Early-Stage Design Criterion for Surf-Riding Susceptibility of Displacement Monohulls

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ABSTRACT

This paper describes the derivation of an early-stage design criterion for avoiding surf-riding. The form of the proposed criterion associates Froude number with ship length at the condition identified as critical, accounting for ship size in relation with wave height and wave length.

The described method also accounts for the likelihood of encountering a wave capable of causing the surf-riding.

The method is illustrated on the basis of a population of seventeen vessels of different types and sizes (fifteen commercial and two naval).

KEY WORDS

Surf-riding, ship, stability, waves, Melnikov

1.0 INTRODUCTION

The danger of surf-riding for a vessel moving with moderate to high-speed is associated with high likelihood of loss of directional control (the phenomenon known as broaching-to) and the development of a large heel. Interest for deriving a simple criterion reflecting ship susceptibility to surf-riding was renewed recently due to the development of the second generation intact stability criteria by IMO, where surf-riding/broaching-to is one of the considered modes of stability failure.

The occurrence of surf-riding depends critically on the relation between surge wave force, ship resistance, and thrust. If the surge wave force is large enough to accelerate the ship up to wave celerity, two equilibria are created: unstable (near to the wave crest) and stable (near trough) see Fig.1.

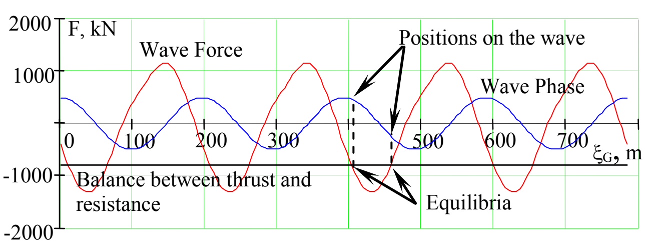

Fig. 1. Surf-riding equilibria for 100 m high-speed vessel, wave height 6 m, speed 24 kn, wave length 200 m (Belenky et al 2008)

Notably, the stable equilibrium near wave trough may still be unstable or weakly attracting in yaw. This could lead to broaching-to, a violent uncontrollable turn which occurs despite applying maximum steering effort. Appearance of the surf-riding equilibrium is a necessary but not sufficient condition of surf-riding, as there is a certain range of speeds where surf-riding can coexist with surging. Then, realization of a certain mode of motion depends on initial conditions. Beyond a certain propeller rate ("nominal Froude Number") however, all initial conditions lead to surf-riding. This is known as the second threshold (or "higher" threshold; as opposed to the first or "lower" threshold where the surf-riding equilibria first appear).

Fig. 2 shows how phase plane orbits of surging motions change with the increase of speed settings for a regular sea environment. The cosine function of ship position on the wave was used in order to present the steady periodic surging motion as a closed cycle. As the origin of the coordinate system is fixed relatively to the wave, surf-riding equilibria appear as points. As the speed approaches the second threshold, the surging cycle is distorted until it "touches" the equilibrium and ceases to exist (Spyrou 1996).

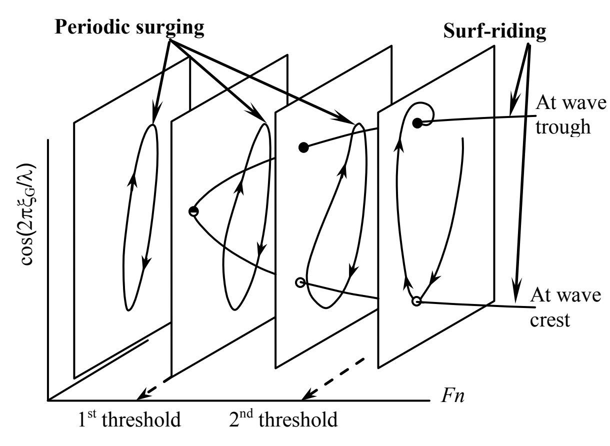

Fig. 2 Change of surging and surf-riding behavior with increasing speed settings (based on Spyrou 1996)

IMO's operational guidance MSC.1/Circ 1228 recognizes that there is danger of surf-riding in following seas when the Froude number exceeds 0.3. This simple criterion was reportedly obtained as an approximation of the second threshold. It is known that a wave capable of causing surf-riding must be steep and have length comparable with ship length. However, the chance of encountering such a wave in a realistic seaway decreases as the ship length is increased. Therefore, the real danger of surf-riding should be less for larger ships even if they were capable of Froude number 0.3 in calm water.

Consideration of the likelihood of encountering waves that can cause surf-riding renders the critical Froude number a function of ship length. To proceed along this direction however, requires determining the second threshold for several waves. While such an analysis might be appropriate for a more advanced design stage, it may be too labor-intensive for early stage design. A method of deriving an approximate formula is described below. However, we shall first review some basic aspects of the dynamics of surf-riding.

2.0 THE SECOND THRESHOLD

Phase planes of surging motions illustrating transition through the second threshold are shown in Fig. 3. The origin of the coordinate system is located at a wave crest (surf-riding equilibria still are shown as points.) Periodic surging is seen as a trajectory heading to the negative direction. If the surging velocity is less than the wave celerity, the wave overtakes the ship in periodic surging mode.

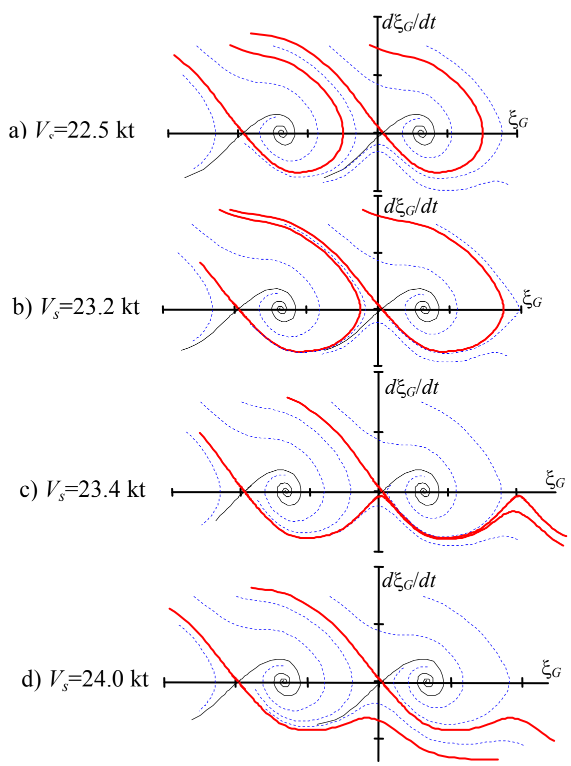

Fig 3 Phase plane with surging and surf-riding, for 100 m high-speed vessel, wave height 6 m, wave length 200 m

Fig 3a shows a typical phase plane between the first and the second threshold characterized by coexistence of surging and surf-riding. Boundaries between the two modes are shown in solid thick lines (red in the color version of the Figure). Fig 3b shows the phase plane just below the second threshold. One can see clearly that the boundaries have come closer to each other. Once the second threshold has been crossed the boundaries fold over eliminating the surging domain completely as seen in Fig3c. The further increasing of speed setting does not produce any significant changes in the topology of the phase plane.

The decreasing distance between the boundaries can be used in order to find the critical speed setting corresponding to the second threshold. The so called Melnikov's method provides an approximate expression for the distance between the two boundaries. The critical condition is encountered when the Melnikov function below (here parameterized with respect to the propeller rate n) obtains the zero value, meaning that the distance between the boundaries becomes zero (Spyrou 2006)

$$M(n) = -\frac{r(n)}{q} - \frac{4}{\pi} p_1(n) + 2p_2 - \frac{32}{3\pi} p_3 \qquad (1)$$

Here coefficients p_i, and r are derived from cubic approximation of the curves of thrust and resistance [other polynomial expressions could also be used with simple adaptation of the Melnikov function (1)], while q is a coefficient depending on the amplitude of wave surging force, see Appendix 1 for details.

Melnikov's method fairly accurate when the damping is low; but it seems to provide a consistently reasonable prediction for the cases when the damping is in the realistic range. Due to its nature of considering a perturbation of Hamiltonian dynamics, the Melnikov's method yields a lower value for the threshold, in comparison with the direct method (numerical integration of the surging equation). Indeed, Fig 4 shows the difference between the direct method and Melnikov's method, calculated for a population of sample vessels of different types and sizes (see also Appendix 2.). As this difference is always positive, it seems reasonable to characterize Melnikov's method as "conservative".

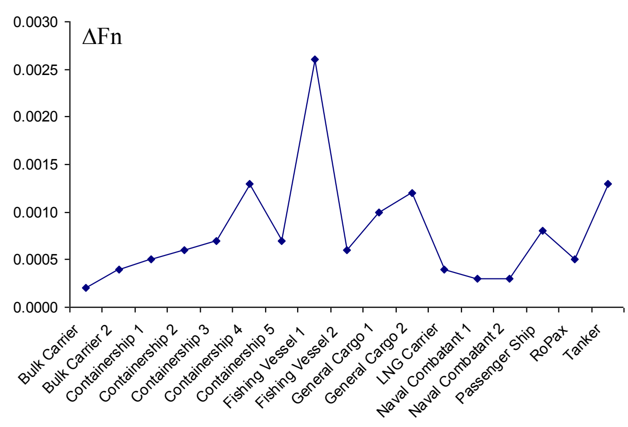

Fig. 3 Difference in terms of nominal Froude number between Melnikov's method and direct calculation for the second threshold

3.0 RELATION BETWEEN THE SECOND THRESHOLD AND WAVE STEEPNESS

The first step is to determine the dependence of nominal Froude number corresponding to the second threshold on wave steepness. The most straight forward way to do this is to calculate of the wave surging force, estimate resistance and thrust, and then apply Melnikov's method as it was described by Spyrou (2006).

Fig. 4 shows the result of such a calculation of the critical nominal Froude number that corresponds to the second threshold carried out for the sample ship population and a range of wave steepness. A brief description of the sample ship population has been placed in the Appendix 2.

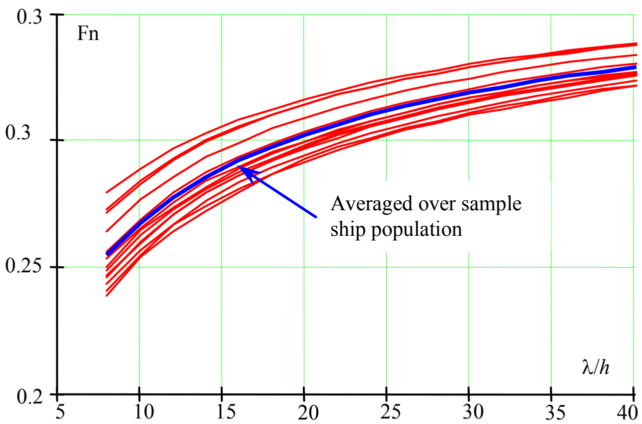

Fig. 4 -Threshold Froude number, as a function of wave steepness

As it can be clearly seen from Fig. 4, the values of Froude number change almost equidistantly with the wave steepness (except perhaps for the sub-range of extremely steep waves). Also, the variation of Froude number values is not that significant, considering the diversity of the sample ship population. The averaged curve could be approximated as

$$Fn(\lambda/h) = 0.2324\,\sqrt[3]{\lambda/h} - 0.07364\,\sqrt{\lambda/h} \qquad (2)$$

where λ is wave length and h is the wave height. In Fig. 5 the approximate curve is plotted against the points of the average curve of Fig 4.

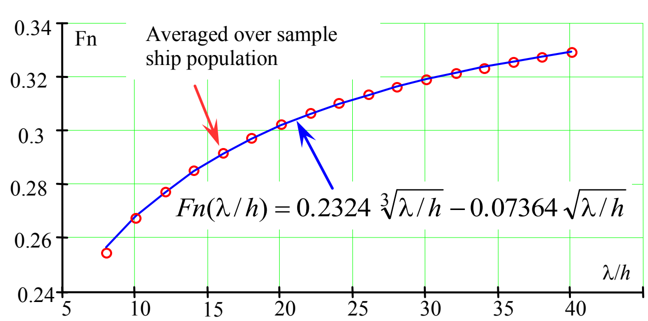

Fig. 5 Approximation of critical Froude number, as a function of steepness

For evaluation of probability, it is convenient to use the inverse function of (2). However, instead of transforming (2) into a cubic equation and then solving it, it is easier to fit another approximation to the already inversed data:

$$h/\lambda = 0.0310\,Fn^{-3} + 0.06226 \qquad (3)$$

Note that the steepness parameter is now expressed as h/λ. The points and the curve are shown in Fig. 6.

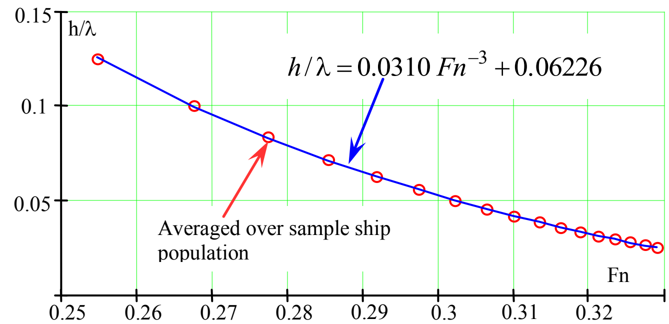

Fig. 6 Approximation of wave steepness as a function of the nominal Froude number, corresponding to the second threshold (averaged over the sample ship population)

4.0 CRITERION ACCOUNTING FOR SHIP LENGTH

The probability of encountering a wave of length equal to the ship length and capable of causing surf-riding becomes a function of Froude number

$$P(Fn) = \int_{a_{cr}(Fn)}^{a_{\lim}} f\left(a \mid k = \frac{2\pi}{L}\right) da \qquad (4)$$

Here a_{lim} is the hydrodynamic limit of the wave amplitude, while a_{cr} is critical amplitude that is determined from (3):

$$a_{cr} = 0.5L(0.0310\,Fn^{-3} + 0.06226) \qquad (5)$$

The conditional distribution density $f(a|k)$ can be found using known joint distribution for wave number and amplitude based on Longuet-Higgins (1957), see Appendix 3.

The formula (4) reflects the known fact that the increase of the speed leads to an increase of the probability of surf-riding. As it can be seen from Fig. 6, increasing the Froude number reduces the steepness; this leads to a decrease of the critical amplitude (5) and to an increase of the range of integration in (4). Since the conditional PDF is always positive, the value of the integral in (4) must increase with the increase of the Froude number.

To avoid complexity for the early-stage design analysis, it has been proposed to limit the consideration of wave lengths only equal to ship length (Umeda, 2010). A more general approach would be of course to consider wave lengths in the realistic range, but not fixed, and evaluate probabilities accordingly. The effect that such a consideration might have on the probability level will be evaluated in a future study. Expression (4) for probability refers to a specific condition of the seaway as well as to a specific ship length and service Froude number.

To account for the variability of the sea state several options are available. For example, one could consider long term statistics and take the weighted average of some suitable statistic of the sea state. To simplify this, Umeda (2010) proposed to average the probability over a scatter diagram, like the one in IACS Recommendation 34 (IACS 2001)

$$P(L, Fn) = \frac{1}{N_{Tot}} \sum_{H_S} \sum_{T_Z} P_{Ref}(H_S, T_Z) N(H_S, T_Z) \quad (6)$$

Formula (6) has a meaning with reference to a scatter diagram and expresses, to some approximation, a probability (averaged over annual storm statistics) of encountering a wave that is equal to the length of the ship and capable of causing surf-riding to a ship heading with specified Froude number. $N(H_S,T_Z)$ is the number of observations of a sea state with significant wave height H_S and mean period of zero-crossing T_Z, while N_{Tot} is the total number of observations available.

While this value cannot be interpreted as the actual probability of surf-riding, it could be used as a rough measure of the danger of surf-riding, if speed and length were considered as fixed. Fig. 7 shows a graphical representation of formula 7.

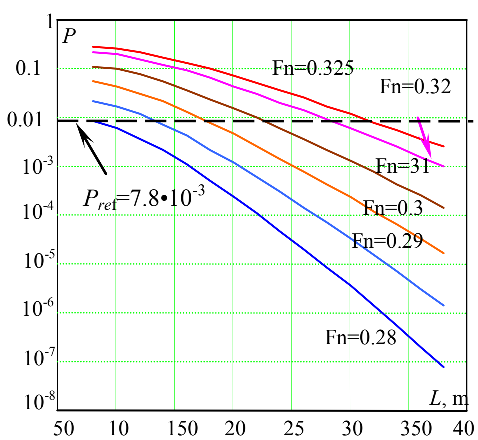

Fig. 7 Probability of Encounter of a Wave Capable of Causing Surf-riding for as a Function of Ship Length for Different Nominal Froude Numbers

As can be seen from Fig. 7 the probability decreases with an increase of the length and increases with the increase of Froude number. The observed tendency is consistent with operational experience. The danger of surf-riding is less for longer ships and increases with increasing speed.

To derive a benchmark probability level, a reference ship length and Froude number have been assumed. More specifically, further calculations were performed for the reference values L=80 m and Fn=0.28 (providing also material for additional scrutiny and discussions). The obtained reference probability is calculated below and shown in Fig. 7.

$$P_{ref} = P(L = 80, Fn = 0.28) = 7.8 \cdot 10^{-3} \quad (7)$$

Keeping the reference probability fixed allows for the expression of the Froude number as a function of length:

$$Fn(L) = Q(L, P = P_{ref}) \quad (8)$$

Here Q is an inverse function for probability (6).

Fig. 8 presents the results of the calculation of the formula (8) depicted as circles, as well as the linear regression through these points:

$$Fn(L) = 0.0000181 \cdot L + 0.282 \quad (9)$$

The formula (10) relates Froude number with the ship length under the condition of encountering a wave with the length equal to ship length and steep enough to cause surf-riding. This line has a positive slope, meaning that a larger vessel should sail with higher speed, in order to have the same probability of encountering a dangerous wave as a smaller vessel. This approach can be used to “give a credit” for larger ships, in terms of the likelihood of experiencing surf-riding and broaching-to for the same speed.

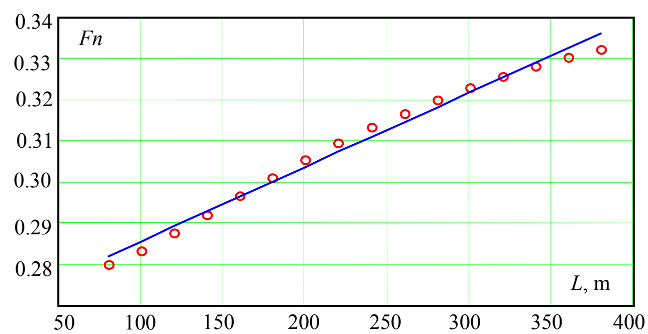

Fig. 8 Froude Number as a Function of Length, Under the Condition of the Equivalent Probability of Encountering a Wave Capable of Causing Surf-Riding

If one accepts Fn=0.28 as a standard for a ship with length equal or less 80 m, then the entire criterion can be formulated as:

$$\begin{aligned} &Fn > 0.28 \quad if \quad L \le 80\ m \\ &Fn > 0.0000181 \cdot L + 0.282 \quad if \quad L > 80\ m \end{aligned} \quad (10)$$

As observed, the criterion (10) relates Froude number with the length of the ship. If the service Froude number exceeds the value (10), surf-riding may be a problem for the new design and more detailed scrutiny of ship behavior in waves is recommended.

5.0 SUMMARY

Broaching-to is a dangerous phenomenon associated with high-speed sailing in following and quartering seas. The most common mechanism of broaching-to starts with surf-riding, a condition of dynamical equilibrium that may be unstable in the yaw direction.

The objective of this work was the derivation of a simple criterion for surf-riding that accounts for ship size and thus goes beyond IMO's empirical guidance MSC.1/Circ 1228.

The proposed criterion is based on the second threshold of surf-riding that separates surf-riding under selected initial conditions from the surf-riding under any initial conditions. The applied prediction "tool" was Melnikov's method which was found to provide sufficiently accurate results for all 17 ships examined (consistently on the slightly conservative side which is a welcomed feature for a criterion).

However, as calculation of the Froude number corresponding to the second threshold may not be practical for general application, it was considered to derive the criterion using regression based on a series of calculations over a representative population of ships representing the modern fleet. Therefore, the second threshold is evaluated for each ship of this population, for a range of wave steepness and then it averaged over the sample ship population. The result reflects the dependence between the Froude number corresponding to the second threshold and wave steepness. This dependence is the key to finding a probability of encounter of a wave capable of causing surf-riding, as a function of ship length, for different speed settings. A reference probability is derived for a ship with length and speed placing her in the marginal category in terms of surf-riding susceptibility, according to experience. Requesting this probability level to be satisfied for ships of higher length leads to the formulation of the criterion.

REFERENCES

Belenky, V., de Kat, J.O. & Umeda N. (2008), "Towards Performance-Based Criteria for Intact Stability," Marine Technology Vol. 45, No 2 pp.101-123.

Bishop, R. C., W. Belknap, C. Turner, B. Simon, and J. H. Kim, (2005) *Parametric Investigation on the Influence of GM, Roll damping, and Above-Water Form on the Roll Response of Model 5613*. Report NSWCCD-50-TR-2005/027.

IACS (2001) "Standard Wave Data", IACS Recommendation No 34, London.

Longuet-Higgins, M. S., 1957, "The Statistical Analysis of a Random, Moving Surface," *Philosophical Transactions A*, 249.

Spyrou, K.J. (1996), "Dynamic Instability in Quartering Seas: The Behavior of a Ship During Broaching," J. Ship Res., Vol. 40, No. 1.

Spyrou, K.J. (2006), "Asymmetric Surging of Ships in Following Seas and its Repercussions for Safety," Nonlinear Dynamics, Vol. 43, pp. 149-172.

Todd, F. H. (1953), "Some Further Experiments on Single-Screw Merchant Ship Forms," *Trans. SNAME*, Vol. 61, pp. 516-589.

Umeda, N. (2010) Private communication with Vadim Belenky, Windsor, UK.

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APPENDIX 1 MELNIKOV'S METHOD

Following Spyrou (2006), consider the surging equation:

$$(m+m_x)\cdot\ddot{\xi}_G + R(c+\dot{\xi}_G) - T(c+\dot{\xi}_G, n) + A_{Fw}\sin(k\xi_G) = 0 \quad \text{(A1)}$$

Here, m is the mass of the vessel, m_x is longitudinal added mass; R is resistance in calm water, T is thrust in calm water, c is wave celerity, and A_{Fw} is amplitude of Froude-Krylov wave force. The symbol ξ_G stands for the distance between the wave crest and the center of gravity of the vessel. Finally, n is the commanded number of revolutions of the propeller— this is an independent parameter. This equation also uses the assumption that the encounter frequency is small.

To apply Melnikov's method in an analytical context, thrust and resistance need to be expressed with elementary functions. The solution available from the above reference uses polynomial approximations for thrust and resistance in the following form:

$$R(V_S) = r_1 V_S + r_2 V_S^2 + r_3 V_S^3 \quad \text{(A2)}$$

$$T(V_S, n) = \tau_0 n^2 + \tau_1 V_S n + \tau_2 V_S^2 \quad \text{(A3)}$$

Here r_1, r_2, r_3 are polynomial coefficients for resistance that can be evaluation with standard regression methods.

The coefficients τ_0, τ_1, τ_2 for thrust are defined as

$$\tau_0 = c_0(1-t_p)\rho D^4 \quad \text{(A4)}$$

$$\tau_1 = c_1(1-t_p)(1-w_p)\rho D^3 \quad \text{(A5)}$$

$$\tau_2 = c_2(1-t_p)(1-w_p)^2\rho D^2 \quad \text{(A6)}$$

Here t_p is the coefficient for thrust deduction, while w_p is the wave fraction coefficient. Both coefficients are evaluated for calm water. D is the propeller diameter and ρ is mass density of water. Coefficients c_0, c_1, c_2 came from polynomial presentation of the coefficient of thrust K_T:

$$K_T = c_0 + c_1 J + c_2 J^2 \quad \text{(A7)}$$

Where J is the advance ratio

$$J = \frac{V_S\left(1-w_p\right)}{nD} \tag{A8}$$

Then the balance between the resistance and thrust can be expressed as:

$$\begin{aligned} &R(c+\dot{\xi}_G) - T(c+\dot{\xi}_G, n) = \\ &\quad A_1(c,n)\dot{\xi}_G + A_2(c)\dot{\xi}_G^2 + A_3\dot{\xi}_G^3 + R(c) - T(c,n) \end{aligned} \tag{A9}$$

Here:

$$A_1(c,n) = 3r_3c^2 + 2(r_2-\tau_1)c + r_1 - \tau_1 n \tag{A10}$$

$$A_2(c) = 3r_3c + 2(r_2-\tau_1) \tag{A11}$$

$$A_3 = r_3 \tag{A12}$$

For convenience equation (A1) is transformed into the following non-dimensional form:

$$x'' + p_1x' + p_2x'^2 + p_3x'^3 + \sin x = \frac{r}{q} \tag{A13}$$

Here:

$$x = k\xi_G \tag{A15}$$

k is the wave number (spatial frequency)

$$q = \frac{k \cdot A_{Fw}}{m+m_x} \tag{A16}$$

A_{Fw} is amplitude of surging wave:

$$A_{Fw} = \rho g k \zeta_A \int_{-0.5L}^{0.5L} A_0(x)\cdot\cos(kx)dx \tag{A17}$$

$$A_0(x) = 2\int_{d(x)}^{0} y(x,z)\exp(kz)dz \tag{A18}$$

Here, x, y and z are the coordinates of points on the surface of the hull, expressed in ship-fixed coordinate system; $y(x, z)$ is the half-breadth on a station with coordinate x at the depth z; $d(x)$ is draft of a station at longitudinal position x; k is the wave number; ζ_A is the wave amplitude; and ρ is mass density of water.

Coefficients p_1, p_2, p_3 represent the resistance and thrust:

$$p_1 = p_1(n) = \frac{A_1(c,n)}{\sqrt{kA_{Fw}\left(m+m_x\right)}} \tag{A19}$$

$$p_2 = \frac{A_2(c)}{k\left(m+m_x\right)} \tag{A20}$$

$$p_3 = \frac{A_3\sqrt{A_{Fw}}}{\left(\sqrt{k\left(m+m_x\right)}\right)^3} \tag{A21}$$

The coefficient r (without any index) reflects the difference between resistance and thrust at the wave celerity

$$r(n) = \frac{k\left(T(c,n) - R(c)\right)}{\left(m+m_x\right)} \tag{A22}$$

Finally, equation (A13) is written in the non-dimensional time, expressed as

$$\tau = \sqrt{q}\, t \tag{A23}$$

The Melnikov's function for the equation (13) is expressed as:

$$M(n) = -\frac{r(n)}{q} - \frac{4}{\pi}p_1(n) + 2p_2 - \frac{32}{3\pi}p_3 \tag{A24}$$

APPENDIX 2 SAMPLE SHIPS

A sample population of 15 commercial and 2 naval ships was used for testing and evaluation of vulnerability criteria for three of the identified intact stability failure modes (pure loss of stability, parametric roll, and surf-riding). The general characteristics of these ships are given in Table A1.

Table A1 Ship Types and General Characteristics

Type	Note	L (m)	L/B	B/d	D/d
Bulk Carrier		275	5.85	2.67	1.36
Bulk Carrier 2		145	6.34	2.21	1.45
Containership 1	Post-panamax	322.6	7.07	3.05	1.65
Containership 2	Post-panamax	376	6.53	3.57	2.36
Containership 3	Post-panamax	330	7.24	3.55	2.26
Containership 4	Panamax	283.2	8.80	2.51	1.70
Containership 5	Post panamax C11-type	262	6.55	3.12	1.93
Fishing Vessel 1	Japanese purse seiner	34.5	4.53	2.87	1.16
Fishing Vessel 2		21.56	3.40	2.53	1.21
General Cargo 1	Series 60 CB=0.7	121.9	7.50	2.51	1.60
General Cargo 2	C-4 type	161.2	7.05	2.73	1.61

Table A1 (Cont/) Ship Types and General Characteristics

Type	Note	L (m)	L/B	B/d	D/d
LNG Carrier		267.8	6.39	3.57	2.29
Naval Combatant 1	ONR topside series –flared	150	8.19	3.42	3.09
Naval Combatant 2	ONR topside series –tumblehome	150	8.19	3.42	3.09
Passenger Ship		276.4	8.04	4.03	1.75
RoPax		137	6.76	3.64	3.24
Tanker		320	5.52	2.76	1.48

Containership 5 is the C11-class containership. General Cargo Ship 1 is Series 60 hull form, C_B=0.7 variant (Todd, 1953). General Cargo Ship 2 is the C4 type. Naval Combatants 1 and 2 are the ONR Topsides Series, flared and tumblehome configurations, respectively (Bishop, *et al.*, 2005).

APPENDIX 3 DISTRIBUTION OF WAVE NUMBERS AND AMPLITIDES

To evaluate a likelihood of encounter a wave with certain length and height, joint probability density function (PDF) is needed. The joint PDF of amplitude and wave number is used for this purpose. It is based on the envelope theory (Longuett-Higgins 1957):

$$f(a,k) = \frac{a^2}{\sqrt{k_2^2 - k_1^2}\sqrt{2\pi\sigma_\zeta^6}} \exp\left(-\frac{a^2}{2\sigma_\zeta^2}\right) \times \left(\exp\left(-\frac{a^2}{2\sigma_\zeta^2}\frac{(k-k_1)^2}{(k_2^2-k_1^2)}\right) + \exp\left(-\frac{a^2}{2\sigma_\zeta^2}\frac{(k+k_1)^2}{(k_2^2-k_1^2)}\right)\right) \quad \text{(A25)}$$

Where k_1 is the mean wave number, k_2 can be interpreted as a mean spectrum bandwidth in terms of wave number, and σ_ζ is the standard deviation of wave elevations:

$$k_1 = \frac{1}{\sigma_\zeta^2}\int_0^\infty \frac{\omega^2}{g} s(\omega) d\omega \quad \text{(A26)}$$

$$k_2 = \frac{1}{\sigma_\zeta}\sqrt{\int_0^\infty \frac{\omega^4}{g^2} s(\omega) d\omega} \quad \text{(A27)}$$

where g is the acceleration constant due to gravity.

Appearance of the joint PDF (A25) is shown in Fig. A1.

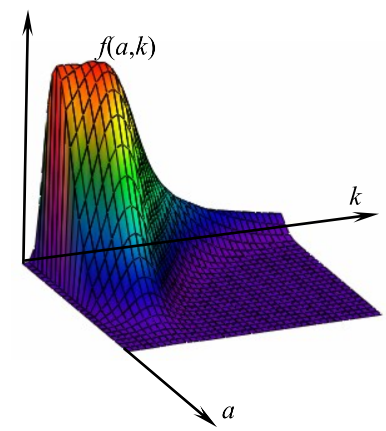

Fig. A1 Joint Distributions of the Amplitude and the Wave Number