nonlinear dynamics based on some simplified model of roll motion, to simulation studies that rely on a fuller set of motion equations, incorporating interactions with the other degrees of freedom. So far, these two ‘worlds’ are quite separate although they target the same problem. Our longer term research goal is to work towards bridging the two: i.e. to set up an environment enabling rigorous nonlinear dynamics analysis of ship roll and capsize on the basis of a detailed coupled mathematical model. At this stage we are focusing on the development of a detailed roll-sway-heave model for regular and irregular beam seas. Some preliminary predictions from this model are reported in this paper. There have been already a few attempts to investigate nonlinear roll dynamics with couple models [Thompson et al., 1992], [Chen et al., 1999], [Mc Cue & Troesch, 2003]. These however did not consider the lateral drift motion. This aspect was tackled by [Kuroda & Ikeda, 2002] who found that the drift (a combined outcome of the wave excitation and the lateral resistance of the ship) could incur sudden jumps of roll amplitude due to the change of the encounter frequency. The behaviour in the shorter wave range where there could be more complex interactions between roll, sway and heave has not been investigated yet.

Limitations of the one degree model and the effective gravitational field

After Froude it has been quite common to investigate ship rolling with a single-degree of-freedom model, assuming that the ship tends to follow the rotational motion of water particles. This allows use of the concept of "effective gravitational field" where the effective gravity force can be assumed to work always perpendicular to the instantaneous water surface [Thompson et al., 1992]. The concept is valid for long incident waves relatively to the beam of the ship and it implicitly assumes a slave variation of heave in order to maintain a constant immersed volume. However, these assumptions have limitations as the wave steepness is increased. Furthermore, in the shorter wave range (length comparable to the ship's beam) the method deviates and may not be fully representative of the physical system.

When a body follows the circular motion of water particles on a sinusoidal wave in deep water, a time varied gravitational acceleration $\vec{g}_e(t)$ act on it, which is often assumed as nearly perpendicular to the wave slope. Fig. 1 shows the circular motion of a particle on the wave surface in deep water. We assume that a surface particle stays on the surface. Initially the particle is at position 1 (see Fig. 1) and after time *t*, it will be transferred to position 2 having run an angle $\omega_w t$ on a circular trajectory.

The accelerations that act on the wave particle at position 2 are: the acceleration of gravity $\vec{g}$ and the centrifugal acceleration $\overrightarrow{-\omega_w^2 A}$. The vector sum of these two is the effective gravitational acceleration $\vec{g}_e(t)$, which forms an angle β with $\vec{g}$ (Fig. 1). The effective gravitational acceleration is calculated then as follows:

$$g_e = g - A\omega_w^2 \cos(\omega_w t) \tag{1}$$

where A, ω_w are, respectively, wave amplitude and frequency. The slope n_y of a sinusoidal wave is:

$$\eta_y = A\frac{\omega_w^2}{g}\sin(\omega_w t) \tag{2}$$

According to Fig. 1, the condition of perpendicularity between the effective gravitational acceleration and the wave slope is expressed as follows:

$$\gamma = \beta - n_y = 0 \tag{3}$$

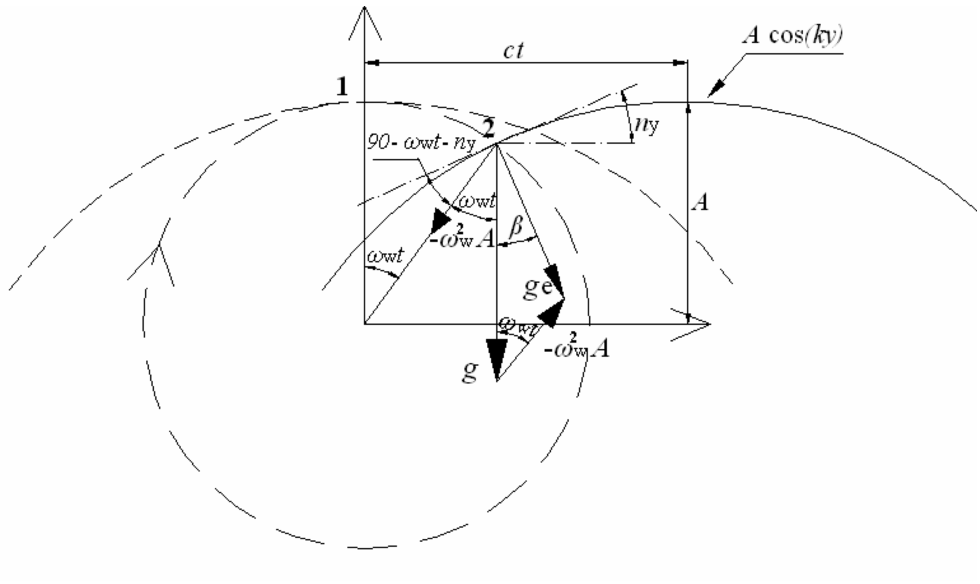


Figure 1**:** The rotational motion of a water particle for a wave in deep water.

where γ is the angle that characterizes the error from perpendicularity and the angle β is calculated as follows:

$$\beta = \arcsin\left(\frac{\omega^2_w A\sin(\omega_w t)}{\sqrt{g^2 + A^2\omega^4_w - 2gA\omega^2_w\cos(\omega_w t)}}\right) \tag{4}$$

In Figs. 2 and 3 are shown the variation of the angle γ (scaled over the wave slope Ak for more enlightening presentation of the results) with the wave steepness H/λ and the ratio λ/B (as usual k, λ are respectively wave number and length; B is the beam of the ship). It becomes clear that the assumption of perpendicularity is not valid (the error is greater than 10% of Ak) when the wave steepness is above 1/20; and also, when the wave length is shorter than about five times the beam (customarily, the long wave assumption refers to a wave that is longer by at least six times to the ship beam). If the body is not following the motion of water particles then it should be drifting, i.e. dynamic sway coupling should be taken into account. Furthermore, in shorter waves the ship should also be heaving. As a matter of fact, a roll-sway-heave model should be more suitable for investigating ship dynamic response in beam seas, especially for shorter waves.

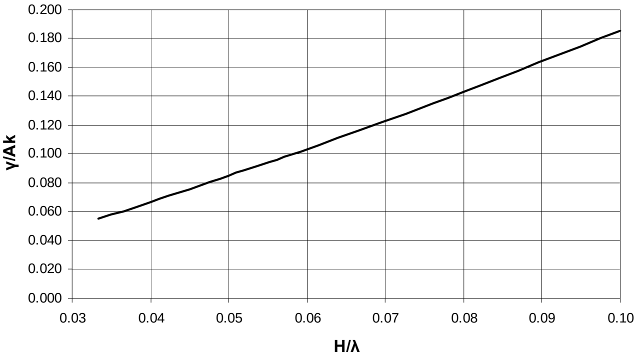


Figure 2: Effect of wave steepness.

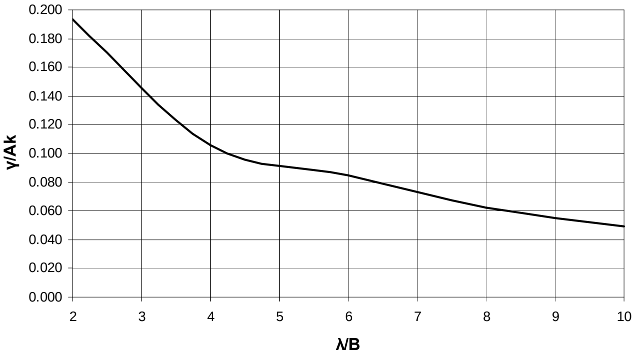


Figure 3: Effect of wavelength.

The mathematical model of the coupled roll motion

From kinematics and in accordance to Fig. 4, the equations of motion in heave, sway and roll are written as follows:

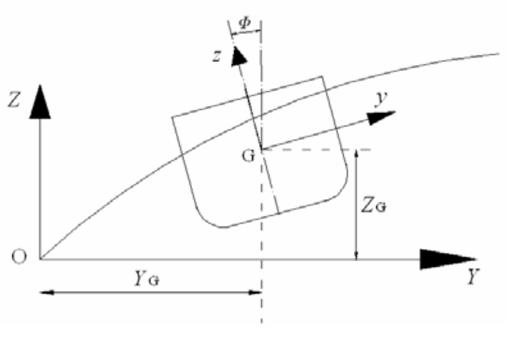


Figure 4: The inertial (O*YZ*) and body fixed (G*yz*) coordinate systems.

$$m(\dot{v}-\dot{\phi}w)=\sum F_y \tag{5}$$

$$m(\dot{w}+\dot{\phi}v)=\sum F_z \tag{6}$$

$$I_G\ddot{\phi}=\sum M_G \tag{7}$$

where v, w are the sway and heave velocity of the ship's center of gravity and $\dot{\phi}$ is the roll angular velocity, m and I_G are, mass and mass moment of inertia around x. The transformation between the earth and body fixed coordinate systems is well known:

$$\begin{pmatrix}\dot{Y}_G\\ \dot{Z}_G\\ \dot{\phi}\end{pmatrix}=\begin{pmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}v\\ w\\ \dot{\phi}\end{pmatrix} \tag{8}$$

The two forces and the moment that appear at the right-hand-side of (5)-(7) can be analyzed as follows:

$$\sum F = F_{Hs}+F_W{}^{FK}+F_W{}^{D}+F_R+F_V \tag{9}$$

F_{Hs} is hydrostatic, F_W^{FK} is Froude – Krylov, F_W^{D} is diffraction, F_R is radiation and F_V is the viscous force.

In linear wave theory, the total wave velocity potential is the sum of the potentials of incident wave, diffraction and radiation. The hydrostatic and Froude – Krylov (hydrodynamic) forces are estimated by the integration of the incident wave pressure (static and dynamic respectively) over the wetted surface of the ship. For regular waves, the incident wave potential is calculated from:

$$\Phi_I=\frac{Ag}{\omega_w}e^{kZ^*}\sin(kY-\omega_w t) \tag{10}$$

$$Z^*=Z-A\cos(kY-\omega_w t) \tag{11}$$

From Bernoulli's equation the pressure is:

$$P=-\rho\left(gZ^*+\frac{\partial\Phi_I}{\partial t}+\frac{1}{2}\nabla\Phi_I\nabla\Phi_I\right) \tag{12}$$

The hydrostatic and Froude - Krylov forces are repetitively:

$$F_{HSi}(t)=-\rho g\iint\limits_{S(t)} Z^*\vec{n}_i ds\text{, for } i\text{=2, 3, 4} \tag{13}$$

$$F_W^{FK}(t)=-\iint\limits_{S(t)}\rho\frac{\partial\Phi_I}{\partial t}\vec{n}_i ds\text{, for } i\text{=2, 3, 4} \tag{14}$$

where i = 2, 3, 4 correspond to sway, heave and roll motion, ρ is seawater density and $S(t)$ is the instantaneous wetted surface. We should mention that the integration is performed over the instantaneous wetted surface and pressures are calculated from the exact wave elevation. As a matter of fact, the nonlinear part of the forces is taken into account, which is important for the accurate simulation of the large motions of the ship. The nonlinear Froude-Krylov force has a nonzero mean, so a steady drift force is present, which may introduce a bias to the rolling motion.

The diffraction force should also bring about steady drift. It is well known that this steady drift force is proportional to the square of the wave amplitude and it increases in the short wave range where the reflection of waves and relative heave motion are more intense (e.g. [Maruo, 1960], [Newman, 1967]). Kuroda & Ikeda (2002) have focused on this force in their investigation of ship roll with drift. This component is currently implemented in our model.

The radiation forces are frequency dependent. In order to study transient behaviour it is necessary to transform these from the frequency domain to the time domain. Using the impulse response function, obtained as the Fourier transform of the frequency dependent radiation transfer function, the radiation forces will be [Cummins, 1962]:

$$F_{Rj}(t) = -a_{jk}(\infty)\ddot{s}_k - \int_0^{+\infty} K_{jk}(\tau)\dot{s}_k(t-\tau)d\tau \text{ , for } j,\ k{=}2,\ 3,\ 4 \tag{15}$$

$$K_{jk}(\tau) = \frac{2}{\pi}\int_0^{\infty} b_{jk}(\omega_e)\cos(\omega_e \tau)d\omega \tag{16}$$

The convolution integral is the well-known memory effect. a_{jk}, b_{jk} are the added mass and damping coefficients. $\dot{s}_k$, $\ddot{s}_k$ are velocity and acceleration of the ship in the *k* direction of motion and ω_e is the encounter frequency. In our model we use a state-space approximation of the radiation force in order to maintain the mathematical model in the form of a system of o.d.e.s which enables easier consideration of nonlinear dynamics.

Our model calculates also the sway drag force, roll damping, and cross coupling forces between sway, heave and roll. For example, the drag force due to bilge keels is calculated as follows (see Fig. 5):

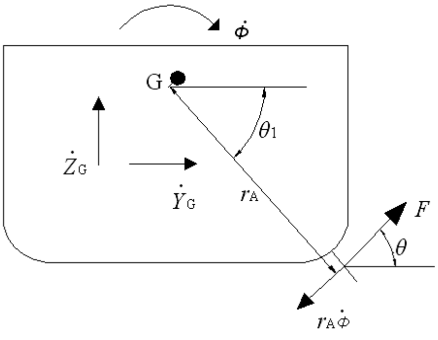


Figure 5: Bilge keel damping forces

$$F_{BY} = \frac{1}{2}\rho(\dot{Y}_G - r_A\dot{\phi}\cos(\theta) - u_2)\left|\dot{Y}_G - r_A\dot{\phi}\cos(\theta) - u_2\right| C_D A_{BK} \qquad (17)$$

$$F_{BZ} = \frac{1}{2}\rho(\dot{Z}_G - r_A\dot{\phi}\sin(\theta) - u_3)\left|\dot{Z}_G - r_A\dot{\phi}\sin(\theta) - u_3\right| C_D A_{BK} \qquad (18)$$

$$M_B = -r_A\,[F_{SZ}\cos(\theta_1) + F_{SY}\sin(\theta_1)] \qquad (19)$$

C_D is the drag coefficient and A_{BK} the total bilge keel area. Other symbols are explained in Fig. 5.

The method takes into account the local relative velocities along the hull, using the sway, heave and roll velocities ($\dot{Y}_G, \dot{Z}_G, \dot{\phi}$), the wave particle velocities u_2 and u_3 (eqn. 20 and 21) as well as the detailed geometry of the hull.

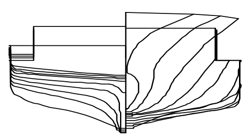
$$u_2 = \frac{\partial \Phi_I}{\partial y} \qquad (20)$$

$$u_3 = \frac{\partial \Phi_I}{\partial z} \qquad (21)$$

To calculate the forces and solve numerically the system of differential equations we have used the program Mathematica. As said, the system contains only ordinary differential equations (the convolution integrals are approximated by sets of o.d.e.s). Input data are as follows: concerning the ship, the hull geometry, her mass and the distribution of mass. For the incident wave, its height and frequency. The code creates panels over the hull whereon the static and dynamic pressures are calculated at successive time steps, as well as the angle between the horizontal plane and the normal vector of the panel.

Application of the mathematical model

As application we have used a Japanese fishing vessel whose body plan and basic particulars are shown in Fig. 6 [Umeda et al., 1995]. Her panelization is shown in Fig.7.



L_{bp} (length)	34.5 *m*	GM (metacentric height)	0.75 *m*
B (beam)	7.60 *m*	T_0 (natural roll period)	9.7 *s*
D (depth)	3.07 *m*	b_{BK} (breadth of bilge keels)	0.35 *m*
T (mean draft)	2.65 *m*	KG (vertical position of centre of gravity above keel)	3.36 *m*
C_{b} (block coef.)	0.597		

Figure 6: Body plan and main particulars of investigated ship.

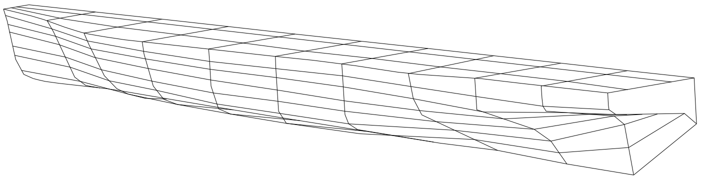


Figure 7: Half-hull panelization.

Characteristic output of the code is summarised below: the GZ curve in calm water as calculated numerical from the code, is shown in Fig. 8. Simulation of a roll decay test from extreme angle of release, approximately 85% of the angle of vanishing stability, is illustrated in Fig. 9. In Fig. 10 is shown a numerical simulation of roll response near resonance, for moderate wave steepness. ω_o, ω_w are respectively the natural roll frequency and the wave frequency. H is the wave height. In Fig. 11 is shown the corresponding sway response, and it is apparent that there is significant drifting motion that cannot be neglected. We have examined also the effect of wave steepness on the amplitude of steady roll and also on the mean roll angle (Figs. 12 & 13). There seems to be a near linear relationship between the amplitude of roll and wave steepness. Furthermore, we notice that there is an increase of a similar nature concerning the mean roll angle (towards the weather side), which, for $H/\lambda = 1/12$, can become about 16% of the roll amplitude. We believe that the mean roll angle comes from the lateral wave drift force combined with the lateral resistance force and the pair produces an extra roll moment that tends to rotate the ship to the weather side. This bias in rolling motion should be seriously taken into account as it may reduce disproportionally the dynamic stability of the ship [Thompson, 1997].

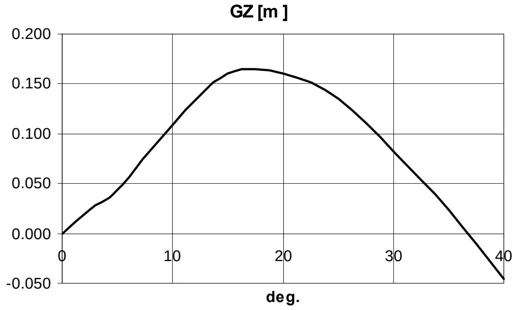


Figure 6: The GZ curve.

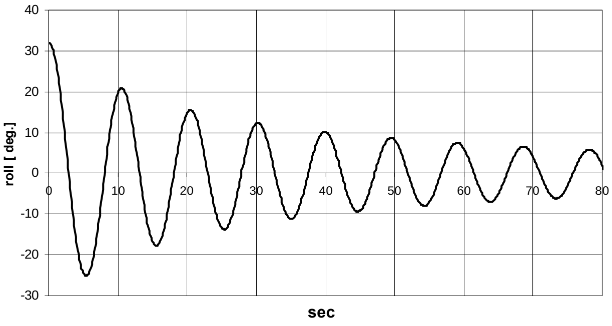


Figure 9: Roll decay test

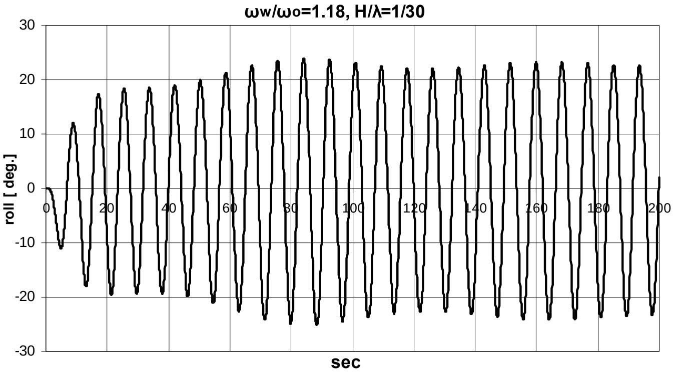


Figure 10: Roll response for $\omega_w / \omega_o = 1.18$ and $H/\lambda = 1/30$.

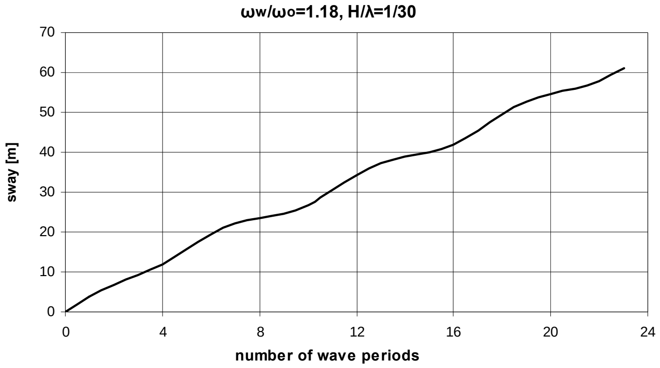


Figure 11: Sway response.

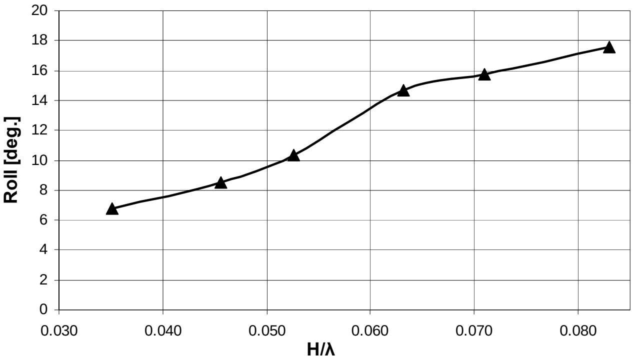


Figure 12: Effect of wave steepness on roll amplitude ($\omega_w / \omega_o = 1.312$).

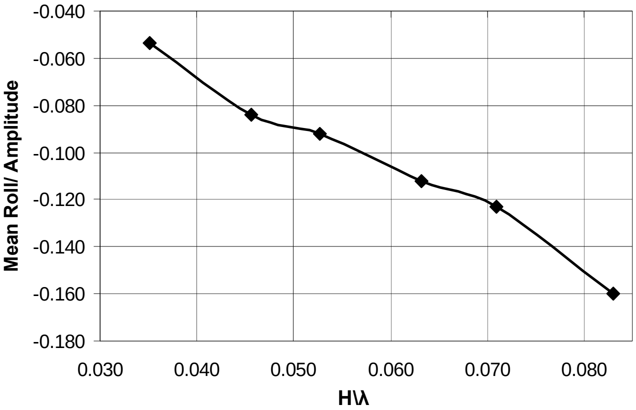


Figure 13: Effect of wave steepness on the mean roll angle ($\omega_w / \omega_o = 1.312$).

The drift motion tends to lower the encounter frequency and, as a result, larger amplitude motion appears later in terms of wave frequency. This effect is shown in Fig. 14 where we present points of the roll response curve taking into account the mean drift velocity in the calculation of encounter frequency.

It is well known that different initial conditions can affect the amplitude of transient (and some times also of steady-state) motion. In order to investigate this effect, we examined various scenarios for the initial roll angle and the lateral initial position of the ship in waves. When there is an initial heel to the lee side the obtained max roll is significantly higher than that of heel to the weather side and therefore the propensity to capsize is affected (Fig. 15). Concerning the initial lateral position of the ship on the wave, the max transient roll angle is reached when we place the ship on a trough (Fig. 16). We should note that in steady state the roll amplitude was the same for all scenarios.

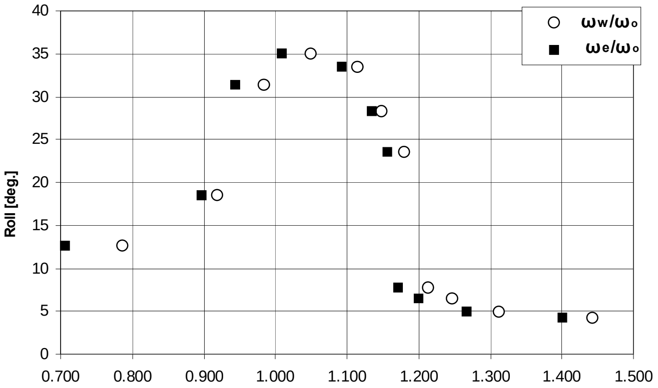


Figure 14: Roll response diagram for constant wave steepness $H/\lambda = 1/30$.

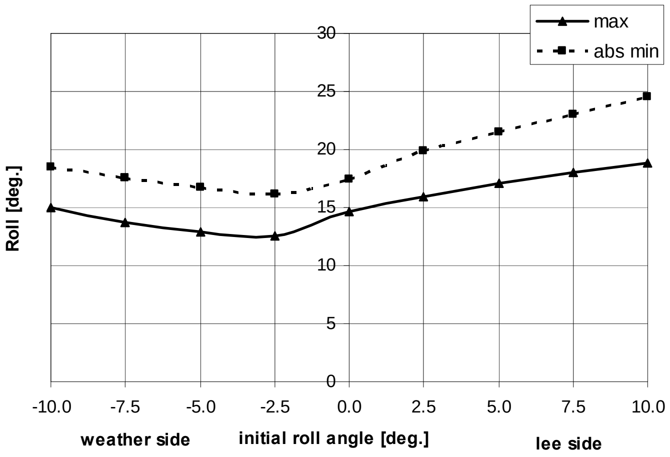


Figure 15: Maximum and minimum roll angles (transient response) for different initial heel ($\omega_w / \omega_o = 1.18$, $H/\lambda = 1/18.5$).

Concluding remarks

The limitations of the one degree of freedom model as well as the need for a more realistic and insightful investigation of roll dynamics have led us to initiate development of detailed mathematical model of ship rolling in beam waves that takes into account the coupling with heave and sway motions. Preliminary results are shown for a fishing vessel.

The implementation of a rigorous nonlinear dynamics investigation based on this model and the integration of these within a “risk – assessment” context are the next formidable tasks of our research.

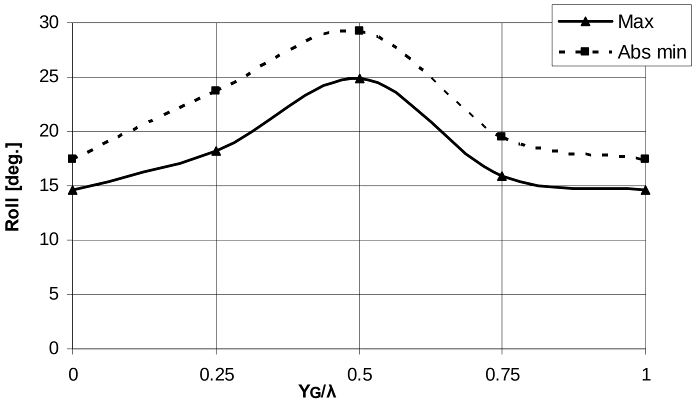


Figure 16: Effect on transient response of the initial lateral position ($\omega_w / \omega_o = 1.18$, $H/\lambda = 1/18.5$).

References

Chen, S., Shaw, S.W., Troesch, A.W. (1999): A systematic approach to modelling nonlinear multi – DOF ship motions in regular seas. *Journal of Ship Research*, **25**, 25 – 37.

Cummins W. E. (1962): The impulse response function and ship motions. *Schiffstechnik*, **9**, 47, 101 – 109.

Francescutto A. (2002): Intact stability – the way ahead. *Proceedings*, 6th International Ship Stability Workshop, Webb Institute, Long Island.

Kuroda, T., Ikeda, Y. (2002): Extreme roll motion in wide frequency range due to rapid drift motion. *Proceedings*, 6th International Ship Stability Workshop, Webb Institute, Long Island.

Lloyd, A.R.J.M (1989): *Seakeeping: Ship Behavior in Rough Weather*. Ellis Horwood Series in Marine Technology, ISBN 0-7458-0230-3.

Maruo, H. (1960): The drift of a body floating on waves. *Journal of Ship Research*, **4**, 3, 1 – 10.

McCue, L.S., Troesch, A.W. (2003): The effect of coupled heave/heave velocity or sway/sway velocity initial conditions on capsize modelling. *Proceedings*, 8th International Conference on the Stability of Ships and Ocean Vehicles, Madrid, 521 -529.

Newman, J.N. (1967): The drift force and moment on ships in waves. *Journal of Ship Research*, **11**, No. 1, 51 – 60.

Perez-Rojas, L. (2003) *Proceedings* 8th International Conference, Stability of Ships and Ocean Vehicles, Madrid, ISBN 84-688-2742-8.

Thompson, J.M.T., (1997): Designing against capsize in beam seas: Recent advantages and new insights. *Applied Mechanics Review*, **50**, 307-325.

Thompson, J.M.T, Rainey, R.C.T., Soliman M.S. (1992): Mechanics of ship capsize under direct and parametric wave excitation. *Philosophical Transactions of the Royal Society*, **338**, 471 – 490.

Umeda N., Hamamoto M., Takaishi Y., Chiba Y., Matsuda A., Sera W., Suzuki S., Spyrou K., Watanabe K. (1995): Model Experiments of ship capsize in astern seas. *Journal of the Society of Naval Architects of Japan*, **177**, 207 – 217.