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# Limits of controllability of a Ro-Pax in wind

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## Abstract

The directional stability of a modern Ro-Pax vessel in wind has been investigated. A novelty of the approach is that the mathematical model is coupled with a state-of-the-art continuation algorithm of nonlinear dynamical systems that can trace periodic as well as stationary responses. This allows swift analysis of steady-state ship motions and bifurcation patterns that arise when one or more environmental or ship parameters are varied (wind speed and direction, rudder area, trim, etc). The nature of oscillatory responses in head wind of the ship, for fixed rudder, is clarified. An operational diagram that can guide the Master in selecting safe speed and heading, as a function of wind's speed and direction, is shown. The developed methodology can be used also by designers to explore limits of controllability and to assess in a systematic way the effect of modification of design parameters on manoeuvring behaviour.

## Introduction

Strong wind can hinder the operation of vessels especially in or near ports. In harsh weather ships entering or leaving a port may be vulnerable to wind destabilizing forces and moments that, combined with loads from waves and currents, can diminish controllability. Passenger ships in particular, due to their large windage areas, can become prone to undesirable sway and yaw motions and face extreme difficulties in keeping or correcting the heading. Ship masters should be aware of the limitations of their ships under such unfavourable conditions and select effective strategies in order to either keep the desired course or turn.

Investigations of ship controllability in wind are not new. Ogawa (1969), Inoue & Ishibasi (1972, 1973a&b), Ishibasi (1975), Tanaka *et al.* (1980), Martin (1980), Asai (1981), Yoshimura & Nagashima (1985) and Eda (1986) determined the equilibrium conditions of a ship's course relatively to the wind, and checked stability properties for various operating conditions. Saddle-type and oscillatory course instabilities were found to arise, depending on the relative wind direction and velocity. Strong beam winds set the severest requirement on the rudder angle for keeping a steady course. In an open sea an autopilot can increase significantly the limits of controllability.

Under wind action, instability of the horizontal plane can be generated from two bifurcation scenarios [Spyrou, 1995]: In wind from the stern, a fold bifurcation marks the transition to saddle-type instability. In head wind a supercritical Hopf bifurcation gives rise to stable self-sustained oscillations around a mean heading. Spyrou coupled a modular model of manoeuvring in wind with a continuation analysis algorithm of stationary states [Kubicek & Marek, 1983] which automated the identification of branches of stable and unstable headings. He named the resultant diagram of rudder angle versus heading as *wind steering diagram*.

In the current work we are taking a few further steps along this direction: A more advanced continuation algorithm ("*MatCont"*), is integrated with our mathematical model in a *MATLAB* environment, setting-up a powerful tool of dynamical analysis for ship manoeuvring and course-keeping. A new capability is the continuation of periodic states, self-sustained, as they arise in wind, or the result of periodic environmental loads, as in waves. A key finding that has accrued is the clarification of the layout of the region of oscillatory yawing in head wind.

## Mathematical model

The basic manoeuvring model is a modular one, following the ideas of the Japanese Manoeuvrability Group of the 80s [Spyrou, 1990 & 1995]. Surge, sway, yaw and roll motions are considered, with non-linear terms included in all 4 degrees-of-freedom. Definitions of symbols are collected in the Nomenclature. The coordinate systems that we use are shown in Fig. 1. Concerning the calculation of wind loads, in the first instance we continued using the basic model of Aage (1971), as expanded from 3 to 4 degrees by McCreight (1986) in order to include the wind roll moment. The wind is assumed stationary and the gradient of wind velocity above water is neglected. In parallel however, we implemented an alternative wind model [Blendermann 1995 & 1996]. This model is based on experimental data of characteristic above-waterline-profiles of ships and hence it should be capable to provide the wind forces and moments with higher accuracy. Preliminary comparison between the two models is reported later in the paper (detailed analysis based on Blendermann's model is still in progress). In the paper we are dealing only with 'fixed controls' dynamics of the ship.

## Nomenclature

$A_R$: total rudder area
$A_X$ , $A_Y$ : front, lateral windage area of ship
CP: cusp point
$H$ : Hopf bifurcation
$H_R$: rudder height
$Hs$: vertical centre of lateral windage area.
$L_{BP}$: ship length
LP: limit (turning) point
NS: neutral saddle point
$r$: yaw rate
$T$: ship draft
$u$: surge velocity
$U$: resultant ship speed
$U_{RW}$: wind velocity relatively to ship
$U_W$: wind velocity
$v$: sway velocity
$X_{ijk}$ , $Y_{ijk}$ , $N_{ijk}$ , $K_{ijk}$ : manoeuvring derivatives
$\delta$: rudder angle
$\varphi$: roll angle
$\psi$: heading of ship
$\psi_w$: wind angle
$\psi_{RW}$: wind angle relative to ship

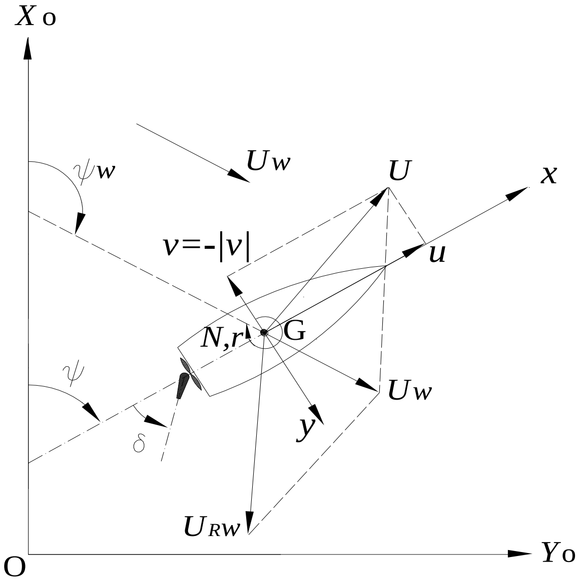

Figure 1: Coordinate Systems

**Principle of continuation**

The numerical technique of continuation can be applied on a mathematical model of a dynamical system in order to compute loci ('branches') of steady-state solutions as one or more independent parameters of the system are varied. Through continuation we are also able to identify possible *bifurcation points* where system behaviour changes qualitatively. Branches of unstable solutions can be followed with no more difficulty than following stable solutions. Continuation methods often use the length of the solution curve ('*pseudo-arc-length continuation'*) as a parameter of the numerical scheme, in order to pass successfully over bifurcation points, where, for example, the solution curve may fold back or continue in more than one direction.

*Matcont* (Dhooge *et al.*, 2003) uses *Moore-Penrose* continuation which is a variant technique of pseudo-arc-length continuation. In order to detect and locate singularities such as bifurcation points, *Matcont* calls a range of test-functions. When the requirements of a test-function are met, then the software diagnoses the type of bifurcation. For the calculation of various singularities different test-function*s* are used and the detection of a bifurcation point may possibly require the satisfaction of conditions of more than one test-function. Through *Matlab*'s Graphic User Interface *Matcont* is capable to show not only the branches of the identified steady solutions, but also the location of identified bifurcations. From there, possibly emerging new branches of stationary or periodic responses may then be traced with no difficulty.

## The investigated ship

The selected ship is a typical modern Ro-Pax with a 25 *m* high superstructure. The principal characteristics of the ship are summarized in Table 1. The ship (shown schematically in Fig. 2) was designed in Greece during the ’90s in the context of a national research project (the participants were, the National Technical University of Athens, Eleusis Shipyards, Hellenic Register of Shipping and MARTEDEC S.A.) but has not been built yet. It is suitable for operation in the Aegean Sea. Two propellers thrust the ship to a service speed of 28 *kn* and two rudders are located directly behind the propellers to assure satisfactory controllability at low speed.

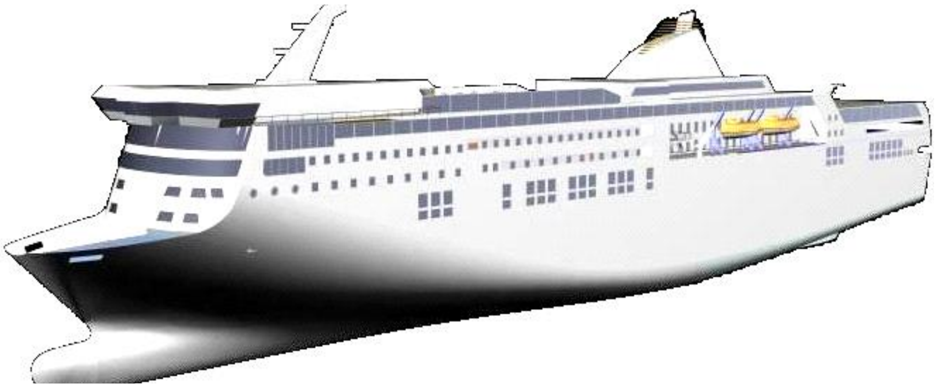

Figure 2: A rendered view of the investigated Ro-Pax

Table 1: Ship particulars

$L_{BP}$ = 172 *m*
$B$ = 25 *m*
$T$ = 6.2 *m*
$V_S$ = 28 *kn*
$H_S$ = 24.017 *m*
$A_R$ = 22.0 $m^2$
$A_X$ = 714 $m^2$
$A_Y$ = 4563.34 $m^2$

Table 2: Manoeuvring derivatives

$X_{vr}$ = -6,085,731 *kg*
$X_{\dot{u}}$ = -1,344,541.5 *kg*
$Y_{\dot{v}}$ = -13,445,415 *kg*
$Y_{\dot{r}}$ = -79,951,391 *kg·m*
$Y_v$ = -133,935.4 *kg/m*
$Y_r$ = 5,728,382.3 *kg*
$Y_{vv}$ = -329,472.08 *kg/m*
$Y_{vr}$ = -792,814,380 *kg*
$Y_{rr}$ = -24,018,970.7 *kg·m*
$N_{\dot{v}}$ = 10,049,649.4 *kg·m*
$N_{\dot{r}}$ = -23,557,572,000 $kg{\cdot}m^2$
$N_r$ = -560,810,620 *kg·m*
$N_v$ = -6,210,522 *kg*
$N_{rr}$ = -121,608,192,000 $kg{\cdot}m^2$
$N_{rrv}$ = -1,717,998,400 $kg{\cdot}m^2$
$N_{vvr}$ = -3,442,848,000 *kg·m*
$N_{\varphi}$ = -714,193.5 *kg*
$N_{v\varphi}$ = -10,657,255.9 *kg*
$N_{r\varphi}$ = 1,323,513,000 *kg·m*
$K_{\varphi}$ = -546,667,620 $kg{\cdot}m^2/s^2$
$K_p$ = -173,636,609 $kg{\cdot}m^2/s$
$K_{\dot{p}}$ = -146,875,828 $kg{\cdot}m^2$

We assessed numerically the ship's directional stability in calm sea at various operating conditions. The calculated manoeuvring derivatives are summarised in Table 2. At even keel the ship is marginally directionally stable (Fig. 3). As expected, when trimmed by the bow it becomes directional unstable. The instability starts forming from the Cusp point (CP) and is magnified as the bow trim is increased.

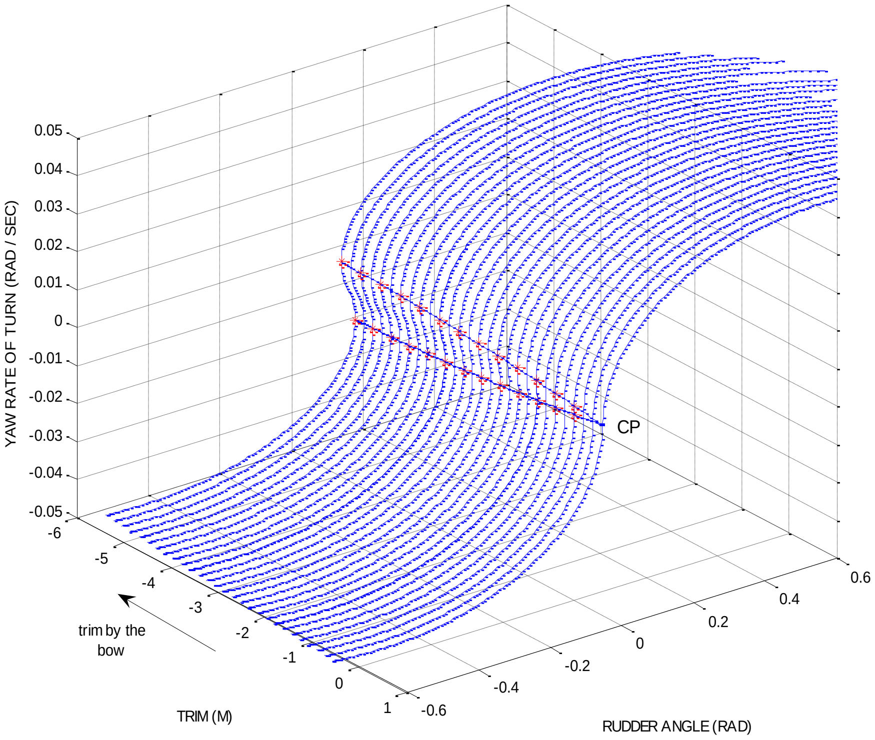

Figure 3: Steering diagrams of the Ro-Pax for varying trim.

## Behaviour in wind

In previous work it had been observed that a vessel might show controllability problems when the ratio of wind to ship velocity becomes around 4. Therefore, we focused initially around that region. In Fig. 4 are presented wind steering diagrams at low ship speed ($U$ = 12 *kn*) for different wind velocities (0 *rad* heading corresponds to wind from the stern). Figs. 5 & 6 are supplementary to Fig. 4: they show the surge, sway velocities and heel angle that correspond to any stationary heading, given the rudder angle. The turning points of the curves of Figs. 5-6 signify the so called *fold bifurcations*. A steady wind velocity of 26 *m/s*, which underlies IMO's "weather criterion", requires the rudder to be deflected at 23° for the ship to maintain a straight-line course; or equivalently, that a rudder angle beyond 23° should be set in order to manage a controlled turning manoeuvre. Moreover, when the wind is increased from 15 to 20 *m/s* the required rudder angle in beam wind is doubled. Further increase from 20 to 26 *m/s* quadruples the rudder angle. The critical wind where the max rudder is reached is about 28 *m/s*. Thereafter the ship cannot perform an intended turn. At points noted with H (Hopf bifurcations) the real parts of two conjugate eigenvalues of the solution turn positive. These are important because they give rise to undesirable self-sustained oscillations while the stationary heading turns unstable.

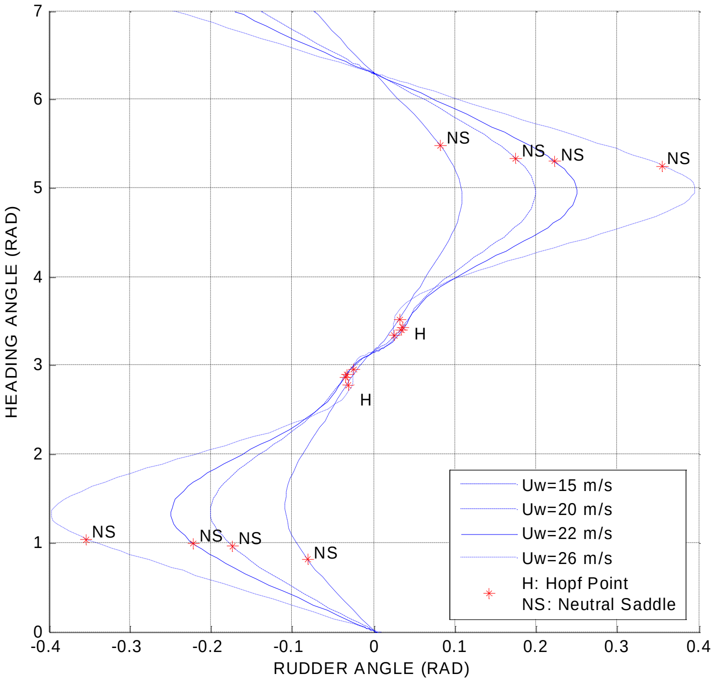

Figure 4: Wind steering diagram of the Ro-Pax at low speed ($U = 12$ *kn*).

As the ship is marginally stable in calm condition, she shows saddle-type directional instability immediately once placed in following wind (even of small intensity). As a matter of fact, active control is needed to maintain the heading. In nearly head wind the ship is stable; but near exact head wind the ship performs stable limit-cycles keeping the course only in the mean. These oscillations arise even at low wind speed. Assuming the ship to be suddenly exposed to wind from the stern and the rudder angle to be fixed, she should turn towards head wind, adopting either the stable stationary heading or the oscillatory one (Fig. 7). Simulations of the two possible transitions are shown in Fig. 8. Notably, the location of the Hopf bifurcation shows little sensitivity on the wind speed. The critical rudder angle for the onset of yaw oscillations can be seen in Fig. 9, as the wind speed is slowly raised. While up to wind speed 20 *m/s* the rudder angle is increased, thereafter it is reduced.

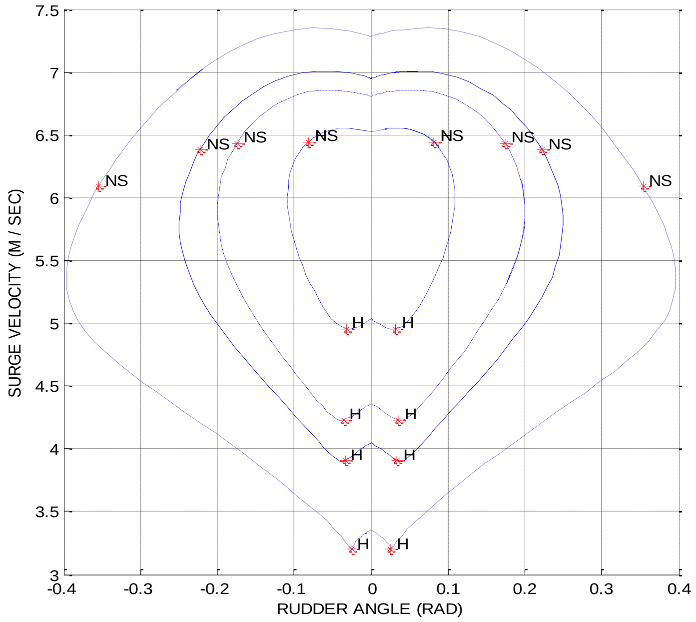

Figure 5: Surge velocity for different wind speeds.

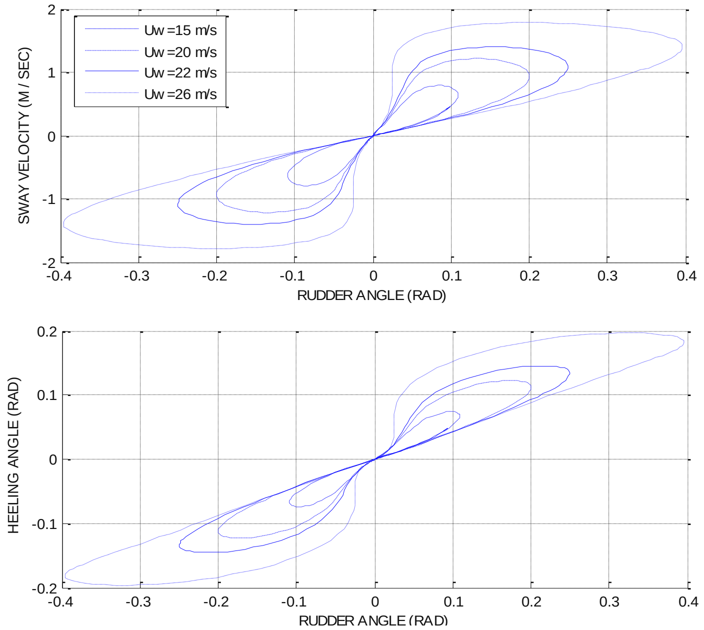

Figure 6: Sway velocity (upper) and heel (lower) versus rudder angle.

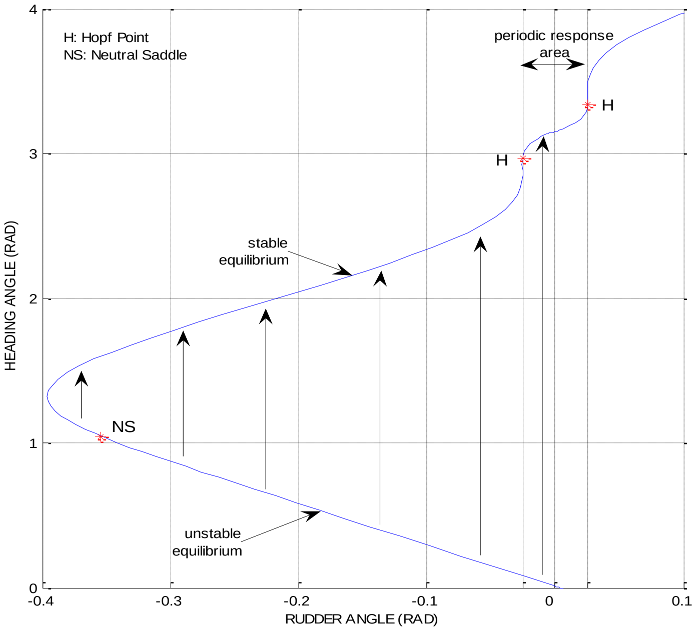

Figure 7: Jumps from following to head wind.

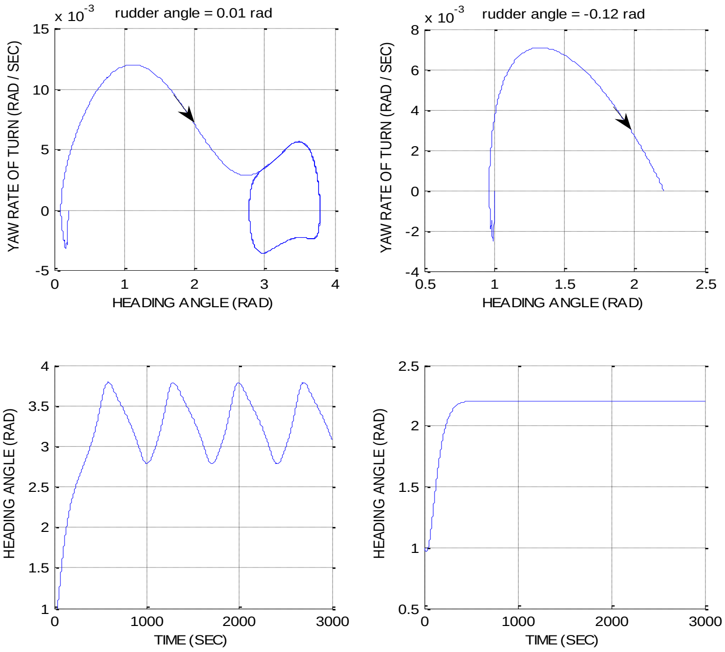

Figure 8: Simulations started from following wind.

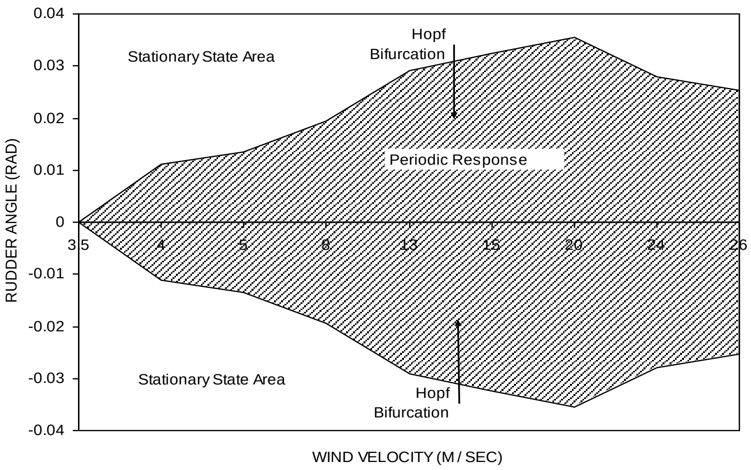

Figure 9: Critical rudder angle leading to oscillations as function of wind velocity.

## Continuation of the periodic response

In this work we were keen to understand the character of the evolution of the periodic responses from birth to extinction, with free parameter either the heading or the rudder angle. A 3-D view of the entire domain of heading oscillations is shown in Figure 10. Their arrangement relatively to the stationary responses (represented by the continuous line that penetrates the volume of periodic responses - inside the volume the stationary responses are unstable) can also be seen. It is observed that the occurring Hopf bifurcations are of supercritical type. The amplitude of oscillation increases smoothly and becomes maximum at about zero rudder angle, which corresponds to intention of operating in perfectly head wind. To produce this picture on a notebook with a Pentium M1600 processor and 768 MB of RAM required approximately 40 *mins*, depending also upon the minimum step-size, the tolerance of error and the sought degree of accuracy of the Jacobian matrix. Our strategy was to determine firstly a periodic response for some rudder angle and then to carry out from there forward and backward continuation until the periodic response fades away.

## Effect of rudder area

Continuation is a powerful tool also for assessing the effect of design modifications on ship behaviour. To demonstrate this capability, here we assess the effect of rudder area on the max rudder angle (i.e. we identify the turning point of the wind steering diagram) which should be applied for course-keeping. This can be interpreted also as the smallest rudder angle that should be applied for executing a turn. To deduce that efficiently, we carried out 2-dimensional continuation, i.e. we determined the locus of the turning point by varying simultaneously two suitable control parameters: rudder

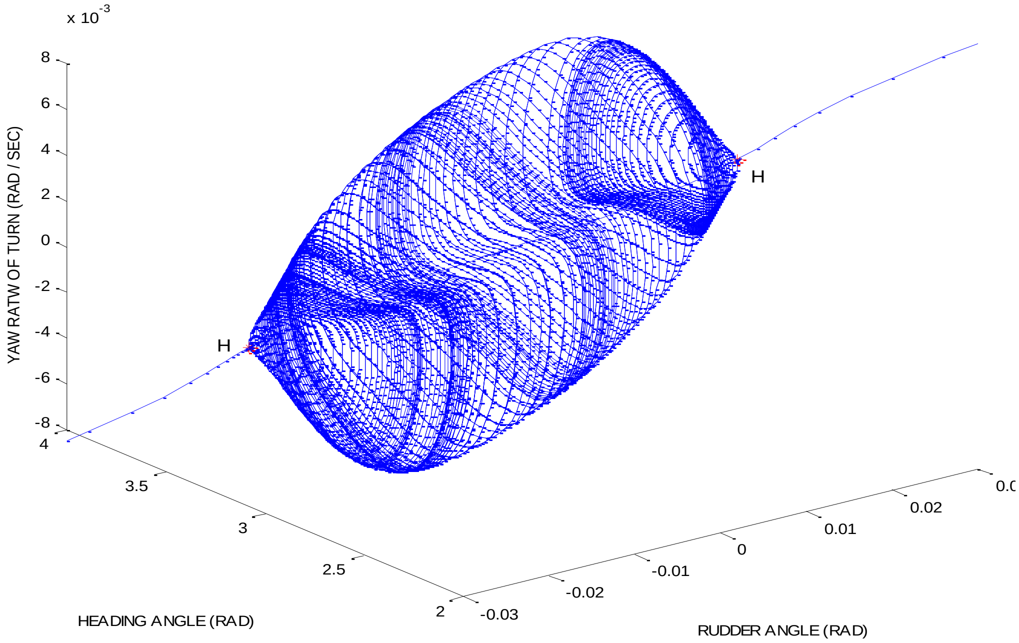

Figure 10: 3-d plot of periodic yawing at $U_w$ = 22 *m/s*, as obtained from continuation. Stationary solutions are also shown.

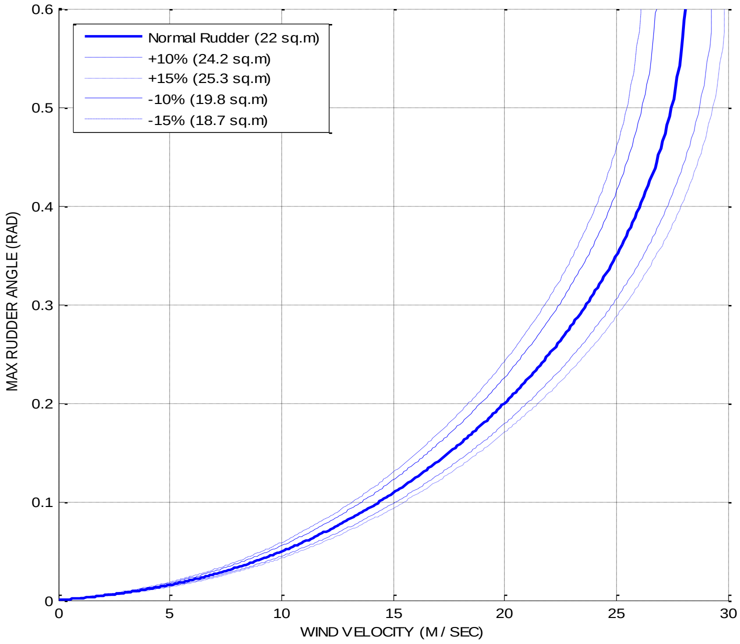

Figure 11: Change of limiting rudder angle with rudder's area.

angle and wind speed. The exercise was repeated for different rudder areas (Fig. 11). Of course, for a thorough investigation a careful look of stern shape would be necessary. It is noted that the maximum rudder angle increases quickly as the wind velocity is raised. At very strong wind, a small increase of rudder area incurs almost infinite increase of the max rudder (the curve becomes nearly vertical). The ship will not be able to turn in beam wind for wind speed higher than, approximately, 28 *m/s*. A 10% increase of rudder area improves the range of wind speed only by about 1 *m/s*.

## Operational guidance

We have produced the diagram of Fig.12 as a possible operational diagram for the Master, to guide him about the minimum rudder angle that should be applied in beam wind, given the wind speed and the operated propeller revolutions of his ship.

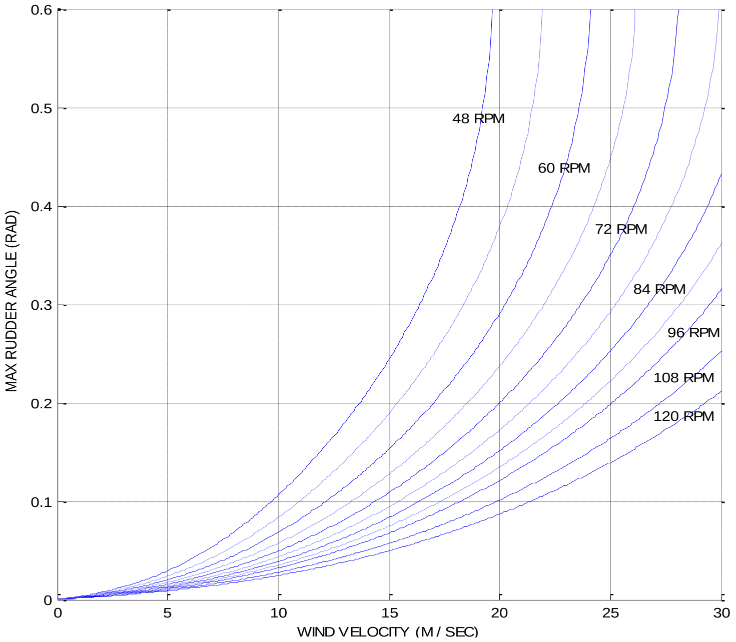

Figure 12: Max rudder angle at beam wind

## Comparison between different wind models

In addition to Aage’s model, we implemented also the wind model of Blendermann (1995 & 1996). In it, wind loading is based on wind tunnel tests that were carried out on ship models. From the tests, the wind loading coefficients are measured for different angles of attack in uniform flow. The non-uniformity of the air-flow with a gradient is then taken into account by a proper definition of the effective dynamic pressure which is responsible for each load component.

The graphs of Fig. 13 illustrate the behaviour of the Ro-Pax for two different rudder angles when the wind loads are calculated with each one of the methods. Ship speed was fixed at 12 *kn* and the wind was 25 *m/s.* Starting from an unstable following wind position, the ship “jumps” to either a head-wind equilibrium or to a steady limit cycle, depending only upon the rudder deflection angle when all other parameters are kept constant. The models seem to yield quantitatively slightly different results concerning vessel responses. However, qualitatively the behaviour is similar.

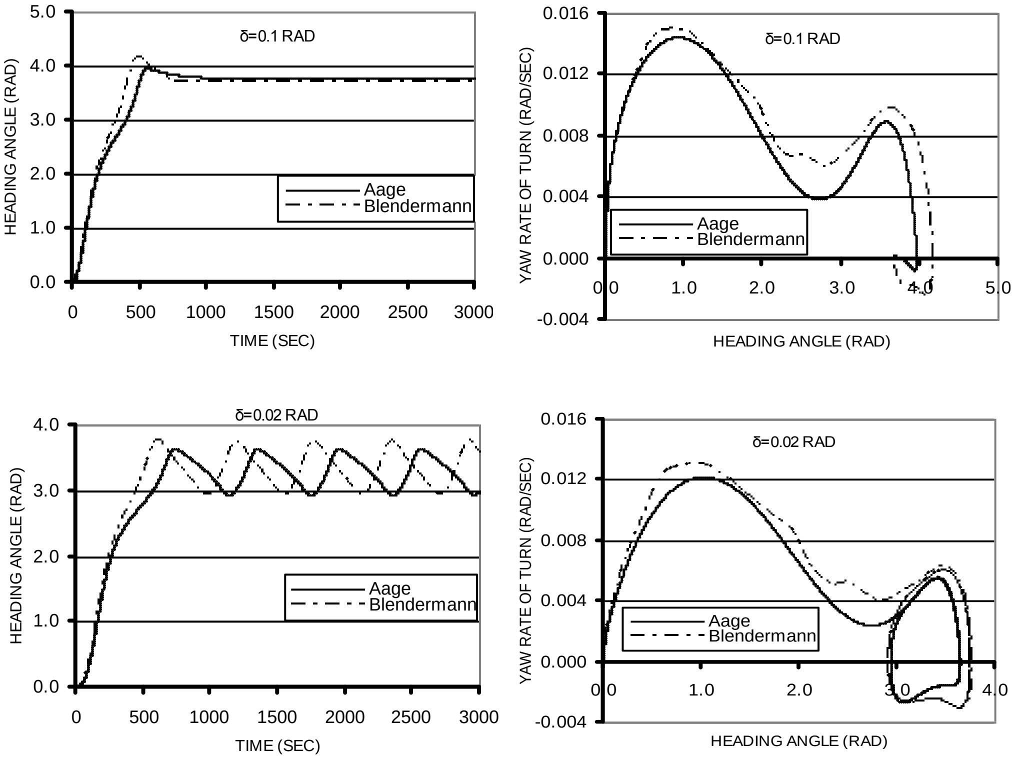

Figure 13: Comparison of responses using Aage’s and Blendermann’s wind models.

## Concluding remarks

We investigated the ‘fixed-controls’ directional stability of a modern Ro-Pax in strong wind. Advanced continuation techniques have been interfaced with a 4-degree-of-freedom modular manoeuvring model. The evolution of the self-sustained yaw-rudder oscillations in head wind, as one (or more) of selected ship or environmental parameters is varied, is clarified.

Our hope is to lay the foundations of a robust design tool for ship motions, combining numerical simulation with advanced numerical tools of nonlinear systems analysis. This could guide efficiently designers to detect and prevent the occurrence of dangerous non-linear phenomena, by altering appropriately the corresponding ship design parameters, and hence optimise behaviour for a variety of environments.

## Acknowledgments

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## References

Aage C. (1971): *Wind coefficients for nine ship models*. Hydro-og Aerodynamisk Laboratorium Report No. A-3, Denmark.

Asai S. (1981): A study on check helms for course keeping of a ship under steady external forces. *Journal of the Society of Naval Architects of Japan*, **150**, 245-253.

Blendermann W. (1995): *Estimation of wind loads on ships in wind with a strong gradient*. *Proceedings*, OMAE, Vol. I-A, Offshore Technology, 271-277.

Blendermann W. (1996): *Wind loading of ships – Collected data from wind tunnel tests in uniform flow*. Report No. 574, University of Hamburg, Shipbuilding Institute.

Dhooge A., Govaerts W., Kuznetsov Yu.A., Mestrom W. and Riet A.M. (2003): CL_*MATCONT: A Continuation Toolbox in Matlab*. Report of Universiteit Gent, Belgium and Utrecht University, Netherlands.

Eda H. (1968): Low speed controllability of ships in wind. *Journal of Ship Research*, **12**, 3, 181-199.

Inoue S. and Ishibasi Y., (1972, 1973a, 1973b): The effects of wind on ship manoeuvrability (I, II & III). *Transactions of the West-Japan Society of Naval Architects*, **44**, **45** & **46** (in Japanese).

Ishibashi, Y. (1975) On course stability and manoeuvrable range of wind velocity in the uniform wind. *Journal of the Society of Naval Architects of Japan*, **138**, (in Japanese).

Kubicek M. & Marek, M. (1983): *Computational Methods in Bifurcation Theory and Dissipative Structures*. Springer-Verlag.

Martin L.L. (1980): Ship manoeuvring and control in wind, *SNAME Trans.*, **88**, 257-281.

McCreight W.R. (1986): Ship manoeuvring in waves. *Proceeedings*, 16th Symposium on Naval Hydrodynamics, Berkeley, 456-469.

Ogawa, A. (1969) Calculations on the steered motion of a ship under the action of external forces. (Part 1): Coursekeeping and turning of a ship in uniform wind and flow. *Journal of the Society of Naval Architects of Japan*, **126**, 117-120 (in Japanese).

Spyrou K. (1995): Yaw stability of ships in steady wind. *Ship Technology Research*, **42**, 21-30

Tanaka M, Yamagami Y., Yamashita Y. and Misumi E. (1980): Ship manoeuvrability in strong winds. *Journal of the Kansai Society of Naval Architects*, **176**, 1-10 (in Japanese).

Yoshimura Y. and Nagashima J. (1985): Estimation of the manoeuvring behaviour of a ship in uniform wind.*Journal of the Society of Naval Architects of Japan*, **158**, 125-136 (in Japanese).

Spyrou K.J. (1990): A new approach for assessing ship manoeuvrability based on dynamical systems theory, PhD Thesis, Dept. of Ship and Marine Technology, University of Strathclyde, Glasgow.